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n POINTS IN THE PLANE CAN DETERMINE $n^{3/2}$ UNIT CIRCLES

G. ELEKES

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In [1] P. Erdős proposed the following problem: *Given n points in the plane (no three collinear) draw a circle through each triple. How many different circles can have unit radius?*

Denoting the maximum by $f(n)$, he proved

(1) $$c_1 n \leqq f(n) \leqq n(n-1).$$

The aim of this note is to give a construction showing

(2) $$c_2 n^{3/2} \leqq f(n).$$

The construction. Denote $\lfloor\sqrt{2n}\rfloor$ by m. Choose a set of m unit vectors $\{v_1, v_2, \dots, v_m\}$ with the property that all the sums of its subsets are different (i.e. the non-trivial linear combinations with coefficients 0, 1 and -1 are not zero).

Let $S=\{v_i+v_j\colon\ 1\leqq i<j\leqq m\}$. Then $|S|=\binom{m}{2}\sim n$. (In fact, $|S|<n$.) Further, let $C=\{v_i+v_j+v_k\colon\ 1\leqq i<j<k\leqq m\}$.

To see (2), we have to show only that each of the $\binom{m}{3}\sim\frac{\sqrt{2}}{3}n^{3/2}$ points of C is the centre of a unit circle containing at least three points of S. But this is obvious, since $v_i+v_j+v_k$ is of unit distance from the points $v_i+v_j,\ v_i+v_k,\ v_j+v_k\in S$.

Remark 1. However, the upper bound in (1) seems to be far from being sharp (probably $f(n)\leqq cn^{3/2}$ holds), but we cannot prove even $f(n)=o(n^2)$.

Remark 2. The original problem can be extended to d-dimensional points and spheres where a construction similar to the previous one gives $c_3 n^{1+1/d}$ as a lower bound.

[1] P. ERDŐS, Some applications of graph theory and combinatorial methods to number theory and geometry, *Algebraic Methods in Graph Theory, Coll. Math. Soc. J. Bolyai* **25** (1981), 137—148.

G. Elekes

Department of Analysis
Loránd Eötvös University
H—1088 Múzeum krt. 6—8, Budapest, Hungary

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