The Expansion Wave Concept—Part II: A New Way to Model Mutual Coupling in Microstrip Arrays

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***Abstract*—A new expansion scheme is introduced to solve the integral equations describing the mutual coupling in microstrip arrays. The scheme is based on the fact that at larger distances the Green's functions in the stratified dielectric medium of the antenna structure can be approximated using analytical expressions. This allows one to describe the waves propagating between the elements thus causing the mutual coupling with a small number of parameters. Since only these parameters have to be determined, the resulting number of unknowns is much smaller than with conventional rigorous techniques. The accuracy of the scheme is illustrated by a comparison of measured and calculated data for both a two-element and a linear eight-element microstrip array antenna.**

***Index Terms*— Green's functions, inhomogeneous media, microstrip arrays.**

I. Introduction

A rigorous, widely used method to analyze microstrip antennas is to solve the integral equations describing the structure using the method of moments. It is well known that this procedure can yield highly accurate results. At first, authors either used a subsectional [1] or an entire domain scheme [2]. Subsectional schemes are the most flexible, but require the largest calculation times. Entire domain schemes with only a few expansion functions do provide a lower calculation time, but the flexibility is completely lost. Only simple shapes can be analyzed. The mixed variant of the combined expansion scheme introduced in [3] combines the two existing techniques, keeping the advantages and eliminating the disadvantages of both. The idea consists of constructing secondary entire domain expansion functions as fixed combinations of primary subsectional expansion functions. The procedure introduced in this paper is, in fact, a powerful generalization of the mixed expansion technique introducing the concept of expansion waves. Instead of constructing secondary entire domain expansion functions at the level of element components (patches, probes, ...), they are constructed at the level of complete elements. Since the number of element expansion functions may be chosen much lower than the number of primary expansion functions on the components of the element, the number of unknowns to be determined can be much lower than if the primary expansion functions were used throughout. This results in a much lower calculation time.

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II. Outgoing Waves

Consider an element of an array in a stratified dielectric medium. The coordinates x and y are parallel to the layer structure and the coordinate z is normal to the layer structure. The observation point is located at (x, y, z). The lateral position of a source point is (x', y'). The lateral position (0, 0) is the reference point of the element considered. An electromagnetic field component F generated by an element consisting of physical components (such as patches, probes, apertures, ...) in general can be written as

$$F(x,y,z) = \sum_j \int_{x'} \int_{y'} S_j(x',y') \cdot G_j^F(x-x', y-y', z)\, dx'\, dy' \qquad (1)$$

where each S_j is a source derived from the currents flowing on the components and G_j^F its spatial Green's function for component F of the electromagnetic field. F can be a lateral or z component of the electric or magnetic field. For isotropic dielectrics, it was proven in [4] that expressions can be found for the electromagnetic field involving Green's functions only depending on the lateral distance between source and observation point. In [5], it was proven that the behavior of a spatial Green's function at larger distances is determined by the dominant singularities of its spectral equivalent. It was shown that taking into account the dominant singularities only, an excellent approximation of the spatial function is obtained, even at relatively small distances from the source. Two types of dominant singularities occur—surface wave poles and branch point singularities—both for the two independent systems of the layer structure of the antenna under consideration—the TE and TM system. The pole positions can differ in both systems. The branch point position is the same in both systems. The positions of the singularities depend on the layer structure only. For the branch point, two square-root singularities are taken into account, both in the TE and TM system. Based on this, it was proven in [5] that the behavior of a spatial Green's function at larger distances can be approximated excellently by

$$\begin{aligned} G_j^F(x-x', y-y', z) \simeq \sum_{P^{\rm TM}} & C^{F,P^{\rm TM},-1}(z)\, C_j^{P^{\rm TM},-1}\, C^{P^{TM},-1}(R) \\ & + C^{F,K^{\rm TM},-0.5}(z)\, C_j^{K^{\rm TM},-0.5}\, C^{K^{\rm TM},-0.5}(R) \\ & + C^{F,K^{\rm TM},+0.5}(z)\, C_j^{K^{\rm TM},+0.5}\, C^{K^{\rm TM},+0.5}(R) \end{aligned}$$

$$+\sum_{P^{\rm TE}} C^{F,P^{\rm TE},-1}(z)\, C_j^{P^{\rm TE},-1}\, C^{P^{\rm TE},-1}(R) + C^{F,K^{\rm TE},-0.5}(z)\, C_j^{K^{\rm TE},-0.5}\, C^{K^{\rm TE},-0.5}(R) + C^{F,K^{\rm TE},+0.5}(z)\, C_j^{K^{\rm TE},+0.5}\, C^{K^{\rm TE},+0.5}(R) \quad (2)$$

where $P^{\rm TM}$, $P^{\rm TE}$, and K are the dominant TM poles, the dominant TE poles, and the branch point, respectively, and $R = \sqrt{(x-x')^2+(y-y')^2}$. For each singularity taken into account, the corresponding term in (2) is the product of a function of z depending on which field component is considered, a constant depending on which source type is considered, and a function of R. For a given Green's function (given field component F and source type j), all three can be determined from the knowledge of the layer structure only. If the observation point is not too close to the source compared to its dimension, using cylindrical coordinates the contribution of a singularity may be approximated using $R \simeq r$ for amplitude and $R \simeq r - r'\cos(\phi'-\phi)$ for phase terms. Insertion of (2) in (1) then yields

$$\begin{aligned} F(x,y,z) \simeq & \sum_{P^{\rm TM}} C^{P^{\rm TM},-1}(r)\, C^{F,P^{\rm TM},-1}(z) \sum_j C_j^{P^{\rm TM},-1} \cdot \int_{x'}\int_{y'} S_j(x',y')\, e^{jP^{\rm TM}r'\cos(\phi'-\phi)}\,dx'\,dy' \\ & + C^{K^{\rm TM},-0.5}(r)\, C^{F,K^{\rm TM},-0.5}(z) \sum_j C_j^{K^{\rm TM},-0.5} \cdot \int_{x'}\int_{y'} S_j(x',y') e^{jKr'\cos(\phi'-\phi)}\,dx'\,dy' \\ & + C^{K^{\rm TM},+0.5}(r)\, C^{F,K^{\rm TM},+0.5}(z) \sum_j C_j^{K^{\rm TM},+0.5} \cdot \int_{x'}\int_{y'} S_j(x',y') e^{jKr'\cos(\phi'-\phi)}\,dx'\,dy' \\ & + \sum_{P^{\rm TE}} C^{P^{\rm TE},-1}(r)\, C^{F,P^{\rm TE},-1}(z) \sum_j C_j^{P^{\rm TE},-1} \cdot \int_{x'}\int_{y'} S_j(x',y')\, e^{jP^{\rm TE}r'\cos(\phi'-\phi)}\,dx'\,dy' \\ & + C^{K^{\rm TE},-0.5}(r)\, C^{F,K^{\rm TE},-0.5}(z) \sum_j C_j^{K^{\rm TE},-0.5} \cdot \int_{x'}\int_{y'} S_j(x',y') e^{jKr'\cos(\phi'-\phi)}\,dx'\,dy' \\ & + C^{K^{\rm TE},+0.5}(r)\, C^{F,K^{\rm TE},+0.5}(z) \sum_j C_j^{K^{\rm TE},+0.5} \cdot \int_{x'}\int_{y'} S_j(x',y') e^{jKr'\cos(\phi'-\phi)}\,dx'\,dy'. \end{aligned} \quad (3)$$

The interesting properties of this last expression are: 1) that for each singularity present, the contributions due to the different sources all have the same r-dependence and z-dependence; 2) that the integrations no longer are dependent on the exact position of the observation point, only on the direction ϕ in which it is located; and 3) that considering the contributions due to the singularities separately, once a single field component of the electromagnetic field is determined using (3), all the other components can be derived from it without having to re-apply (3) to each of them. The physical interpretation is that for each singularity that needs to be considered to obtain a good approximation for the spatial Green's function, the element emits a wave. Both the r dependence and z dependence of this wave only depend on the singularity. They can be determined in advance from the knowledge of the layer structure only. Only the amplitude in each lateral angular direction ϕ depends on the element itself, more specifically on the type of components in the element and on the current flowing on them. It has to be calculated via easy integrations over all the components and a summation of the resulting integrals. For mutual coupling calculations, an element can thus be characterized by a number of outgoing waves from which the outgoing expansion waves will be derived equal to the number of dominant singularities of the layer structure. The r dependence and z dependence of these waves can be determined uniquely once the layer structure is completely defined. They do not depend on the element configuration. Only the ϕ dependence has to be calculated numerically from the currents flowing on the element components.

III. Incoming Waves

Each element of the array considered is excited not only by its own feed, but also by the outgoing waves emitted by the other elements. This, in fact, is mutual coupling. If the element emitting the outgoing wave is not too close to the element receiving it, the amplitude of the incoming wave can be considered constant over the receiving element. Only phase variations have to be taken into account. Since an incoming wave hitting the element from a certain direction is completely known except for its amplitude, the element can be solved for this incoming wave after normalization using any technique available (for example solving the integral equations for the element in question using subsectional expansion functions in a moment method technique). The outgoing waves emitted by the currents induced on the element by the incoming wave can be determined. This means that the relation between the incoming and outgoing waves on the element can be established. This relation does not depend on the array structure, only on the layer structure (which determines the shape of the waves, i.e., the z and r dependence and the interrelation between the wave field components), and the element structure (which determines the amplitude and the ϕ dependence of the waves).

IV. Solution of an Element

In practice, the first step to solve an array is to solve the element from which it is built, simultaneously for the following excitations: the feeds exciting the element directly and the waves coming in from the different lateral directions. For all these excitations, the outgoing waves emitted by the currents induced on the element can be calculated. For a numerical solving procedure, both the outgoing and the incoming waves have to be described with a finite number of parameters. This is done in the following two sections.

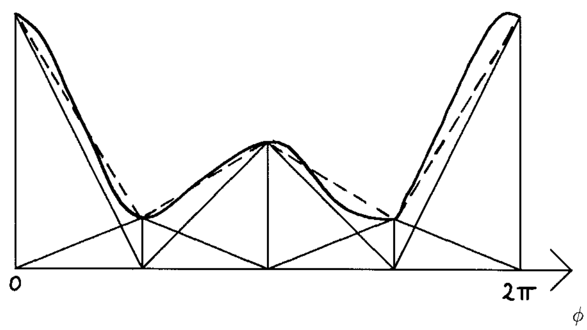

Fig. 1. Discretization of the outgoing waves.

A. Outgoing Expansion Waves

For the outgoing waves, we approximate the function describing the complex amplitude of the wave in terms of the angle ϕ using a number of basis functions. The wave corresponding to such a basis function is called an outgoing expansion wave. Using subsectional expansion, we arrive at a scheme as depicted in Fig. 1. Notice that the value at the end point equals the value at the starting point due to the fact that $\phi = 0$ is the same direction as $\phi = 2\pi$. In case of the subsectional scheme, it is easy to refer to a discrete number of reference directions for the outgoing expansion waves (where the corresponding subsectional basis function has the value one), namely the number N_w. It is evident that other basis functions can be used to describe the complex amplitudes of the outgoing waves. Of course, this leads to other types of outgoing expansion waves.

B. Incoming Expansion Waves

In a numerical solving procedure, it is impossible to consider waves coming in from all directions. Therefore, only waves coming in from a finite number of reference directions are considered. They are defined as the incoming expansion waves. Waves coming in from other directions are approximated by a decomposition into the two neighboring incoming expansion waves. If the number of incoming expansion waves is sufficiently large, the error introduced by this approximation is negligible.

C. Matrix Relations

The relation between outgoing expansion waves on the one hand and primary feeds (feed voltages) and incoming expansion waves on the other hand can be expressed in matrix form

$$W_o^e = W_o^e F_v^e \cdot F_v^e + W_o^e W_i^e \cdot W_i^e \tag{4}$$

where W_o^e, W_i^e, and F_v^e are the column matrices containing the amplitudes of the outgoing expansion waves, the incoming expansion waves, and the feed voltages, respectively, for the element and $W_o^e F_v^e$ and $W_o^e W_i^e$ the matrices containing the coupling coefficients between the outgoing expansion waves and the feed voltages and between the outgoing expansion waves and the incoming expansion waves, repectively, for the element. The superscript ^{e} indicates that the matrices are for a single element. Similarly, the relation between the feed responses (the feed currents) on the one hand and the feed voltages and the incoming expansion waves on the other hand can also be expressed in matrix form

$$F_i^e = F_i^e F_v^e \cdot F_v^e + F_i^e W_i^e \cdot W_i^e. \tag{5}$$

The dimension of both W_o^e and W_i^e is the product of the number of dominant singularities (both TM and TE) and N_w. The dimension of both F_v^e and F_i^e is the number of feeds.

V. Solution of the Array

For an array with N_e elements, it is clear that there are incoming and outgoing expansion waves for each element. The total number of incoming and outgoing expansion waves for the array is thus the product of N_e with the number of expansion waves for a single element. Equations (4) and (5) can be written for the global array

$$F_i = F_i F_v \cdot F_v + F_i W_i \cdot W_i \tag{6}$$

$$W_o = W_o F_v \cdot F_v + W_o W_i \cdot W_i \tag{7}$$

where the global column matrices consist of the column matrices for the different elements and the global coupling matrices consist of the coupling matrices for the different elements on their diagonals. The dimensions of the respective matrices are the dimensions of the corresponding matrices for a single element multiplied with the number of elements. It is clear that for each array geometry there is also a relation between the incoming expansion waves and the outgoing expansion waves. Each outgoing expansion wave emitted by one of the array elements will generate incoming expansion waves on a subset of the total number of elements of the array. In order to establish the relationship, the definition of the concept outgoing expansion waves and the decomposition technique for incoming expansion waves have to be used.

In Fig. 2(a), it is depicted how the amplitudes of the expansion waves emitted by an element in two neighboring reference directions give rise to an amplitude of a wave emitted in an intermediate direction. In Fig. 2(b), it is depicted how a wave hitting an element from a certain direction can be decomposed into the two neighboring incoming expansion waves. Using these two principles the relation between the discrete number of incoming expansion waves and the discrete number of outgoing expansion waves in the array can be written as

$$W_i = W_i W_o \cdot W_o. \tag{8}$$

It is important to emphasize that most of the elements in the matrix $W_i W_o$ are zero. Solving (6)–(8) yields

$$F_i = (F_i F_v + F_i W_i \cdot (U - W_i W_o \cdot W_o W_i)^{-1} \cdot W_i W_o \cdot W_o F_v) \cdot F_v \tag{9}$$

where U is the unit matrix. The matrix between brackets in (9) is the admittance matrix of the feed structure of the array calculated including full mutual coupling.

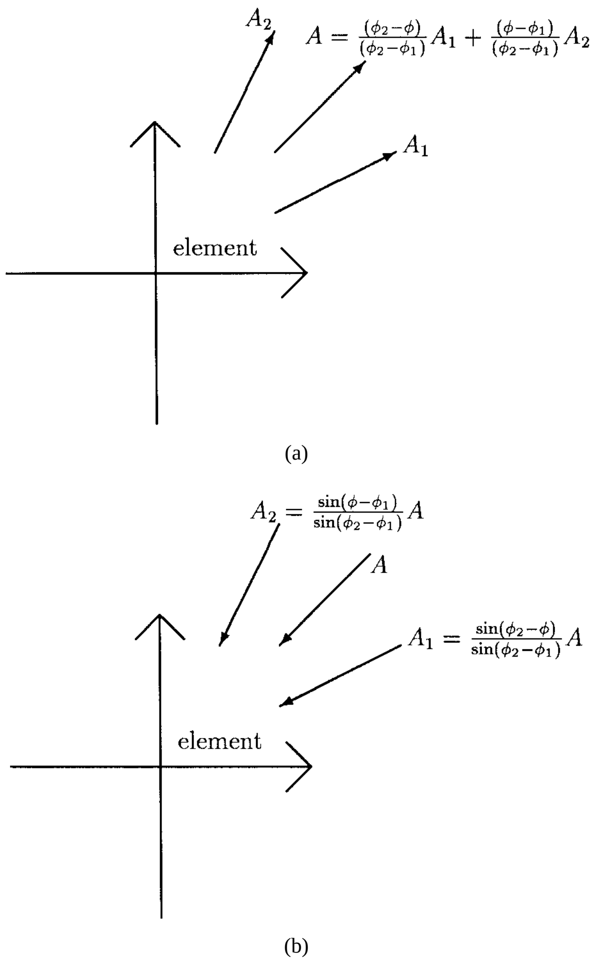

Fig. 2. (a) Interpolation for outgoing expansion waves. (b) Decomposition of an incoming expansion wave.

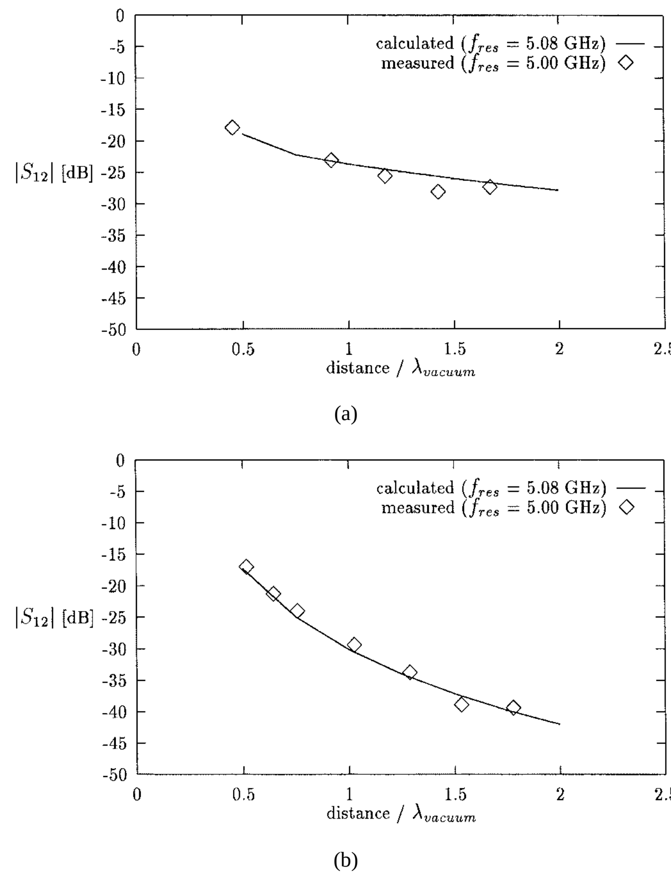

Fig. 3. The calculated and measured amplitude of S_{12} at the calculated and measured resonant frequency for both (a) the E-plane and (b) the H-plane case as a function of the normalized distance between the reference points of the two elements.

VI. Numerical Results

In order to prove the capabilities of the new technique, it was applied to the double scalar form of the mixed-potential integral expressions for the electromagnetic field in a stratified dielectric medium derived in [4]. The calculations were performed using the subsectional expansion technique of [4] to model the element and the expansion wave concept to model mutual coupling in the array. On the patches 10×10 subsections and for each element, eight reference directions were used $(\phi = 0, \pi/4, \pi/2, \cdots)$.

The first antenna structure involved is the array of two elements given in [6]. Each element consists of a coaxially fed microstrip patch. The dielectric layer between ground plane and patch has permittivity 2.55 and thickness 1.57 mm. The patch has x dimension 16.93 and y dimension 16.00 mm. The coaxial feed is located at 5.5 mm from the center of a y-directed patch edge. For this antenna the S parameters were calculated for both the E-plane and the H-plane configuration for several separations between the elements at the calculated frequency of the element (5.08 GHz). The results are given in Fig. 3.

The second structure is the linear eight-element microstrip array antenna of [3]. The layer configuration consists of a first dielectric layer (made of foam) with a thickness of 6.35 mm and with a relative permittivity of 1.03 sandwiched between a ground plane and a second dielectric layer, made of 3M Cuclad, with a thickness of 0.50 mm and with a relative permittivity of 2.17. The patch configuration is located on top of the second layer and consists of eight square patches with dimension 31.9 mm. Each patch is fed by a coaxial feed located 4.95 mm from the center of an edge. The used connectors are of the SMA type. The patches are positioned in each other's E or H plane. The distance between the patch centers is 60 mm. For this antenna the S parameters calculated between the feed of the first patch and the feeds of the seven other patches are compared to the measured ones both for the E-plane and the H-plane configuration at three frequencies. The results are given in Figs. 4 and 5.

Both examples illustrate the accuracy of the expansion wave technique. It yields results which agree very well with the measured results concerning both amplitude and phase.

VII. Discussion

Theoretically, the accuracy of the expansion wave technique as presented here is determined by three approximations. The first one is the approximation of the exact Green's functions by the superposition of their dominant pole and branch point contributions. The accuracy of this approximation is discussed in the first part of this paper. The second one is the transversal

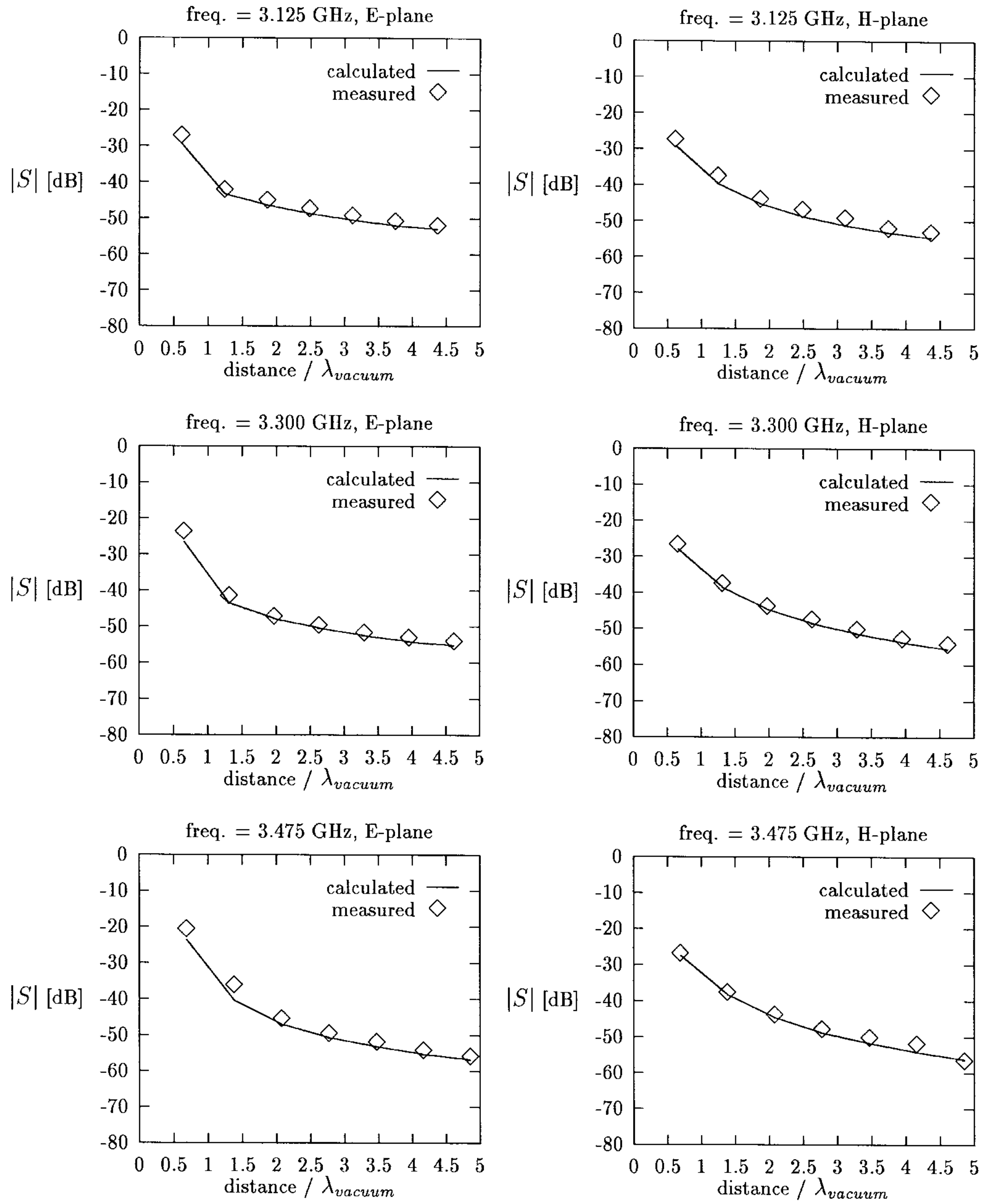

Fig. 4. The calculated and measured amplitudes of the S parameters between the feed of the first element and the feeds of the seven other elements at the frequencies 3.125 GHz, 3.3 GHz (the resonant frequency), and 3.475 GHz, both for the E-plane and the H-plane case as a function of the normalized distance between the reference point of the first element and the reference points of the seven other elements.

"far-field" approximation used in deriving (3) from (1) and (2). The error caused by this depends on the ratio of the size of the element emitting the waves and the distance from the observation point. The third is the transversal "far-field" assumption that the wave incident on an element can be regarded as having a constant amplitude over the element and plane wave phase variation over the element. It has to be emphasized that these "far-field" approximations are fundamental. The inaccuracy introduced by them cannot be overcome by increasing the number of expansion wave directions N_w. The traditional considerations concerning accuracy of "far-field" approximations hold (in case of an element of size $D = \lambda_{\text{vacuum}}/2$; the well-known threshold $2D^2/\lambda_{\text{vacuum}}$ yields a distance of $\lambda_{\text{vacuum}}/2$).

In order to allow smaller unit cells, the accuracy of the expansion wave scheme has to be improved. In our view, this can be reached by using entire domain expansion both for the outgoing and the incoming waves. The functions $\cos(m\phi)$ with $m = 0, 1, 2, \cdots$ and $\sin(n\phi)$ with $n = 1, 2, \cdots$ instead of rooftops can be used to form the outgoing wave of Fig. 1. The same functions can be used to describe the incoming waves. It is evident that this modification to the expansion wave technique has no effect on the solving procedure described in this paper. Only the elements in the coupling matrices will change. The advantage of this alteration is that by using a cylindrical decomposition of the waves actually present in the structure, the "far-field" approximations are not necessary any more. The accuracy of the description can be increased just

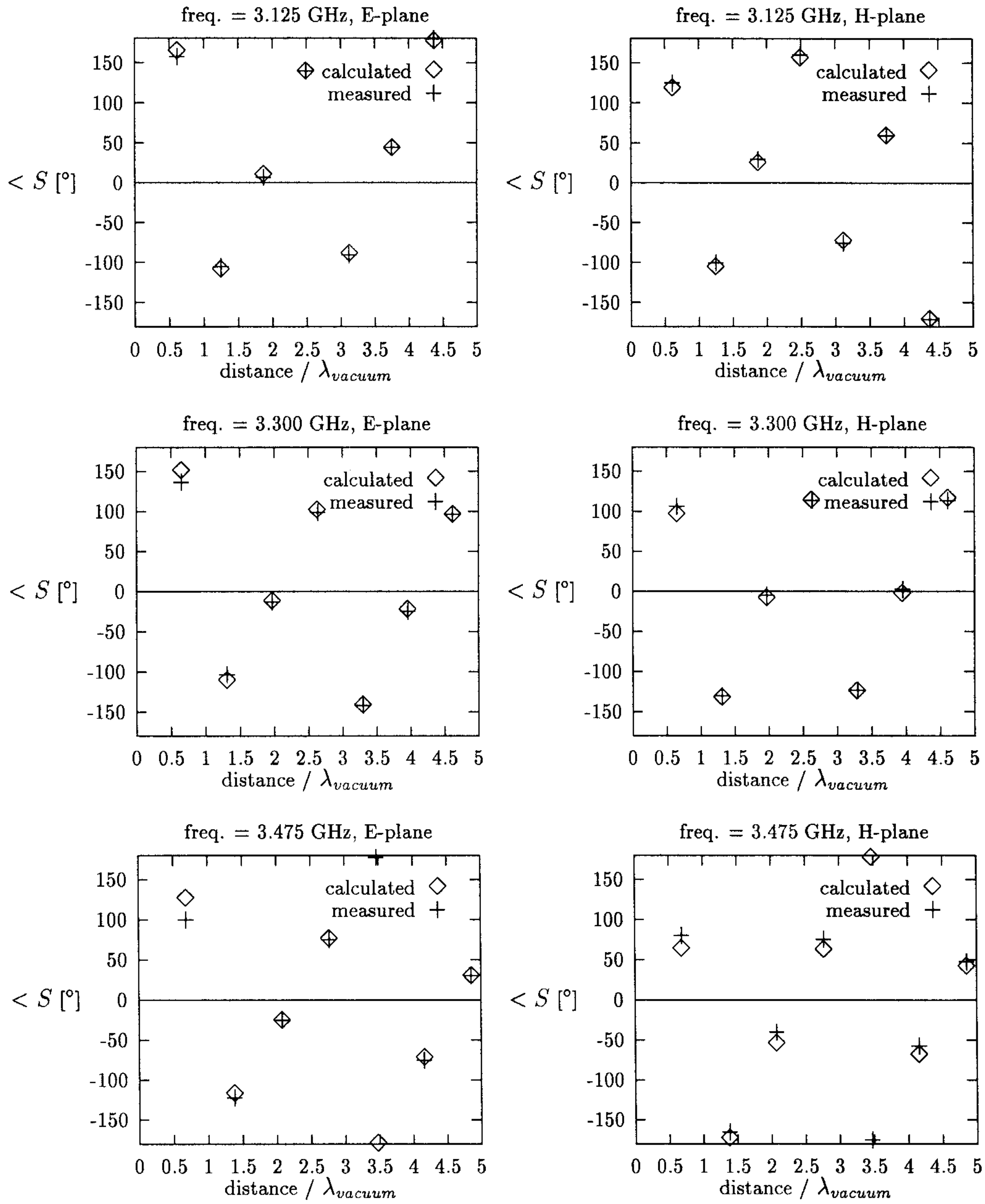

Fig. 5. The calculated and measured phases of the S parameters between the feed of the first element and the feeds of the seven other elements at the frequencies 3.125 GHz, 3.3 GHz (the resonant frequency), and 3.475 GHz, both for the E-plane and the H-plane case as a function of the normalized distance between the reference point of the first element and the reference points of the seven other elements.

by increasing the maximum m and n used thus by using more expansion waves. Only the first approximation remains.

Very important is that in the numerical results section, it is proven that the new theoretical concept of the expansion-wave technique (introduced in the sections before) does work for practical antennas. Comparing calculations with measurements, the technique yields very good agreement. However, the expansion-wave technique is not just an alternative to more rigorous techniques. In our view, it eliminates efficiently and elegantly the problem that arises when one wants to use the flexible subsectional expansion technique for arrays of arbitrary elements—the large number of unknowns necessary to describe mutual coupling. To get good accuracy, only eight reference directions per element were needed in the numerical results section compared to 10 $\times$ 10 subsections per patch if subsectional expansion was used also at the array level.

Although in this paper the technique was only applied to arrays of probe fed single patch elements, from the theoretical line of reasoning, it is clear that it can be applied also in the case of other feed types, stacked patch configurations, and even more complex elements. We expect that for all these elements the total number of eight reference directions will be sufficient to get high accuracy. For very complex elements, involving multiple patches with arbitrary shapes, probes, and other possible components, this will result in an enormeous reduction of the number of unknowns at the array level. The calculation time on the array level will be a constant independent of the configuration of the element.

The software available at this moment always takes into account all singularities discussed in this paper. For larger arrays, this still results in a large number of unknowns. Currently, we are working on a scheme to decide for a given layer structure and array configuration what singularities as a minimum have to be taken into account in order to get accurate results. When this is finished, a comparison between measured results and results calculated using the expansion wave technique will be possible for the 7 × 7 array given in [7].

VIII. Conclusions

Based on the results of Part I, a new way is derived to model mutual coupling in microstrip arrays. It is shown how the new technique yields a number of unknowns much lower than for conventional subsectional expansion techniques without changing the accuracy. The technique opens the way to model larger arrays of arbitrary elements in a more acceptable calculation time.

References

[1] J. R. Mosig and F. E. Gardiol, "General integral equation formulation for microstrip antennas and scatterers," *Proc. Inst. Elect. Eng.*, vol. 132, pt. H, pp. 424–432, Dec. 1985.

[2] D. M. Pozar, "Input impedance and mutual coupling of rectangular microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 1191–1196, Nov. 1982.

[3] G. A. E. Vandenbosch and A. R. Van de Capelle, "Use of a combined expansion scheme to analyze microstrip antennas with the method of moments," *Radio Sci.*, vol. 27, pp. 911–916, Nov./Dec. 1992.

[4] G. A. E. Vandenbosch and A. R. Van de Capelle, "Mixed-potential integral expression formulation of the electric field in a stratified dielectric medium—Application to the case of a probe current source," *IEEE Trans. Antennas Propagat.*, vol. 40, pp. 806–817, July 1992.

[5] F. J. Demuynck, G. A. E. Vandenbosch, and A. R. Van de Capelle, "The expansion wave concept—Part I: Efficient calculation of spatial Green's functions in a stratified dielectric medium," *IEEE Trans. Antennas Propagat*, this issue, pp. 397–406.

[6] A. H. Mohammadian, N. M. Martin, and D. W. Griffin, "A theoretical and experimental study of mutual coupling in microstrip antenna arrays," *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 1217–1223, Oct. 1989.

[7] D. M. Pozar, "Finite phased arrays of rectangular microstrip patches," *IEEE Trans. Antennas Propagat.*, vol. AP-34, pp. 658–665, May 1986.

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Filip J. Demuynck (M'95), for photograph and biography, see this issue, p. 406.