

PROCESS CHARACTERISTICS OF SCREW IMPELLERS WITH A DRAUGHT TUBE FOR NEWTONIAN LIQUIDS. THE FLOW MODEL OF HOMOGENATION

Pavel SEICHTER

Research Institute of Chemical Equipment, Brno

Received May 28th, 1980

A flow model has been proposed of a mixing vessel with a screw impeller and a draught tube. The volume of the mixed batch has been divided into a non-ideally mixed part, a so-called laminar distributor, and a stagnating volume. Convective exchange of mass takes place by recirculation of liquid between the non-ideal mixer and the laminar distributor. By the Laplace transform of the frequency functions for both regions and their summation, the time dependence has been determined of the local change of concentration in an arbitrary position within the system.

Using results of measured pumping capacities and mixing times, relationships have been obtained between the model parameters and simplexes of geometrical similarity of the mixing system.

Theoretical prediction of the course of homogenation process in a mixed highly viscous batch encounters the problem of insufficient knowledge of the velocity field. This question has been discussed already in the earlier papers¹⁻³.

An alternative way to tackle this problem in a simplified manner is the use of a suitable flow model. From the standpoint of this work as interesting appear models applicable to homogenation in a mixed batch, *i.e.* a discontinuous arrangement of the mixed system.

Numerous models pertain to clearly turbulent flows and low-viscosity liquids⁴⁻¹⁰. Only the papers of O'Shima, Yuge¹³ and Takamatsu and Sawada^{11,12} have proposed models describing mixing of highly viscous liquids. In both cases^{11,13} though the impellers used were ill-suited for homogenation mixing. These conditions of mixing and the character of the flow in the batch were then reflected in both models in that a substantial role in the homogenation was played by molecular diffusion¹¹⁻¹³. In all papers mentioned⁴⁻¹³ one of the principal parameters of the models was the pumping capacity of the impeller, or the volume flow rate through the impeller.

An attempt to solve the problem of homogenation in a vessel with a screw impeller and a draught tube has been made by Chavan and coworkers¹⁴. Their approach has been based on the theory of laminar mixing. The required degree of segregation, according to this solution¹⁴, is achieved by repeated (recirculatory) shearing of the mixed liquids. The result of the solution was, however, only a relation expressing indirect proportionality between the time of homogenation and the frequency of revolution of the impeller.

ANALYSIS OF THE PROBLEM

Rotation of a screw impeller in an immobile draught tube induces recirculatory flows of highly viscous batch in the mixing vessel. A relatively intensive mixing takes place in the region of the screw channel, partly also below and above the impeller, where the flow possesses all three velocity components. Low intensity mixing prevails in the annular space, where the flow is essentially unidirectional, namely in the axial direction with a strong velocity profile². The "corners" of the mixed system near the bottom and the level are off limits of the flow and the very slow concentration equalization is controlled by molecular diffusion¹. From the standpoint of experimental technique of detecting the course of homogenation the latter region may be regarded as an unmixed region.

The above analysis of the conditions of mixing in a vessel with a screw impeller (Fig. 1a) provided guidelines for the division of the mixed batch into three regions entirely different as far as their intensity of mixing is concerned (Fig. 1b). These regions are referred to as: The non-ideal mixer (*m*), the laminar distributor (*d*) and the stagnant region (*s*).

It must be noted that by mixing it is understood only axial mixing. The model assumes that perfect homogeneity has been achieved in the lateral direction.

Limiting cases of axial mixing are: The ideal mixer and the plug flow¹⁵. Their frequency function of the residence times, or the probability density of the residence time, $E(t)$ or $E(s)$ (in the Laplace transform) are generally known^{15,16}. Also for the case of the laminar flow in a tube the hydrodynamic distribution functions are available^{17,18}. Numerous alternative models have been proposed in the litera-

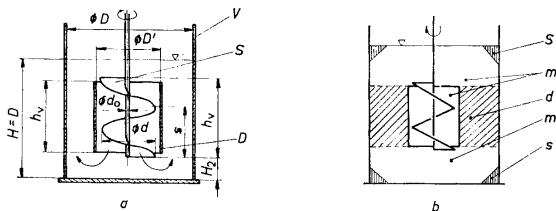


FIG. 1

Sketch of the mixing system. *a*: screw impeller with a draught tube; *S* screw; *D* draught tube; *V* mixing vessel. *b*: mixing regions; *m* non-ideal mixing; *d* laminar distributor; *s* stagnating region

ture^{15,16,19-27} to describe a non-ideal mixer. From the analysis of individual solutions it follows that the frequency distribution function, which for the given systems may be taken to be linear stationary operators of the examined regions, are mostly fairly complicated functions. Their use of the solution of the given system with recirculation would bring along considerable computational difficulties even with the aid of a computer. This fact has lead us to a number of simplifications in the flow scheme of the mixed batch.

The scheme of such a model is shown in Fig. 2. The non-ideal mixing in the region I is represented by a cascade of ideal mixers, where β is an arbitrary number greater than unity. The frequency distribution function of such a system (I) is expressed in the Laplace space^{19,20} as

$$E_M(s) = \frac{1}{(1 + s\bar{\tau}_M)^\beta} \quad (1)$$

The time constant $\bar{\tau}_M$ may be expressed in terms of the volume of the non-ideal mixer, V_M , and the pumping capacity of the screw impeller, \dot{V}

$$\bar{\tau}_M = \frac{V_M}{\beta \dot{V}} \quad (2)$$

The frequency distribution function of the non-ideal mixer of the given type is of the following form^{19,20}

$$E_M(t) = \frac{t^{\beta-1}}{\Gamma(\beta) \bar{\tau}_M^\beta} e^{-t/\bar{\tau}_M} \quad (3)$$

To express the frequency distribution function of the laminar distributor, two plug flow regions connected in parallel are considered. The frequency distribution function of this region (II) is given as a sum of both frequency functions (summation

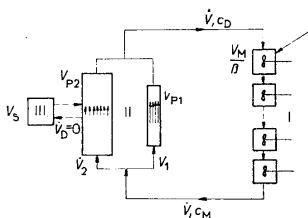


FIG. 2

Scheme of the flow model of homogenation of a highly viscous batch. I non-ideal mixer; II Laminar distributor; III stagnating region

of the probability of passage)

$$E_D(s) = \frac{\dot{V}_1}{\dot{V}} \exp(-s\bar{\tau}_{p1}) + \frac{\dot{V}_2}{\dot{V}} \exp(-s\bar{\tau}_{p2}), \quad (4)$$

where $\bar{\tau}_{p1}$, $\bar{\tau}_{p2}$ are mean residence times in the respective plug flow regions. This division into two subsystems agrees to some extent with the measurement of homogenation in the mixed system¹: a small portion of the overall flow rate passes through the probe located in the proximity of the maximum of the velocity (fast recirculation). The greater portion of liquid flows outside the probe while the time of recirculation is here substantially longer. After introducing the parameters α_D and k_τ by

$$\alpha_D = \frac{V_{p1}}{V_D} = 1 - \frac{V_{p2}}{V_D} \quad (5)$$

$$k_\tau = \frac{\bar{\tau}_D}{\bar{\tau}_{p1}} \quad (6)$$

$$\frac{\bar{\tau}_D}{\bar{\tau}_{p2}} = \frac{1 - k_\tau \alpha_D}{1 - \alpha_D}. \quad (7)$$

Here $\bar{\tau}_D$ is the mean residence time in the laminar distributor

$$\bar{\tau}_D = \frac{V_D}{\dot{V}}. \quad (8)$$

On introducing the parameters α_D and k_τ into Eq. (4), the expression for $E_D(s)$ may be arranged to give

$$E_D(s) = \alpha_D k_\tau \exp\left(-\frac{s\bar{\tau}_D}{k_\tau}\right) + (1 - \alpha_D k_\tau) \exp\left(-\frac{(1 - \alpha_D)s\bar{\tau}_D}{1 - \alpha_D k_\tau}\right). \quad (9)$$

The frequency distribution function of the volume V_s need not be searched for as the exchange of mass between this region and the remaining parts of the batch is taken to be zero.

The initial condition for the solution is given by the manner of feeding the tracer into the mixed system¹. The inlet part of the non-ideal mixer of volume V_M/β is fed at the time $t = 0$ the amount of the sample ΔV of concentration c_{MAX} . Then we may write

$$t = 0 \quad c_0(0) = \frac{c_{MAX} \Delta V \beta}{V_M}. \quad (10)$$

The transient change of the outlet concentration is given, with the knowledge of the operators E_M , E_D , by the convolution integral of the Laplace transform^{16,28}

$$C_e(s) = E(s) C_i(s). \quad (11)$$

In the solution for the system with a recycle and two regions of mixing with the corresponding functions $E_M(s)$ and $E_D(s)$, the response, C_e , of the first region represents the inlet function for the second system, C_i .

The response of the non-ideal mixer in the first recycle is given by

$$C_M^I(s) = E_M(s) c_0(0) \quad (12)$$

and in the m -th recycle by

$$C_M^m(s) = c_0(0) E_M^m(s) E_D^{m-1}(s). \quad (13)$$

The resulting concentration is given by the sum of partial concentration for the given region in the first up to the m -th recycle (summing of the probability functions). After substituting from Eqs (1) and (9) one may write, for instance, for the non-ideal mixer the following relation

$$C_M(s) = c_0(0) \sum_{m=1}^{\infty} \alpha_D k_{\tau} \exp\left(-\frac{s\bar{\tau}_D}{k_{\tau}}\right)^{m-1} + (1 - \alpha_D k_{\tau}) \exp\left(-\frac{(1 - \alpha_D) s\bar{\tau}_D}{1 - \alpha_D k_{\tau}}\right)^{m-1} / (1 + s\bar{\tau}_M)^{m\beta}. \quad (14)$$

The corresponding time dependence of the outlet concentration from the non-ideal mixer $c_M(t)$, was found by the inverse transform^{28,29}. The resulting relationship is as follows

$$c_M(t) = \sum_{m=1}^{\infty} \frac{c_0(0)}{\bar{\tau}_M^{m\beta-1} \Gamma(m\beta)} \sum_{j=1}^m \{K_{(m-1)}(\alpha_D k_{\tau})^{m-j} (1 - \alpha_D k_{\tau})^{j-1} T_{mj}^{m\beta-1}\} \cdot \exp\left(-\left(\frac{T_{mj}}{\bar{\tau}_M}\right)\right) \cdot H(T_{mj}). \quad (15)$$

The time variable T_{mj} is given by

$$T_{mj} = t - \left[(m-j) \frac{\bar{\tau}_D}{k_{\tau}} + (j-1) \frac{1 - \alpha_D}{1 - k_{\tau} \alpha_D} \bar{\tau}_D \right] \quad (16)$$

and $K_{(m-1)}$ is the binomial coefficient of the $(m-1)$ -th order.

In spite of all the simplifications the number of parameters is much too large: V , V_D , V_M , β , α_D , k_τ . For the investigated mixed system with a screw impeller and a draught tube the following assumptions have been introduced: 1) The recycle flow rate \dot{V} equals the pumping capacity of the screw impeller. 2) The volume of the laminar distributor, V_D , equals the volume of the annular space of the length equaling the height of the draught tube. 3) The values of α_D and k_τ depend on the velocity profile of the flow in the laminar distributor. A large change of both values causes only a minor change of the overall rate of homogenation. Taken: $\alpha_D = 0.33$, $k_\tau = 1.9$ (expresses fast recirculations of liquid, usually detected by the conductivity probe¹). 4) The stagnating volume as a fraction of the total volume, V_s/V , depends on the relative size of the impeller only. 5) The degree of non-ideality β in the volume V_M depends only on the shape of the impeller (not on its size).

Given the geometrical configuration of the mixed system, including the frequency of revolution of the impeller and on taking the volume of the batch, V , as the sum of the volume of the non-ideal mixer, the laminar distributor and the stagnating region, the volume V_D and the pumping capacity \dot{V} may be computed in advance². The remaining problem is to find two remaining parameters of the model V_s and β .

THE RESULTS OF CALCULATION OF TIME VARIATION OF THE CONCENTRATION

The basis for the calculation of the transient change of the concentration at the exit from the non-ideal mixer were Eqs (15) and (16) and the assumptions 1–3.

For the given geometrical configuration of the system with the screw impeller and a draught tube and the given size of the system ($D = 0.29$ m) we have computed

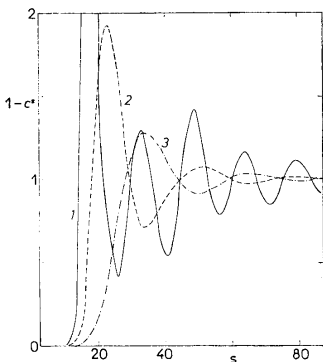


FIG. 3

Transient development of mixing in a vessel of $D = 0.29$ m at $\dot{V} = 6 \cdot 10^{-4} \text{ m}^3 \text{ s}^{-1}$ ($\beta = 8$). 1 $V_M/V = 0.2777$ $V_s/V = 0.318$ (—); 2 $V_M/V = 0.471$ $V_s/V = 0.239$ (---); 3 $V_M/V = 0.877$ $V_s/V = 0.072$ (-.-.-)

the volume of the laminar distributor (see assumption 2). For all calculations we took a constant value of the recirculating flow rate $\dot{V} = 6 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$ (in view of the assumption 1, this value should be different for various configurations, impeller diameters and speeds of revolution²). This way we have fixed also the value of the time constant τ_D (see Eq. (8)).

The shape of the curves $c_M(t)$ was considerably influenced especially by the relative size of the non-ideal mixer, V_M/V , see Fig. 3. The courses of all dependences display oscillatory behaviour; the effect of the variable parameters shows in the change of the "amplitude", spacing of the peaks and corresponding "simulation" mixing times $t_{0,2}$, $t_{0,1}$, $t_{0,05}$.

For the geometrical configuration of the mixed system used for the homogenation tests¹ we have computed simulation mixing times as functions of the model parameters V_s/V and β .

As the basis for the evaluation of the relationship between the model parameters and the simplexes of geometrical similarity served the experimentally found mixing times¹. These data were "unified" by the constant pumping capacity for all geo-

TABLE I

Mean mixing times t (s) in the creeping flow region (according to ref.¹) at the pumping capacity of the screw impeller $\dot{V} = 6 \cdot 10^{-4} \text{ m}^3 \text{ s}^{-1}$

D/d	s/d	$c^* = 0.2$	$c^* = 0.1$	$c^* = 0.05$
1.59	0.93	47.0	69.8	86.3
1.98	1.00	61.7	69.1	77.5
2.00	1.00	54.0	65.9	85.9
2.13	0.93	47.8	63.0	79.1
2.16	0.93	62.6	72.4	81.4
2.20	0.93	49.7	63.6	77.5
2.30	0.33	26.6	33.4	47.0
2.30	0.46	33.6	36.6	54.5
2.30	0.60	32.6	55.9	63.9
2.30	0.75	40.0	51.6	65.7
2.30	1.00	34.7	47.1	61.5
2.30	1.33	30.1	47.4	64.8
2.30	1.50	31.5	40.3	56.2
2.69	1.00	40.2	49.7	62.9
2.73	1.00	48.5	71.3	85.1
3.14	1.00	58.9	77.9	95.8
3.19	1.00	47.6	61.5	77.4
3.37	1.00	47.3	61.3	69.3

metrical configurations: $\dot{V} = 6 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$. Thus found values of the mixing times are shown in Table I.

Using the simplifying assumptions 4 and 5 and on the basis of comparison of the simulation and experimental mixing times $t_{0,2}$, $t_{0,1}$ and $t_{0,05}$, statistical methods were applied to evaluate the dependence $\beta = f(s/d)$ and $V_s/V = f(D/d)$. These dependences are plotted in Figs 4 and 5.

The course of the function $\beta(s/d)$ shows that the value of the simplex s/d (keeping D/d constant) affects little the degree of non-ideality. At the same time the mean values of β range between 7 and 10, which indicates that longitudinal mixing in the non-ideal mixer is far from the ideal mixing.

The diagram 5 shows that the fraction of the stagnating volume strongly increases with the decrease of the relative size of the screw impeller in the vessel. On the contrary, the effectively mixed fraction of the batch markedly diminishes. Around $D/d = 3$ the fraction of the stagnating volume amounts to about 25%. The dependence in Fig. 5 may be expressed by ($\beta = 10.1$)

$$\frac{V_s}{V} \sim \left(\frac{D}{d}\right)^{3.85}.$$

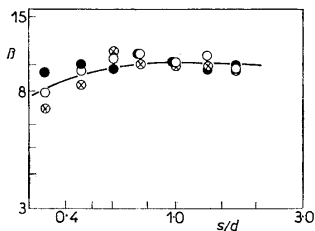


FIG. 4

The degree of nonideality β as a function of the relative lead of the screw impeller s/d at $V_s/V = 0.125$ (corresponds to $D/d = 2.3$). \circ $c^* = 0.2$; \bullet $c^* = 0.1$; \otimes $c^* = 0.05$

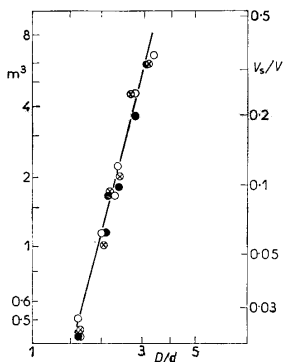


FIG. 5

Fractional volume of the stagnating region V_s/V as a function of the relative size of the impeller D/d at $\beta = 10.1$ (corresponds to $s/d = 1.0$). \circ $c^* = 0.2$; \bullet $c^* = 0.1$; \otimes $c^* = 0.05$. On the axis x is $\text{m}^3 \cdot 10^{-3}$

At the same time the degree of inhomogeneity c^* practically does not affect the magnitude of the stagnating region, nor the degree of non-ideality in the non-ideal mixer.

DISCUSSION

With the aid of the graphical dependences for the parameters β and V_s/V of the flow model (Fig. 4 and 5) and after introducing the assumption 2 we were able to calculate from Eq. (15) the mixing times for the selected degrees of inhomogeneity $c^* = 0.2$; 0.1 and 0.05 for the geometrical configuration from the earlier published work¹. These values were compared with the experimental data. A comparison of the computed and experimental values of the mixing times related to the constant pumping capacity $\dot{V} = 6 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$ is furnished in Table II.

The experimental values agree relatively well with those computed for the degree of inhomogeneity $c^* = 0.2$ and 0.1. Somewhat higher deviations occur for $c^* = 0.05$ and the configuration with the ratio $D/d \leq 2$. It may be therefore concluded that the proposed model and the relation of its parameters to the simplex of geometrical similarity of the mixing system satisfactorily describe the concentration change at the exit from the non-ideal mixer.

High values of the parameter β , and hence also the deviation from ideal mixing, indicate that the supplied power for the mixing is primarily used for the lateral mixing.

TABLE II

A comparison of the mixing times computed from the flow model (FM) with the experimental data (EXP) for the screw impeller with a draught tube in the creeping flow region (taking $\dot{V} = 6 \cdot 10^{-4} \text{ m}^3 \text{ s}^{-1}$)

Configuration		$t_{0,2}, \text{ s}$		$t_{0,1}, \text{ s}$		$t_{0,05}, \text{ s}$	
D/d	s/d	FM	EXP.	FM	EXP.	FM	EXP.
2.30	0.33	23.4	26.6	36.3	33.4	48.9	47.0
2.30	1.00	35.2	34.7	48.1	47.1	60.9	61.5
1.59	0.93	45.3	47.0	61.3	69.8	65.8	86.3
2.00	1.00	42.2	54.0	59.0	65.9	73.3	85.9
2.69	1.00	35.7	40.2	42.5	49.7	59.4	62.9
3.37	1.00	41.0	47.3	62.6	61.3	77.4	69.3

The magnitude of the volume of the stagnating region significantly increases with growing ratio D/d . This increase is even higher than the relative decrease of the volume of the screw impeller.

The results of this study may therefore be summarized into the following findings:

- the relative magnitude of the impeller considerably affects the magnitude of the fraction of the fraction of the stagnating region
- longitudinal mixing in the non-ideal mixer is low and also little affected by the shape of the screw impeller
- the energy supplied for mixing is used up primarily for lateral mixing.

LIST OF SYMBOLS

ϵ^*	degree of inhomogeneity
c	concentration of mixed component
C	Laplace transform of concentration c
D	internal diameter of vessel
D'	internal diameter of draught tube
d	impeller diameter
$E(s)$	Laplace transform of the frequency function
$E(t)$	frequency function (operator)
H	height of liquid level in vessel
$H(t)$	Heaviside unit function
k_τ	model parameter given by Eq. (6)
$K_{(m-1)}$	binomial coefficient of the $(m-1)$ -th order
m	number of recycles
s	lead of screw impeller
s	Laplace variable
T_{mj}	time variable defined by Eq. (16)
t	time
V	volume
ΔV	added volume of solution
\dot{V}	volume flow rate, pumping capacity of the screw
α_D	model parameter, given by Eq. (5)
β	model parameter expressing the degree of non-ideality of mixing
$\bar{\tau}$	mean residence time

Subscripts

D	laminar distributor
M	non-ideal mixer
P	plug flow region
e	exit
i	inlet
o	initial value
MAX	maximum value
1	fast recirculation
2	slow recirculation

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Translated by V. Staněk.