Blindness Removal in Arrays of Rectangular Waveguides Using Dielectrically Loaded Hard Walls

Sergei P. Skobelev, *Member, IEEE*, and Per-Simon Kildal, *Fellow, IEEE*

***Abstract*—We analyze an infinite planar array of open-ended rectangular waveguides in which the E-plane walls are loaded with dielectric slabs in order to create a hard boundary condition. The analysis is done by using the mode-matching method with Floquet modes in the exterior region. The results show that the hard walls remove the blind spots otherwise present in the element patterns of wide-angle scanning arrays. This occurs both for rectangular and for triangular lattices. In addition, the array becomes better matched to free-space in a rectangular lattice. In limited-scan arrays of large square waveguides, the use of the hard walls increases the array gain by 0.9 dB, it reduces the grating lobe level in the H plane by 11 dB and considerably decreases the difference between the array characteristics in the E and H planes.**

***Index Terms*—Antenna arrays.**

I. Introduction

ONE of the most convenient elements for phased antenna arrays is the empty open-ended rectangular waveguide. However, it is well known that these elements have very different scan characteristics in the E and H planes, and that scan blindness occurs for wide-angle scanning [1]. These features are not improved if the waveguides are completely filled with dielectric material.

A simple method, which can be used to improve the array scanning properties, is to partially load the waveguides with dielectric plates. This loading is simple and provides an additional degree of freedom relative to the complete dielectric filling. The formulation of this problem and a method of solution were given in [2]. Some important numerical results and corresponding conclusions can be found in [3]. The work [4] considers application of the partially filled waveguides in arrays for dual-frequency operation.

An important case of partial filling the rectangular waveguides is to load the E-plane walls with dielectric slabs of thickness t and relative permittivity ε so that

$$t = 0.25\lambda/\sqrt{\varepsilon - 1} \tag{1}$$

where λ is the wavelength. This corresponds to the so-called hard-wall boundary condition [5]. When (1) is satisfied, the propagation constant of the dominant waveguide mode is equal to that of free-space and the amplitude distribution of this mode over the waveguide cross section is uniform in the central empty part of the waveguide. These features allow to expect better matching of the array to free-space, more similar scanning properties in the E and H planes, and increased aperture efficiency and gain [6]–[8].

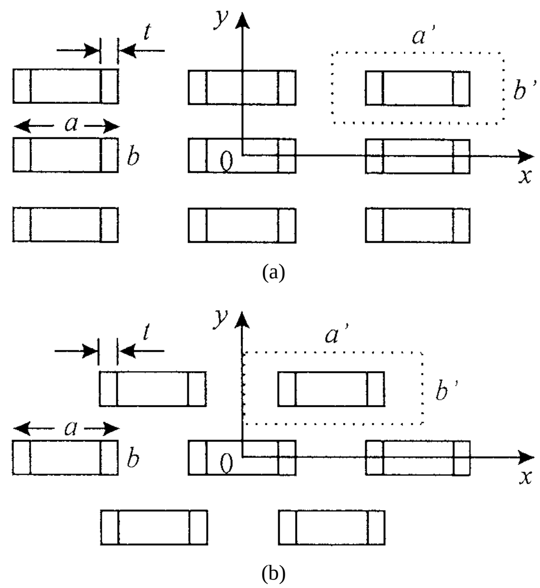

Fig. 1. Geometry of aperture of dielectrically loaded waveguide array. (a) Rectangular lattice. (b) Triangular lattice.

The hard-wall waveguide array has only been indirectly treated in [3] without any quantitative evaluation. Some numerical results characterizing have been presented in [9], but only for a simplified two-dimensional (2-D) case. The purpose of the present paper is to perform the analysis based on a three-dimensional statement of the problem, making the results directly applicable to the design of practical planar antenna arrays.

II. Statement of Problem and Main Formulas

The geometry of the open waveguide array in the plane of its aperture is shown in Fig. 1. Semi-infinite perfectly conducting waveguides of rectangular cross section $a \times b$ are arranged in an infinite rectangular or triangular lattice with element spacings a' and b' in the directions of x and y, respectively. All waveguide openings have a common perfectly conducting flange and their walls in one plane are loaded with dielectric slabs of thickness t and relative permittivity ε.

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We represent the field in the waveguide in terms of longitudinal section electric (LSE) and longitudinal section magnetic (LSM) modes and assume that the waveguides are excited by LSE_{10} modes with unit amplitudes and linearly progressing phases. The phase progression in x- and y-directions determines the direction of the main beam.

The reflected field can be represented as a superposition of the LSE_{mn} and LSM_{mn} modes and the radiated field can be represented as a superposition of vector Floquet harmonics. The coefficients of the two expansions are determined by matching the tangential E and H fields at the waveguide opening. This is commonly referred to as the mode-matching method [10]. The procedure is well known so we omit intermediate expressions and give only the main formulas.

The transverse components of the total electric and magnetic fields in the central waveguide are given by the infinite sums

$$\boldsymbol{E}_\tau(x,\,y,\,-0) = \sum_{m,n}[(\delta_{m,1}\delta_{n,0} + R_{1mn})\gamma_{1mn}\boldsymbol{f}_{1mn} + R_{2mn}k\boldsymbol{f}_{2mn}] \tag{2}$$

$$\boldsymbol{H}_\tau(x,\,y,\,-0) = \sqrt{\varepsilon_0/\mu_0}\sum_{m,n}[(\delta_{m,1}\delta_{n,0} - R_{1mn})k\boldsymbol{g}_{1mn} - R_{2mn}\gamma_{2mn}\boldsymbol{g}_{2mn}] \tag{3}$$

where γ_{jmn} and R_{jmn} are propagation constants and reflection coefficients of the LSE_{mn} $(j = 1)$ and LSM_{mn} $(j = 2)$ modes; δ_{mn} is the Kronecker symbol

$$\boldsymbol{f}_{1mn}(x,\,y) = \boldsymbol{e}_y\Phi_{1m}(x)\sqrt{\frac{2-\delta_{n0}}{b}}\,\cos\,\frac{n\pi(y+b/2)}{b},$$
$$\boldsymbol{g}_{1mn}(x,\,y) = -\boldsymbol{e}_x\chi_{1mn}\Phi_{1m}(x)\sqrt{\frac{2-\delta_{n0}}{b}}\,\cos\,\frac{n\pi(y+b/2)}{b} + \boldsymbol{e}_y\,\frac{n\pi}{kb}\,\frac{d\Phi_{1m}(x)}{k\,dx}\,\sqrt{\frac{2-\delta_{no}}{b}}\,\sin\,\frac{n\pi(y+b/2)}{b}$$

are transverse vector-functions for LSE_{mn} modes, and

$$\boldsymbol{f}_{2mn}(x,\,y) = \boldsymbol{e}_x\chi_{2mn}\Phi_{2m}(x)\sqrt{\frac{2}{b}}\,\sin\,\frac{n\pi(y+b/2)}{b} + \boldsymbol{e}_y\,\frac{n\pi}{kb}\,\frac{d\Phi_{2m}(x)}{k\,dx}\,\sqrt{\frac{2}{b}}\,\cos\,\frac{n\pi(y+b/2)}{b}$$
$$\boldsymbol{g}_{2mn}(x,\,y) = \boldsymbol{e}_y\varepsilon(x)\Phi_{2m}(x)\sqrt{\frac{2}{b}}\,\sin\,\frac{n\pi(y+b/2)}{b}$$

are those for the LSM_{mn} modes; $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are unit vectors of the corresponding coordinate axes, $\chi_{jmn} = (\gamma_{jmn}/k)^2 + (n\pi/kb)^2$, $k = 2\pi/\lambda$, $\varepsilon(x) = \varepsilon$ for the dielectric part and $\varepsilon(x) = 1$ for the empty part of the waveguide, ε_0 and μ_0 are dielectric and magnetic constants of free-space. The orthonormalized functions $\Phi_{jm}(x)$, as well as the corresponding dispersion equations for calculation the propagation constants, are determined from the boundary conditions for the layered waveguide as described in details in [11].

The transverse components of the radiated electric and magnetic fields on the array aperture are represented as infinite sums of the vector Floquet harmonics

$$\boldsymbol{E}_\tau(x,\,y,\,+0) = \sum_{p,q}[T_{1pq}k\psi_{1pq}(x,\,y) + T_{2pq}\Gamma_{pq}\psi_{2pq}(x,\,y)], \tag{4}$$

$$-\boldsymbol{e}_z\times\boldsymbol{H}(x,\,y,\,+0) = \sqrt{\varepsilon_0/\mu_0}\sum_{p,q}[T_{1pq}\Gamma_{pq}\psi_{1pq}(x,\,y) + T_{2pq}k\psi_{2pq}(x,\,y)] \tag{5}$$

where T_{jpq} are amplitudes of the TE $(j = 1)$ and TM $(j = 2)$ harmonics; Γ_{pq} and ψ_{jpq} are propagation constants and orthonormalized transverse vector functions, respectively, defined in [1].

Further, the vector product of the magnetic field (3) and the unit vector $\boldsymbol{e}_z$ must be equal to (5) in the waveguide aperture and the electric field (4) must be equal to the electric field (2) in the waveguide aperture and to zero on the flange. Using the orthogonality property defined in [11] for the waveguide modes in the indicated continuity condition for the magnetic fields, as well as the orthogonality property of the Floquet harmonics on the array cell [1] in the indicated boundary condition for the electric fields, we obtain the following infinite system of linear algebraic equations:

$$\begin{aligned}
&R_{1m'n'}\chi_{1m'n'} + \sum_{p,q}(T_{1pq}S^{11}_{m'n'pq}\Gamma_{pq}/k + T_{2pq}S^{12}_{m'n'pq})\\
&\quad = \delta_{m'1}\delta_{n'0}\chi_{1m'n'}\\
&R_{2m'n'}\chi_{2m'n'}\gamma_{2m'n'}/k + \sum_{p,q}(T_{1pq}S^{21}_{m'n'pq}\Gamma_{pq}/k\\
&\qquad + T_{2pq}S^{22}_{m'n'pq}) = 0,\\
&\sum_{m,n}(R_{1mn}S^{11*}_{mnp'q'}\gamma_{1mn}/k + R_{2mn}S^{21*}_{mnp'q'}) - T_{1p'q'}\\
&\quad = -S^{11}_{10p'q'}\gamma_{110}/k\\
&\sum_{m,n}(R_{1mn}S^{12*}_{mnp'q'}\gamma_{1mn}/k + R_{2mn}S^{22*}_{mnp'q'})\\
&\qquad - T_{2p'q'}\Gamma_{p'q'}/k = -S^{12*}_{10p'q'}\gamma_{110}/k
\end{aligned} \tag{6}$$

for the unknown coefficients R_{1mn}, R_{2mn}, T_{1pq}, and T_{2pq}, where

$$S^{ij}_{mnpq} = \iint_A (\boldsymbol{f}_{imn}\bullet\psi_{jpq})\,dx\,dy \tag{7}$$

and A is the waveguide aperture. Sign * in (6) denotes complex conjugation. The matrix elements (7) are expressed by explicit formulas, which are not given here for the sake of brevity.

After proper truncation, (6) is easily solved by any known numerical method, e.g., the Gauss elimination method. Then, the complex amplitudes of the TE and TM Floquet harmonics of the zeroth order T_{100} and T_{200} are used for calculation of the components of the vector-array element pattern in the spherical coordinates. This characterizes the behavior of the array gain when the main beam is scanned. The magnitudes of these components when normalized for unit power of the incident LSE_{10} mode are expressed by

$$F_\theta(\theta,\,\varphi) = (k/\gamma_{110})^3|T_{200}(u,\,v)|\,\cos\,\theta \tag{8}$$
$$F_\varphi(\theta,\,\varphi) = (k/\gamma_{110})^3|T_{100}(u,\,v)|\,\cos\,\theta \tag{9}$$

where $u = ka' \sin\theta \cos\varphi$, $v = kb' \sin\theta \sin\varphi$.

The incident, reflected, and radiated powers must satisfy the energy balance relationship

$$\gamma_{100}\chi_{110} - \sum_{j=1}^{2}\left(\sum_{m,n}\gamma_{jmn}\chi_{jmn}|R_{jmn}|^2 + \sum_{p,q}\Gamma_{pq}|T_{jpq}|^2\right) = 0 \quad (10)$$

where the sums include only propagating waveguide and Floquet modes.

III. Numerical Results and Discussion

The technique described above has been implemented in a FORTRAN program to calculate the array characteristics. The program has been verified by comparing the results with the numerical and experimental data available in [1] for empty and completely filled waveguides in [3] for layered rectangular waveguides as well as in [9] for the corresponding 2-D case. The latter case corresponds to a planar array of rectangular waveguides arranged in a rectangular lattice and scanning in the H plane, provided that the perfectly conducting waveguide walls which are parallel to the indicated plane have zero thickness. The comparison showed good agreement of the results. Moreover, all the calculations were accompanied by verification of the energy balance relationship (10) and the disbalance was not greater than $2 \cdot 10^{-6}$. After testing the program, different hard-wall waveguide arrays were calculated and the results were compared with those obtained for the corresponding cases of empty waveguides. Some of these results are presented below.

A. Wide-Angle Scanning, Rectangular Lattice

It is well known that the element patterns of arrays of both empty and completely filled rectangular waveguides can have deep dips causing blindness. The blindness effect is most considerable when scanning in the E plane of the array where its elements are located in a rectangular (and, in particular, square) lattice. The influence of the hard walls on the array performance is demonstrated by using an array of square waveguides with $a = b = 0.6305\lambda$ arranged in a square lattice with $a' = b' = 0.6729\lambda$. The results for the array element pattern F_θ ($F_\varphi = 0$ in the E plane) are presented in Fig. 2. The dotted curve in Fig. 2 is the element pattern for the empty waveguide array, which is in a good agreement with the theoretical and experimental data presented in [1]. The element pattern has a deep dip at the angle, which is slightly smaller than that of the main lobe position at which the grating lobe starts to radiate at −90°. The for hard wall waveguides with $\varepsilon = 2$ and 5 are presented in Fig. 2 by solid and dashed curves, respectively. The results show that the use of the hard walls completely removes the dip and the pattern becomes similar to that of an array of empty parallel-plate waveguides excited by TEM waves considered in [9].

Calculations have been also performed in a frequency band. The results show that a reduction of the frequency does not

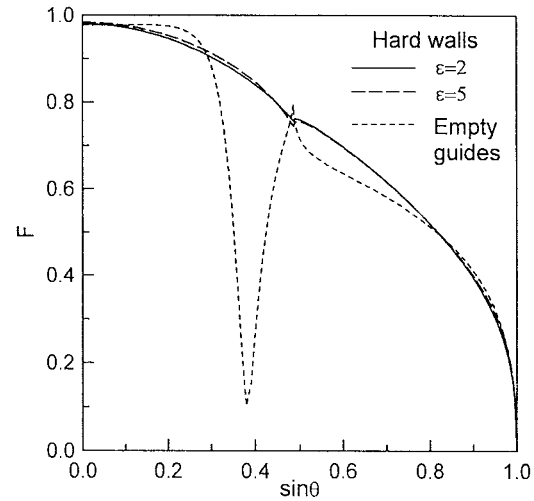

Fig. 2. Element pattern F_θ in E plane of array with rectangular lattice and $a = b = 0.6305\lambda$, $a' = b' = 0.6729\lambda$.

remove the dip in the element pattern of the empty waveguide array, but shifts it to a new angle. Moreover, the reflection coefficient at broadside increases because the dominant TE_{10} mode becomes closer to cutoff. In the case of the hard-wall waveguides, reduction of the frequency also makes the array characteristics worse because the amplitude distribution of the LSE_{10} mode approaches that of the TE_{10} mode of the empty waveguide. However, since the LSE_{10} mode still remains sufficiently far from cutoff, the changes are less considerable than those for the empty waveguides. Note also that the greater slab permittivity is the more frequency-sensitive the array becomes.

When increasing the frequency, the dip in the element pattern of the empty waveguide array decreases and then disappears. In the element pattern of the hard waveguide array, the dip does not appear at all.

Thus, if the hard-wall condition is exactly provided for the array with a rectangular lattice at the lower edge of a specified frequency band, this can guarantee the absence of blindness over the whole band. The behavior of the array characteristics in the H plane is similar to that for the parallel-plate waveguide arrays considered in [9]. The array characteristics in the intermediate planes of scan also do not have considerable anomalies.

B. Wide-Angle Scanning Triangular Lattice

When empty rectangular waveguides are arranged in a triangular lattice, anomalies in the array characteristics can take place both in E and in H planes, but in the latter they are most essential. We consider an array with $a = 0.905\lambda$, $b = 0.4\lambda$, $a' = 1.008\lambda$, and $b' = 0.504\lambda$, which is the same as the examples considered in [1]. The calculated element pattern in H plane is shown in Fig. 3 by a curves with smaller dashes. This result is in a good agreement with the corresponding experimental data presented in [1]. The array-element pattern has a deep dip, which occur much before the main lobe position (at $\sin\theta \approx 0.866$) corresponding to the case when the grating lobe is at the boundary of real space. This significantly reduces the sector of scan.

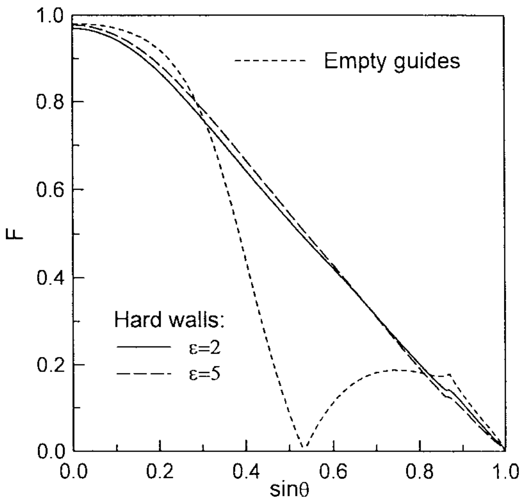

Fig. 3. Element pattern F_φ in H plane of array with triangular lattice and $a = 0.905\lambda$, $b = 0.4\lambda$, $a' = 1.008\lambda$, $b' = 0.504\lambda$.

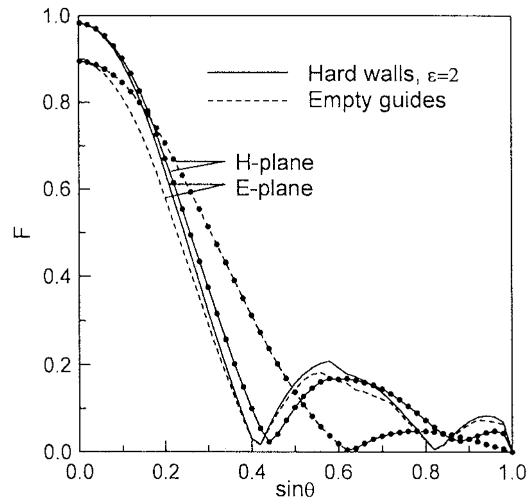

Fig. 4. Element patterns F_θ (E plane) and F_φ (H plane) of array with $a = b = 2.4\lambda$ and $a' = b' = 2.45\lambda$.

The results for the hard-wall waveguides with $\varepsilon = 2$ and 5 ($t = 0.25$ and 0.125, respectively) are shown in Fig. 3 by a solid curve and a curve with larger dashes, respectively. We see, the use of the hard walls have removed the dip from the element pattern. However, the reflection coefficient considerably increases when the main beam is scanned from broadside (from 0.2 or 0.24 at $\sin\theta = 0$ up to 0.45 for $\sin\theta = 0.4$ and up to 0.65 for $\sin\theta = 0.6$).

Thus, in the whole, the use of the hard walls in the rectangular waveguide arrays with a triangular lattice is not so effective as in the arrays with a rectangular lattice, and do not provide such good wide-angle matching as in the arrays using nonsymmetrical irises [12].

C. Array of Large-Aperture Elements

Fig. 4 presents the results of calculation of the element pattern for an array of square waveguides with $a = b = 2.4\lambda$ arranged in a square lattice with $a' = b' = 2.45\lambda$. Arrays with such large elements are common in limited-scan applications and sometimes as feeds in reflector antennas. As we can see, the H-plane element pattern for the case of the empty waveguides considerably differs from that in the E plane. This is as well known and caused by the difference in the dominant mode-field distributions in the two planes. The cosine distribution in H plane makes the pattern in this plane wider than in E plane and shifts the pattern nulls that results in a higher level of the first grating lobes and, as a consequence, in decreased array gain. The hard walls provide the uniform amplitude distribution in the central empty part of the waveguide cross section that makes the grating lobe level in the H plane by about 11 dB lower than for the empty waveguides and increases the gain by 0.9 dB. Moreover, the width of the element pattern in H plane becomes much closer to that in E plane.

Note, that the H plane results obtained in the present study are very close to those presented in [9] for the corresponding array of parallel-plate waveguides because the thin waveguide walls in H plane is negligible when the array is scanned in H plane. For this reason, all the conclusions in [9] for the parallel-plate waveguides in connection with the frequency behavior of the array characteristics are valid for the present rectangular waveguides as well.

IV. Conclusion

The infinite planar arrays of open-ended rectangular waveguides with E plane walls loaded with dielectric slabs satisfying the hard wall boundary condition have been numerically analyzed by using the mode matching method. The results of the calculations have been compared with those obtained for the corresponding arrays of empty waveguides.

When the array is designed for wide-angle scanning, the most attention has been concentrated on the influence of the hard walls on the effects of blindness present in empty waveguide arrays. Blindness is associated with the presence of deep dips in the array element pattern. The obtained results show that the hard walls completely remove the dips from the element pattern when the waveguides are arranged in a rectangular or square lattice. Moreover, if the hard wall condition is satisfied exactly at the lower edge of a specified frequency band, the effects of blindness may be removed over the whole band providing relatively good matching of the array to free-space.

The hard walls also remove the effects of blindness when the waveguides are located in a triangular lattice. However, the reflection coefficient in the H plane considerably increases when the main beam is scanned from broadside.

We have also calculated arrays of square waveguides with large aperture dimension. These results confirmed the results obtained earlier for the corresponding parallel-plate waveguide array, and have shown that the use of the hard walls reduces the grating lobe level in the H plane by about 11 dB and correspondingly increases the array gain by 0.9 dB in comparison with the array of empty waveguides. In addition, the hard walls make the element pattern in H plane much more similar to that in the E plane than what is in the case for empty waveguides.

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