Scattering from Structures Formed by Resonant Elements

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***Abstract*— A two-dimensional (2-D) structure, formed by a finite number of resonant elements, has been considered in this paper, and its scattering characteristics have been analyzed in detail over a wide frequency band including the resonance region. The elements of the structure comprise perfectly conducting thin cylinders with longitudinal slots. It has been demonstrated that the scattered field of such an array exhibits rather remarkable properties at resonance frequencies. For instance, the strong coupling between the elements in the resonance region manifests itself into superdirective radiation in the far field and that this is a common attribute of scatterers formed by coupled resonant elements.**

***Index Terms*—Electromagnetic scattering.**

I. Introduction

RESONANT elements often play an important role as components of many electromagnetic devices, e.g., oscillators. The complexity of these resonant structures, for instance, the presence of internal cavities that couple to their exteriors via apertures, makes them rather difficult to analyze via direct numerical methods, especially over a wide frequency band.

The problem of electromagnetic wave scattering from cavity-backed apertures (CBA's) has received considerable recent attention. Single elements with resonant cavities have been investigated by a number of authors. A thin, cylindrical conducting screen with a longitudinal slot represents a simple example of a two-dimensional (2-D) CBA.

An efficient solution to the problem of scattering from the slotted cylinder with a circular cross section has been obtained by using a rigorous approach based on the solution to the Riemann–Hilbert problem in the theory of functions of complex variables [1]. The method has been successfully applied to the problem of scattering from slotted circular cylinders [2]–[4] and is well-suited for studying the influence of the resonance of the interior cavity on the characteristics of the scattered fields. Recently, the above method has been generalized [5], [6] to the problem of electromagnetic scattering from thin 2-D screens whose cross sections can be arbitrary. It is based on the extraction of the principal singular part of the integral equation kernel and the subsequent inversion of the corresponding residual operator. This partial-inversion approach reduces the initial boundary value problem to an infinite system of linear algebraic equations of the second kind. The fact that the resulting system is a Fredholm system of the second kind contributes to the numerical efficiency of the method. An important feature of this type of second-kind operator is that the condition number of its associated matrix system does not grow with the size of the truncated system and this makes such a semi-analytic approach especially useful for analyzing scatterers of complex shapes in the resonance region.

While the characteristics of single CBA's have been thoroughly investigated in the past, the same cannot be said about multiply-coupled resonant CBA's that have received scant attention. While it is tempting to analyze the scattering properties of finite arrays of CBA's by following techniques typically employed for infinite periodic structures, this is not advisable because the field scattered by a structure formed by a finite array of passive resonant elements have some unique characteristics that are not found in their infinite counterparts. For instance, the field scattered by such structures can be superdirective [8] in a narrow frequency band, despite the fact that the resonant elements are totally passive. This paper shows that the occurrence of the superdirectivity can be attributed to the excitation of modes that induce a phase reversal in the adjacent scatterers and that the amplitude distribution also plays an important role. While the role played by the phase reversal of the fields in the adjacent elements in inducing superdirectivity in the array is obvious, the same cannot be said for the amplitude structure of the field near the array, which turns out to be equally important. This is because an active superdirective antenna is highly sensitive to small changes in both the phase and amplitude distributions of the sources and this sensitivity increases with the increase in the directivity.

The field distribution in an array aperture emanating superdirective radiation is rather complex, though its primary attributes are phase reversals accompanied by high-field amplitudes. In addition, the elements of such an array are located in close proximity to each other, with the spacing considerably less than $\lambda/2$. Although the literature is replete with reports of extensive studies of superdirective antennas [9], the practical realization of these compact antennas appears to be very difficult, if not impossible [10].

An alternate approach to fabricating a 2-D superdirective antenna using a single active element and a reflector comprising of resonant scatterers, has been explored in the past [4], [11]. Physical phenomena responsible for inducing superdirective radiation have been studied in some detail and it has been shown that the interaction between the resonant elements helps excite a mode characterized by a high level of reactive power in the vicinity of a passive structure. In the

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above works, particular attention was paid to the sensitivity of the superdirective antenna and to the constraints on the tolerances of the parameters. It was demonstrated [11] that superdirectivity can be maintained even when the angular width of the slots in one or more open cylinders in a CBA is allowed to vary by ±1%.

The purpose of this paper is to demonstrate that a moderate superdirective property of the scattered field is "natural" for passive scatterers formed by resonant elements. We show how the mutual coupling between the elements can be exploited to generate and control the desired phase and amplitude behaviors of the near field that lead, in turn, to the superdirectivity in the radiated field. This is true despite the fact that all of the elements of the array are passive and this is indeed a remarkable property of the CBA scatterers. Incidentally, even though the structure is totally passive, its amplitude and phase distributions can nonetheless be controlled to enhance the effect of superdirectivity by varying the coupling between the elements through an adjustment of their spacing or mutual orientation.

It should be realized, however, that structures formed by passive elements have their own intrinsic limitation in terms of the level of the superdirectivity that they can achieve, even when we assume that they are perfect conductors. Obviously, the finite dimensions of the elements impose a limit on the closeness of the elements, which, in turn, limits their directivity. However, it is possible to raise this limit by varying the shapes of the elements such that their Q factors are increased.

The numerical results presented in this paper pertain to the simplest, 2-D case of finite number of thin perfectly conducting circular cylinders with longitudinal slots; however, the method of analysis itself is applicable to screens with arbitrary cross sections. The simple geometry was chosen as an illustrative example because an alteration of the cross-sectional shape does not affect the qualitative scattering characteristics of the array.

Before closing this section, it is interesting to point out that the superdirective phenomena are found to exist only in finite arrays since the phase reversal property does not appear to exist in infinite gratings.

II. Analysis of Finite Cavity-Backed Aperture Arrays

In this section, we consider the problem of analyzing a finite array of CBA's comprised of perfectly conducting thin slotted cylinders that are infinite along the z axis. An H-polarized plane wave is incident upon the structure with the wave vector $\vec{k}$ in the plane perpendicular to the z axis. The angle between the $\vec{k}$ and y axis is φ_i (see Fig. 1). For the H-polarized case, we can derive the following integral equation for N cylindrical screens whose cross sections can be arbitrary:

$$\frac{j}{4}\frac{\partial}{\partial n_g^q}\sum_{s=1}^{N}\int_{L_s}\hat{\mu}^s(p_s)\frac{\partial}{\partial n_p^s}H_0^{(2)}(k|p_s-g_q|)\,dl_s = \frac{\partial}{\partial n_g^q}H_z^i(g_q) \qquad g_q\in L_q,\ q=1,\cdots,N.$$

$$F_q(g_q)=\frac{\partial}{\partial n_g^q}H_z^i(g_q) \tag{1}$$

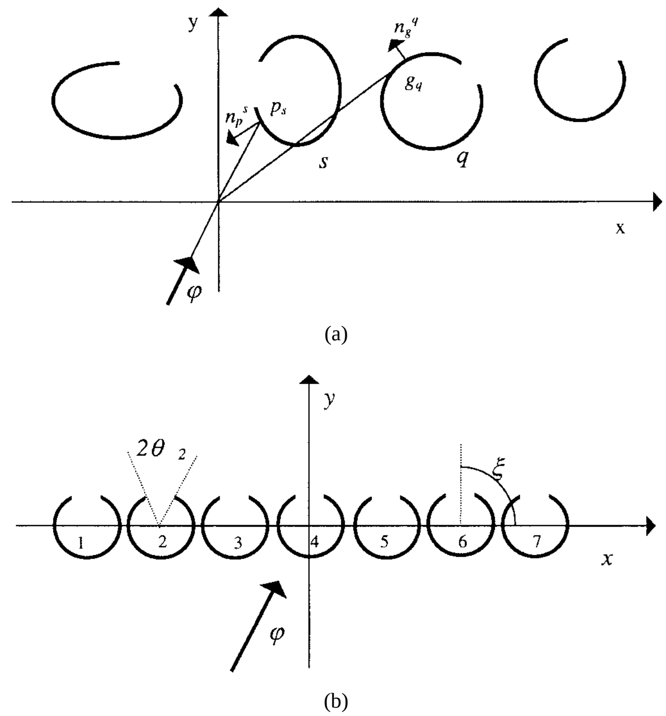


Fig. 1. Cross-sectional views of finite arrays of cavity-backed apertures.

where L_s is the contour of the surface of the sth cylinder, p_s, g_q are the radius vectors from the origin of the system of coordinates to the corresponding points on the surfaces of the sth, and qth cylinders is a normal to the contour of the qth cylinder at the point g_q, $H_0^{(2)}$ is the Hankel function of the second kind, $\hat{\mu}^s(p_s)$ denotes the surface current density, and H_z^i represents the incident H-polarized wave.

The construction of the solution entails the following steps: 1) isolation of the logarithmic singularity from the Hankel function appearing in the kernel of (1); 2) reduction of the integral equation on the curve to a conventional one by following the procedure given in [12]; and 3) Fourier series representation of the functions.

The contour of the sth scatterer is parametrized by the pair of functions $x^s(\tau)$, $y^s(\tau)$, where τ is the parameter in the interval $[-\alpha_s,\ \alpha_s]$, as follows:

$$\mu^s(\tau)=\hat{\mu}^s[\eta^s(\tau)];\ \tau\in[-\alpha_s,\ \alpha_s];\ \eta^s(\tau)=[x^s(\tau),\ y^s(\tau)]. \tag{2a}$$

Using the functions of parametrization the integral (1) can be rewritten as

$$\frac{1}{2\pi}\frac{\partial^2}{\partial\vartheta^2}\int_{-\alpha_q}^{\alpha_q}\mu^q(\tau)\ln\left|2\sin\frac{\vartheta-\tau}{2}\right|d\tau - \frac{1}{2\pi}\sum_{s=1}^{N}\int_{-\alpha_s}^{\alpha_s}\mu^s(\tau)H_{qs}(\vartheta,\tau)\,d\tau = F_q(\vartheta) \tag{2a}$$

$$\vartheta\in[-\alpha_q,\ \alpha_q], \qquad q=1,\cdots,N$$

$$F_q(\vartheta)=l_q(\vartheta)\frac{\partial}{\partial n_g^q}H_z^i[x_q(\vartheta),\ y_q(\vartheta)] \tag{2b}$$

$$l_q(\vartheta)=\sqrt{\left[\frac{\partial x_q(\vartheta)}{\partial\vartheta}\right]^2+\left[\frac{\partial y_q(\vartheta)}{\partial\vartheta}\right]^2} \qquad (2c)$$

$$H_{qs}(\vartheta,\,\tau)=\begin{cases} q=s,\; H_{qq}(\vartheta,\,\tau) \\ \quad =\dfrac{j\pi}{2}\,l_q(\vartheta)l_q(\tau)\dfrac{\partial^2}{\partial n_g^q\partial n_p^q}\,H_0^{(2)}(kr_{qq}) \\ \quad +\dfrac{\partial^2}{\partial\vartheta^2}\,\ln\left|2\,\sin\dfrac{\tau-\vartheta}{2}\right| \\ q\neq s,\; H_{qs}(\vartheta,\,\tau) \\ \quad =\dfrac{j\pi}{2}l_q(\vartheta)l_s(\tau)\dfrac{\partial^2}{\partial n_g^q\partial n_p^s}H_0^{(2)}(kr_{qs}). \end{cases} \qquad (2d)$$

The functions H_{qs} given in (2d) are not smooth and, hence, their Fourier coefficients must be calculated by using special numerical algorithms for efficient computation.

Our next step is to transform the integral equation (2a) into dual-series equations by using the Fourier series representation of the functions given as follows:

$$\mu^s(\tau)=\sum_{n=-\infty}^{\infty}\mu_n^s e^{jn\tau},\; \tau\in[-\pi,\,\pi]$$

$$H_{qs}(\vartheta,\,\tau)=\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}h_{nm}^{qs}e^{jn\vartheta+jm\tau},\; \vartheta,\,\tau\in[-\pi,\,\pi]$$

$$F_q(\vartheta)=\sum_{n=-\infty}^{\infty}f_n^q e^{jn\vartheta},\; \vartheta\in[-\pi,\,\pi] \qquad (3)$$

together with the one for the logarithmic function given in (2). Next, by using the definition $\mu^q(\tau)=0$ if $\tau=[-\pi,\,\pi]\backslash(-\alpha_q,\,\alpha_q)$, we arrive at the following desired equations for Fourier coefficients of the unknown currents induced on the surfaces of the array elements:

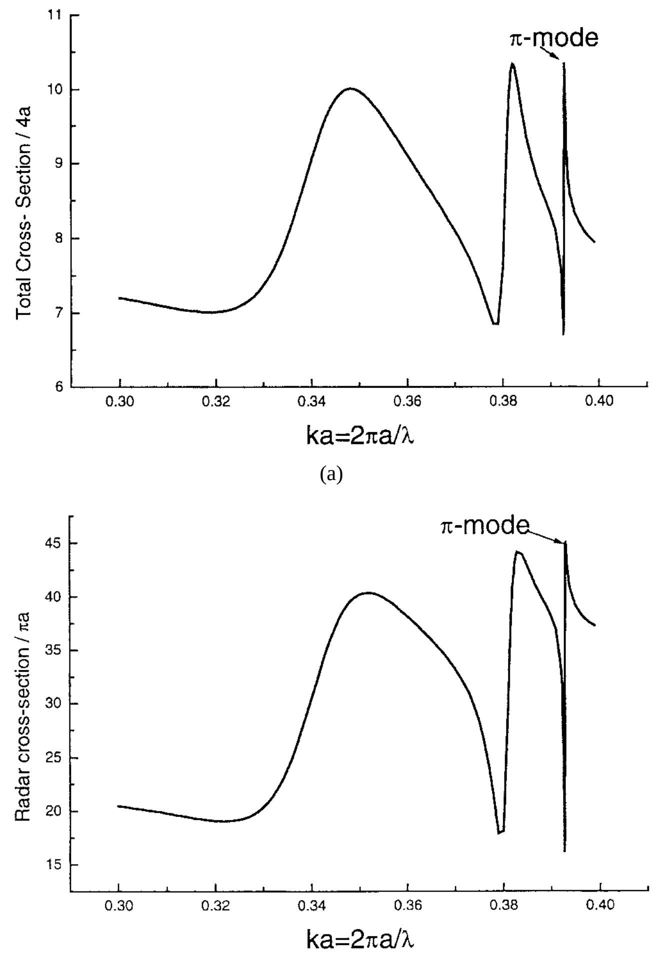
$$\sum_{n\neq 0}\mu_n^q|n|e^{jn\vartheta}-2\sum_{s=1}^{N}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\mu_m^s h_{n,-m}^{qs}e^{jn\vartheta}$$
$$=\sum_{n=-\infty}^{\infty}f_n^q e^{jn\vartheta},\qquad \vartheta\in[-\alpha_q,\,\alpha_q]$$
$$\sum_{n\neq 0}\mu_n^q e^{jn\vartheta}=0,\qquad \vartheta\in(-\pi,\,\pi)\backslash[-\alpha_q,\,\alpha_q]. \qquad (4)$$

Our next step is to reduce the dual-series equations (4), with $q=1,\cdots,N$ to the following N coupled infinite systems of linear algebraic equations via the generalized Reimann–Hilbert technique described in [5]

$$\mu_m^q=2\sum_{s=1}^{N}\sum_{p=-\infty}^{\infty}\mu_p^s\sum_{n=-\infty}^{\infty}h_{n,-p}^{qs}\frac{V_{m-1}^{n-1}(\cos\,\alpha_q)}{m}+2\sum_{n=-\infty}^{\infty}f_n^q\frac{V_{m-1}^{n-1}(\cos\,\alpha_q)}{m} \qquad (5a)$$

$$\vec{\mu}^q\,(1-C^q)-\sum_{s=1}^{N}\vec{\mu}^s M^{qs}=\vec{b}^q \qquad (5b)$$

where $\vec{\mu}^q=\{\mu_n^q\}_{n=-\infty}^{\infty}$, $\vec{b}^q=\{b_n^q\}_{n=-\infty}^{\infty}$, $b_n^q=2\sum_{n=-\infty}^{\infty}f_n^q[V_{m-1}^{n-1}(\cos\,\alpha_q)]/m$ and V_n^m are functions



(b)

Fig. 2. (a) Frequency scan of TCS of a resonant structure formed by seven slotted cylinders. The parameters are: $\theta_1=\theta_2=\cdots=\theta_7=5°$, $\xi_1=\xi_2=\cdots=\xi_7=90°$, angle of incidence of the plane wave $\varphi_i=0°$, distance between the cylinders d is 2.1a, where a is the radius of the cylinders. (b) Frequency scan of the RCS of a resonant structure formed by seven slotted cylinders. The parameters are: $\theta_1=\theta_2=\cdots=\theta_7=5°$, $\xi_1=\xi_2=\cdots=\xi_7=90°$, angle of incidence of the plane wave $\varphi_i=0°$, distance between the cylinders d is 2.1a, where a is the radius of the cylinders.

of Legendre polynomials $P_n(u)$ given by the following:

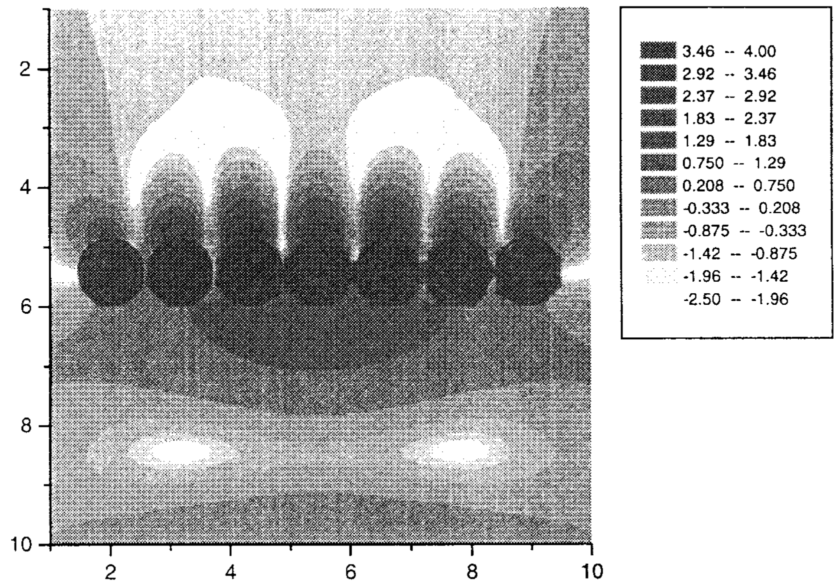
$$V_m^n(u)=\begin{cases}\frac{1}{2}\sum\limits_{p=0}^{n+1}\rho_{n+1-p}(u)P_{p-m-1}(u), & n\geq 0\\ \frac{1}{2}[P_m(u)-P_{m+1}(u)], & n=-1\\ \frac{1}{2}\sum\limits_{p=0}^{-n-1}\rho_{-n-1-p}(u)P_{p+m+1}(u), & n<-1\end{cases}$$

and

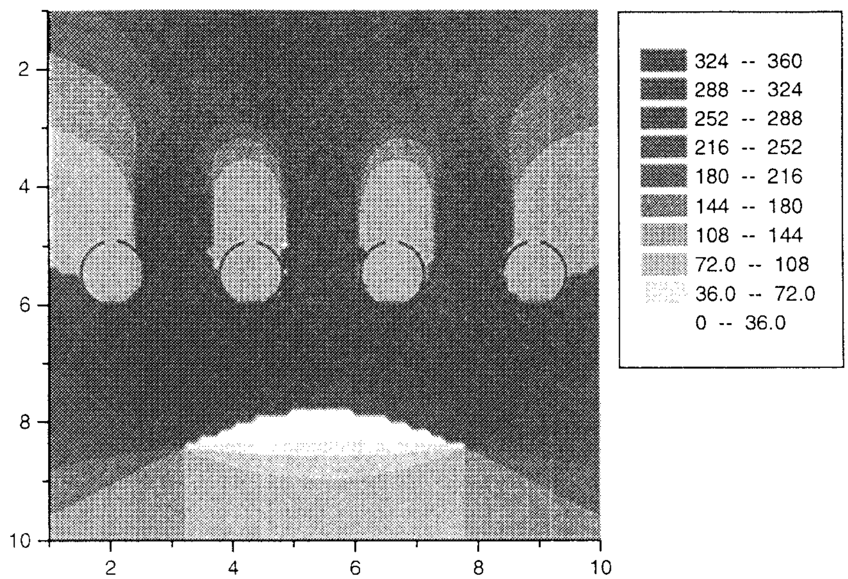
$$\left.\frac{V_{n-1}^{m-1}}{n}\right|_{n=0}=-W_m. \qquad (6)$$

In the above, $\rho_0=1$; $\rho_1(u)=-u$; $\rho_n=P_n(u)-2uP_{n-1}(u)-P_{n-2}(u)$ and

$$W_n(u)=[P_n(u)-P_{n-1}(u)]\frac{1}{2n},\qquad n\neq 0$$
$$W_0(u)=\ln\,\frac{1-u}{2}. \qquad (7)$$



(a)



(b)

Fig. 3. (a) Amplitude distribution of the magnetic field (log$|H_z|$) in the vicinity of the resonant structure described in Fig. 2, with $ka = 0.392\,88$. The total cross section/$4a = 10.34965$. (b) Phase distribution of the H_z field in the vicinity of a resonant structure described in Fig. 2, with $ka = 0.392\,88$.

The scattered field in the near and far zones can both be expressed such that they depend only on the coefficients $\{\mu_n^s\}_{n=-\infty}^{\infty}{}_{s=1\cdots N}$ that are obtained by solving the system (5), which is a cellular matrix. The diagonal blocks of this matrix correspond to the system of equations associated with the single open screen, while the off-diagonal blocks describe the interaction between the various elements. The diagonal and nondiagonal matrices define the Hilbert–Schmidt operators in the l_2 space. Thus, the operator defined by the system matrix is a Hilbert–Schmidt operator in the l_2^N space (Nth degree of the space l_2). The system may be solved after truncating the infinite matrices, which form the main cellular matrix, followed by the use of conventional inversion algorithms. The truncation technique does not limit the range of parameters of the problem and permits a determination of the scattered field with any desired accuracy.

III. Numerical Results

The total cross section (TCS), σ^H, is one of the most desired characteristics of the scattering properties of a structure. We consider a finite array of thin equispaced perfectly conducting and circularly cylindrical screens with longitudinal slots (see Fig. 1) whose axes are coplanar and are parallel to the z axis. For the above configuration, using the representation to the scattered field in far zone [11] and definitions of the TCS [13]

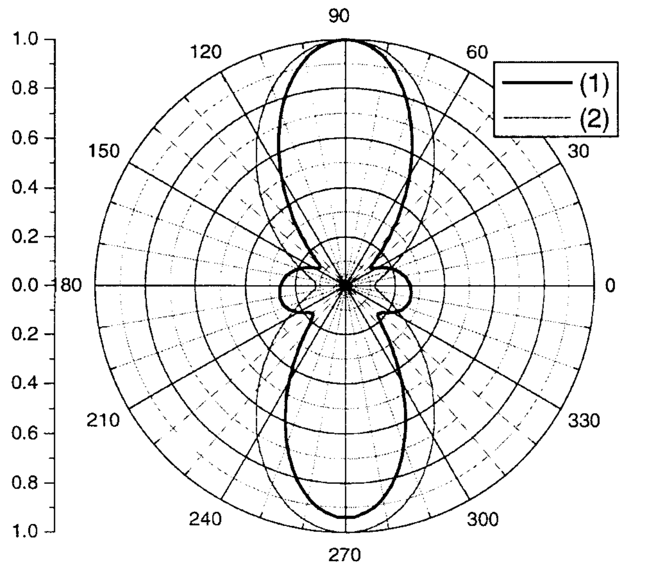


Fig. 4. Scattered far-field patterns of a resonant screen formed by seven slotted cylinders. $ka = 0.393\,02$ (max RCS) π-mode excitation that increases the TCS and the RCS values. $\cdots$ far-field pattern of the field radiated by seven isotropic line sources with equiphase excitation. The collinear sources are equally spaced with a separation of $0.131\,31\lambda$. $D(\varphi) = \sin[7(kd\,\cos\,\varphi)/2]/7\,\sin(kd\,\cos\varphi/2)$.

the expression for σ^H can be written in the form

$$\sigma^H = \frac{4}{k}\sum_{m=-\infty}^{\infty}\Bigg|\tilde{\mu}_m^s J'_m(ka_s) + \sum_{q=1,\, q\neq s}^{N}\sum_{p=-\infty}^{\infty} \cdot\, \tilde{\mu}_p^q J'_p(ka_q) J_{p-m}(kr_{qs}) e^{j(p-m)\varphi_{qs}}\Bigg|^2$$

$$\tilde{\mu}_n^s = \frac{j\pi k a_s}{2}\,\mu_n^s(-1)^n e^{-jn\xi_s} \tag{8}$$

where $(r_{qs},\,\varphi_{qs})$ are the coordinates of the sth cylinder in the system of coordinates associated with the qth cylinder and $\{\xi_s\}_{s=1}^N$ are the angles of orientations of the slots (see Fig. 1). It can be shown analytically and substantiated numerically that the TCS is independent of the index s of the particular cylinder.

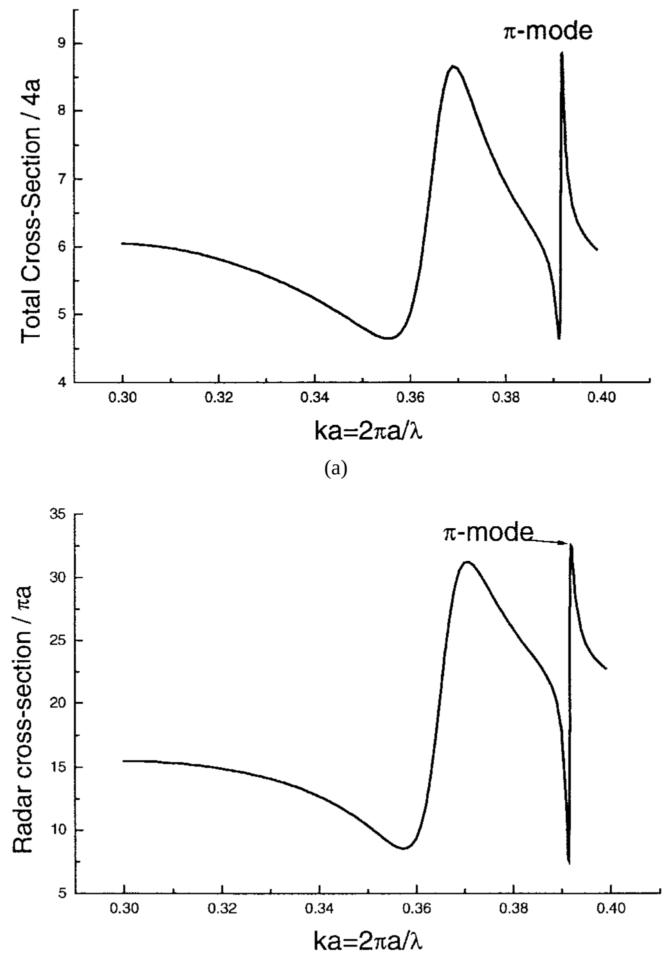
The radar cross section (RCS) of the array can be evaluated by using the following expression:

$$\sigma_B^H = \frac{4}{k}\left|\sum_{q=1}^{N} e^{-jkr_{q0}\sin(\varphi_i+\varphi_{q0})}\sum_{n=-\infty}^{\infty}\tilde{\mu}_n^q e^{-jn\varphi_i} J_{-n}(ka_q)\right|^2 \tag{9}$$

and the far-field pattern (FFP) $\Phi(\varphi)$ can be calculated from the values of the coefficients $\{\mu_n^s\}_{n=-\infty\, s=1\cdots N}^{\infty}$. The expression for the pattern reads

$$\Phi(\varphi) = \sum_{q=1}^{N} e^{jkr_{q0}\cos\varphi-\varphi_{q0}}\sum_{m=-\infty}^{\infty}\tilde{\mu}_m^q J'_m(ka_q) e^{jm[\varphi-(\pi/2)]} \tag{10}$$

where $(r_{q0},\,\varphi_{q0})$ are the coordinates of the qth cylinder.

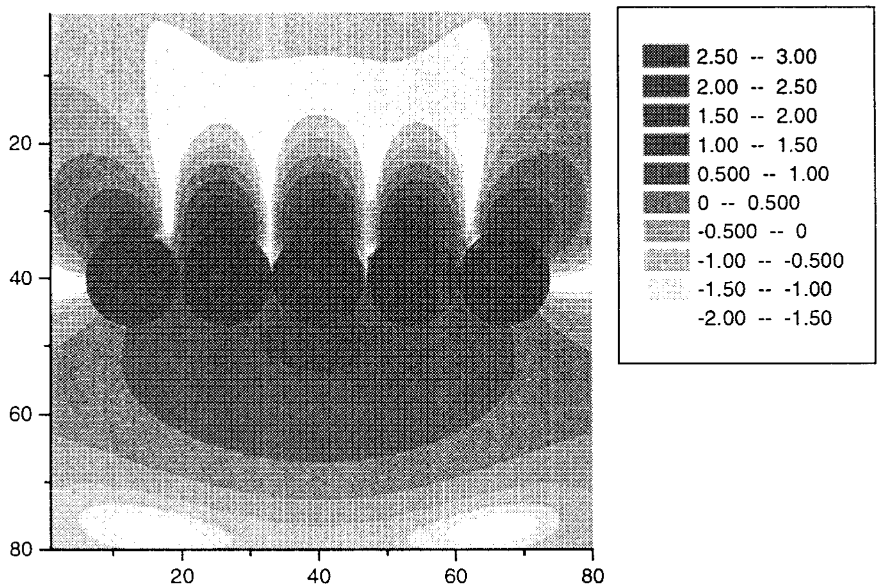


(b)

Fig. 5. (a) Frequency scan of the TCS of the structure formed by five slotted cylinders. $\theta_1 = \theta_2 = \cdots = \theta 5 = 5°$, $\xi_1 = \xi_2 = \cdots = \xi_5 = 90°$, the angle in the incidence of the plane wave $\varphi_i = 0°$, the distance between the cylinders d is $2.1a$. (b) Frequency scan of the RCS of the structure formed by five slotted cylinders.

It should be noted that the solution to the corresponding eigenvalue problem, viz., that of determining the complex-valued eigenfrequencies of the structure, can be solved in an analogous way. It can be shown that in the domain of the complex values of the frequency parameter ka, the matrix operator of the obtained system of linear algebraic equations is of trace class. Thus, it can be proven that the method of truncation can be applied to calculate eigenfrequencies and eigenmodes of the structure. The complex eigenfrequency spectrum forms a discrete set and has a finite multiplicity. Also, for eigenvalues with small imaginary parts, the stored energy in the scattered field rises up considerably at frequencies close to the real parts of these eigenvalues.

In scatterers with internal cavities as, for instance, slotted cylinders or grooves in a conducting surface, an H-polarized incident field excites certain eigenmodes, e.g., the H_{00}-mode in the slotted cylinder or the $\lambda/4$ mode in the rectangular groove. Such excitation occurs at frequencies for which the wavelength of the incident field considerably exceeds the dimensions of the screen. The excitation of such structural



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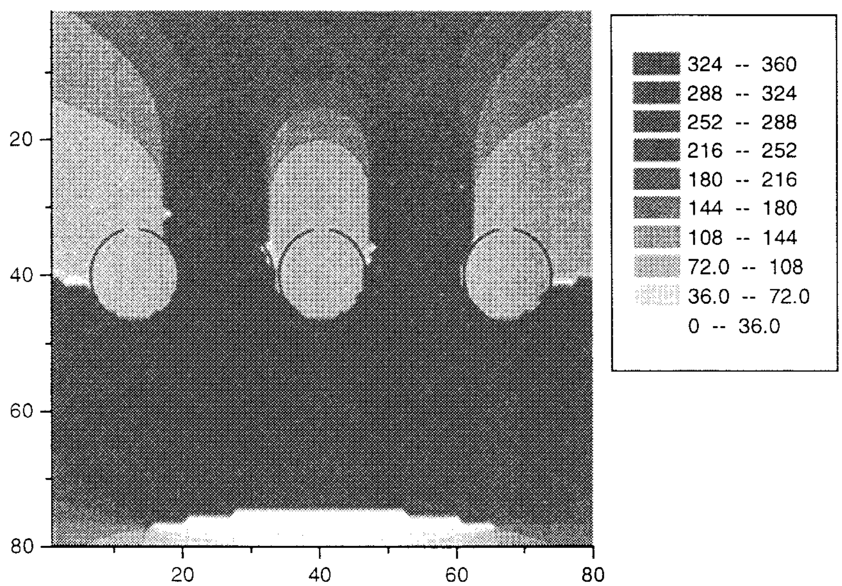

(b)

Fig. 6. (a) Amplitude distribution of the magnetic field ($\log|H_z|$) in the vicinity of the resonant structure described in Fig. 5, for $ka = 0.3919$. Total cross-section/$4a = 8.845$. (b) Phase distribution of the H_z field in the vicinity of a resonant structure described in Fig. 5 for $ka = 0.3919$.

modes leads to a sharp increase of the energy in the scattered field. This, in turn, causes a number of peaks to appear in the frequency response of the total and bistatic cross sections and also leads to a sharp increase in the electromagnetic energy in the near field of the structure.

The eigenfrequencies associated with a single-element split into a set of several when a finite array of these elements is considered. The Q factors of the corresponding modes are different and, typically, the real parts of these eigenfrequencies, i.e., the resonant frequencies of the structure, are in close proximity to each other.

Modes with higher Q factors are characterized by high levels of stored energy accompanied by high-field amplitudes in the structural elements. Consequently, a mode with a high Q factor will typically dominate, although it is possible to suppress a particular mode with a suitable choice of the incident field. For instance, it is obvious that no asymmetrical modes will be excited in the structure for the normal incidence ($\varphi_i = 0°$) case.

The frequency responses of the total and radar cross sections of a structure comprising seven slotted cylinders are presented in Fig. 2. The radii of the cylinders and the slot width are equal to each other ($\theta = 5°$). The slots are oriented at the same angle ($\xi = 90°$). The three maxima of the TCS in Fig. 2(a) correspond to the excitation of three eigenmodes in the scatterer. The strong coupling of the cylinders leads to a considerable difference of the eigenfrequencies and, consequently, to the possibility that the incident plane wave will excite them. Note that the Q factors of the modes are different.

An analysis of the near-field behavior shows that the resonance with the highest Q factor at the frequency $ka = 0.392\,88$ is associated with the excitation of the oscillation mode, which is characterized by a difference of π radians between the phases of the magnetic fields (H_z^{tot}) in the neighboring cylinders. This implies that coupling of energy occurs between the elements of the structure during the process of oscillation. The amplitude and phase behaviors of the near field at the above resonance are shown in Fig. 3.

The high level of the energy in the near zone and the phase reversal of the fields in the adjacent elements are the principal characteristics of superdirectivity and they are evident in the CBA under consideration. It should be mentioned, however, that the amplitude structure of the field also plays an important role in the process of achieving superdirectivity. For instance, by adjusting the coupling between the cylinders, say, by changing the angular orientation of the slots, the amplitude structure of the near field and, hence, the superdirectivity in the far field can be controlled.

The FFP for the scattered field is presented in Fig. 4 for $ka = 0.392\,88$, where a is the radius of the cylinder. It is evident from this figure that the equivalent aperture corresponding to the main lobe is greater than the physical dimensions of the array aperture. The increase in the effective aperture is attributed to an increase in the energy of the electromagnetic field in the vicinity of the structure at the resonant frequency. For comparison, we present in Fig. 4 (curve 2), the far-field pattern of an array of magnetic line sources that have equal phases and amplitudes and are located at the axes of the cylinders.

The presence of other resonances associated with the maxima of the RCS and TCS, shown in the Fig. 2 for $ka = 0.352$ and 0.383, can be attributed to the excitation of oscillation modes with complex phase structures. The first resonance occurs when the phases of the fields in three central cylinders are close to each other and these fields are approximately out-of-phase with those in the cylinders near the edges. Symbolically, the above phase structure can be depicted by $(++-\,-\,-++)$, whereas the second maximum of the TCS is associated with a modal phase structure depicted by $(+-\,-+-\,-+)$.

For a five-element array, the normally-incident wave efficiently excites two modes of oscillations in the frequency range under investigation. The frequency variations of the TCS and RCS of a slotted cylinder are presented in Fig. 5 for the following choice of parameters: $\theta_1 = \theta_2 = \cdots = \theta_5 = 5^\circ$; $\xi_1 = \xi_2 = \cdots = \xi_5 = 90^\circ$, the angle of incidence of the plane wave $\varphi_i = 0^\circ$, and the distance d between the cylinders equal to 2.1a, where a is the radius of the cylinders. The near-field structure at the frequency of the second resonance is presented in Fig. 6. The sharp increases of the energy in the scattered field at frequencies that correspond to the maxima of the TCS are associated with the excitation of the π-mode $(+-+-+)$. The first maximum of the TCS is related to the excitation of the $(++-++)$ mode, for which the field in the cylinder at the center is out-of phase with the rest of the elements. The scattered field patterns at both of the resonant frequencies exhibit the superdirective properties as evidenced by the plot in Fig. 7 for $ka = 0.3919$.

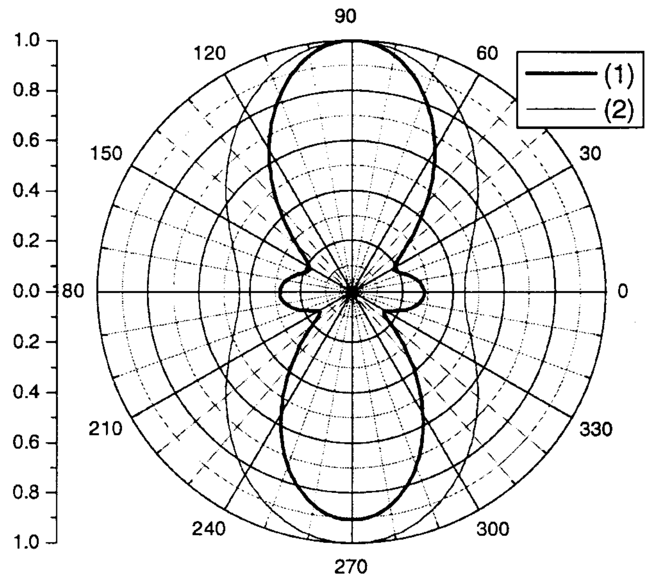

Fig. 7. Scattered FFP's of a resonant screen formed by five slotted cylinders. —— $ka = 0.3919$ and π-mode excitation that produces maxima in TCS and RCS; FFP of the field radiated by five isotropic line sources with equiphase excitation. The collinear sources are equally spaced with a separation of $0.130\,98\lambda$. $D(\varphi) = \sin[5(kd\cos\varphi)/2]/5\sin(kd\cos\varphi/2)$.

IV. Conclusions

In this paper, the scattering properties of a finite array of resonant elements, viz., thin cylinders with longitudinal slots, have been studied for H-polarized incident fields. It has been found that strong mutual coupling between the elements can introduce rather interesting characteristics as, for instance, superdirectivity in the FFP, even for a passive structure.

The resonant frequency is characterized by a differential phase shift of π radians between the neighboring elements, coupled with an amplitude behavior that causes the FFP of the scattered field to exhibit a relatively narrow main lobe, indicating an increase in the effective area of the aperture of the array. The superdirectivity effect can be enhanced by optimizing the parameters of the structure, viz., the distance between the cylinders and the angle of orientation of the slots (ξ_i).

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