Tropospheric Range-Error Corrections for the Global Positioning System

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Abstract— The global positioning system (GPS) is a highly accurate navigation system that has a broad spectrum of military, civilian, and commercial applications. It uses a triangulation scheme based on the time delays of signals from the satellites to the user; these time delays are then equated to distances. However, as the timing signal passes through the earth's atmosphere it undergoes an additional time delay due to the index of refraction. The time delay produced by the troposphere approaches a value corresponding to a range error of about 25 m for an elevation angle of 5° and decreases with increasing elevation angle to less than a few meters at zenith. It has been shown that there is a good correlation between the range error and the surface index of refraction. Worldwide statistics of surface refractivity have been analyzed and shown to be correlated with site latitude, height above sea level, and time of year. Regression lines for range-error corrections based on these parameters are derived. Range-error accuracies vary from about 8% down to 3.7% of the total range error, depending on the amount of information that is available.

Index Terms— Global positioning system.

I. INTRODUCTION

THE global positioning system (GPS) is a highly accurate navigation system that provides timing, velocity, and positioning information. It is a triservice system that evolved from four previous satellite systems: transit, a U.S. Navy system developed by Johns Hopkins Applied Research Lab; Timation, developed at the Naval Research Lab; Project 621B, an Air Force Study Program; and the Defense Navigation Satellite System. By 1972, the best features of these systems coalesced into the system previously known as NAVSTAR global positioning system, now known as simply GPS. It has a broad spectrum of military, civilian, and commercial users. It consists of 21 satellites plus three in-orbit spares in six orbital planes, each in a 10 900 nautical mile circular orbit with a 12 h period. It broadcasts continually at frequencies of 1227.6 and 1575.2 MHz. It uses a triangulation scheme based on the time delays of signals from the satellites to the user; the time delays are then equated to distances. However, as the timing signal passes through the earth's atmosphere it undergoes an additional time delay due to the finite index of refraction of the atmosphere. The larger part of this delay is due to the ionosphere; the other part is due to the troposphere. The additional ionospheric delay can produce a range error of over 30 m. However, because the ionosphere is dispersive, the index of refraction has a strong frequency dependence.

By operating the system at two frequencies, it is possible to accurately correct for the ionospheric range error.

The additional time delay produced by the troposphere is not frequency dependent, so the two-frequency approach does not help. This delay approaches a value corresponding to a range error of about 25 m for an elevation angle of 5° and decreases with increasing elevation angle to less than a few meters at zenith. In order to determine this additional delay, the index of refraction along the ray path must be known; in general, this is not possible. A number of models of the index of refraction as a function of altitude have been used for estimating the tropospheric range error. Based on these results it was shown that there is a good correlation between the range error and the surface index of refraction—a parameter that is generally available. In 1971, Altshuler [1] computed a set of range-error corrections as a function of the surface refractivity, the height of the user above sea level, and the elevation angle to the satellite. The equations used for these calculations could not be easily evaluated in real time with state of the art computers, so, in 1974, Altshuler and Kalaghan [2] converted the previous equations into a set of polynomial-type equations that were more computer friendly. In addition, they also examined world-wide surface refractivity statistics and showed that the refractivity is correlated with the site latitude, height above sea level, and time of year. In this paper, we present an algorithm for estimating the tropospheric range error from the user height above sea level, latitude, day of year, and elevation angle to the satellite from the GPS receiver. A sensitivity analysis of this algorithm is also presented.

II. REFRACTIVITY MODELS

The index of refraction n of the troposphere is only about 1.0003 at the earth's surface, so it is usually represented by a refractivity N where $N = (n-1) \times 10^6$. Numerous refractivity (N) profiles have been obtained over the years from samples measured directly with a refractometer or calculated from measurements of temperature, pressure, and absolute humidity. A number of models have been generated from these data for the purpose of correcting for tropospheric effects on propagation. Historically, the earth's radius model formulated by Schelleng *et al.* [3] was first used for line-of-sight communications problems. It was shown that by assuming an earth having a radius of about 4/3 times that of the actual earth, radio rays could be drawn as straight lines. The 4/3 earth model works well for propagation paths at low altitudes, where ray paths are within about 2 km of the earth's surface, but not for those at higher altitudes. From an examination of many years of N-

profile data for various climates it appears that N decreases linearly from the surface to an altitude of about 1 km above the earth's surface and then exponentially to an altitude of about 9 km at which the variation in refractivity is a minimum, with a range of about only 8 N units and an average value of 104.8 N units [4]. The rocket panel data [5], the ARDC model atmosphere [6], and work by Dubin [7] showed that above 9 km, refractivity again decreases exponentially. These data were used in formulating the model known as the CRPL reference atmosphere 1958 [4], which includes the following expressions for refractivity for three ranges of altitude:

$$N(h) = N_s(h - h_s)\Delta N, \quad h_s < h < h_s + 1 \text{ km}$$

where

$$\Delta N = -7.32 \exp(0.005577)N_s$$
$$N(h) = N_1 \exp[-c(h - h_s - 1)], \quad h_{s+1} < h < 9 \text{ km}$$

where

$$c = \frac{1}{8 - h_s} \ln \frac{N_1}{105}$$

and

$$N(h) = 105 \exp[-0.\ 1424(h - 9)], \qquad h > 9 \text{ km}$$

where

$N(h)$ refractivity at altitude h above sea level;
N_S surface refractivity;
N_1 refractivity at altitude 1 km above the surface;
h_S height of surface above sea level.

To simplify these expressions, the following single exponential (referred to as the CRPL exponential reference atmosphere 1958) was generated:

$$N(h) = N_S \exp[-c_e(h - h_S)]$$

where

$$c_e = \ln \frac{N_S}{N_S + \Delta N}.$$

Although this model is in good agreement with refractivity data below about 8 km, the values it gives at higher altitudes are too low. For example, at an altitude of 9 km the average refractivity is about 105, with minimum and maximum values of 100 and 108, respectively; but, at this altitude, the exponential reference atmosphere gives a value of only 85 for $N_S = 313$ (average surface refractivity in the United States). This model is, therefore, not expected to be very useful for range-error calculations. Hopfield [8] noted that if the refractivity as a function of height is represented by an exponential, it is not integrable in closed form, whereas if it has the form

$$N = k(h_{0\mu} - h)^\mu, \qquad h \leq h_0$$

it is integrable. She showed that representing the dry and wet terms of refractivity by quartic equations ($\mu = 4$) gave good agreement with range error and doppler data [9].

A number of other models of refractivity have been proposed and are summarized in Parkinson and Spilker [10]. They are, for the most part, variations of those mentioned above and

will not be discussed. Probably the best source of refractivity data is the CRPL standard atmospheric radio refractive index sample [11]. It consists of a sample of 77 N profiles selected from thirteen radiosonde stations representative of the major geographic and climatic types of the world.

III. FORMULATION OF SIMPLIFIED EXPRESSION FOR RANGE ERROR

Range errors were calculated for each of the 77 profiles as a function of elevation angle using ray-tracing and least-square regression lines of the form $\Delta R_e = a + bNS$ and were obtained as a function of angle. For elevation angles above 5°, range errors calculated from the CRPL reference atmosphere 1958 were found to be in good agreement with those obtained from this regression line [12]. Millman [13] used the CRPL reference atmosphere 1958 and a ray-tracing program to generate extensive curves for range error as a function of elevation angle, surface refractivity, and height. By selecting a model consisting of stratified layers only 50 m thick from ground level to an altitude of 30 km and taking into account refractive bending, he was able to compute very accurate range errors. Our objective is to generate a simple expression that will provide range errors in close agreement with those computed by Millman [13] for elevation angles of 5° and above. Bean and Dutton [4] had shown that the total range error based on the CRPL standard sample is of the form

$$\Delta R_e = \csc \theta_0(1.4588 + 0.0029611N_s) \tag{1}$$

where

ΔR_e range error in meters;
θ_0 elevation angle;
N_s surface refractivity.

A comparison of range errors obtained from (1) with those computed by Millman [13] (see Fig. 1) shows that the differences increase approximately exponentially with decreasing elevation angle for constant surface refractivity. Plotted as a function of refractivity for constant elevation angle (Fig. 2) the differences are shown to be quadratic in form. Hence, subtracting a term of the form

$$\Delta(\theta, N_s) = \Delta[c_1\theta^{c_2}, c_3(N_s - c_4)^2 + c_5]$$

from (1) provides a best fit to the curve. As shown in Appendix A

$$\Delta(\theta, N_s) = [0.001786(N_s - 360)^2 + 89.6]\theta^{-2.30}. \tag{2}$$

Therefore

$$\Delta R_{eo} = \frac{1.4588 + 0.002961N_s}{\sin \theta} - [0.001786(N_s - 360)^2 + 89.6]\theta^{-.230} \tag{3}$$

where

ΔR_{eo} range error in meters;
θ elevation angle in degrees;
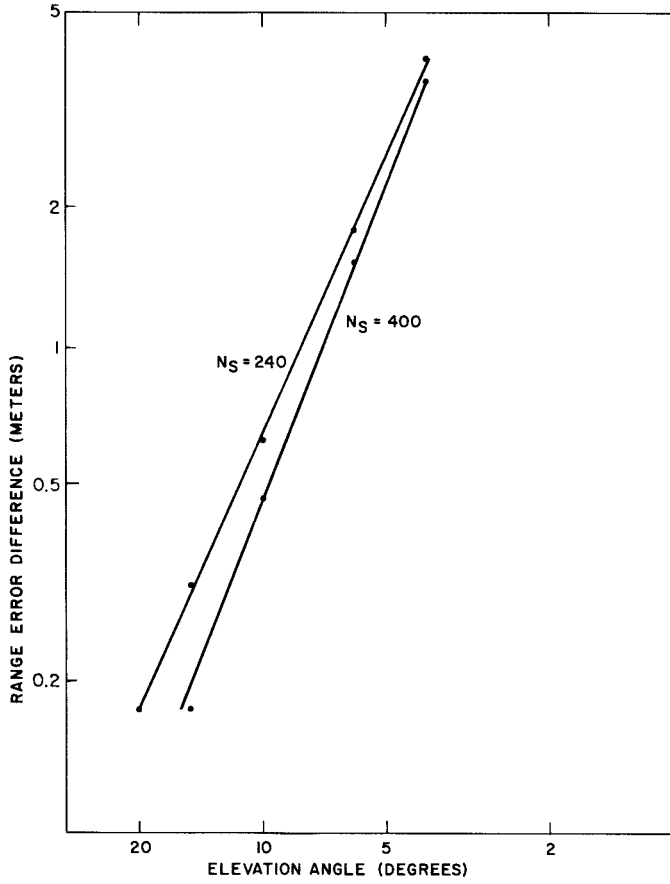N_s refractivity in N units at the surface.

Fig. 1. Differences between approximate and more accurate expressions for range error as a function of elevation angle for constant surface refractivity.
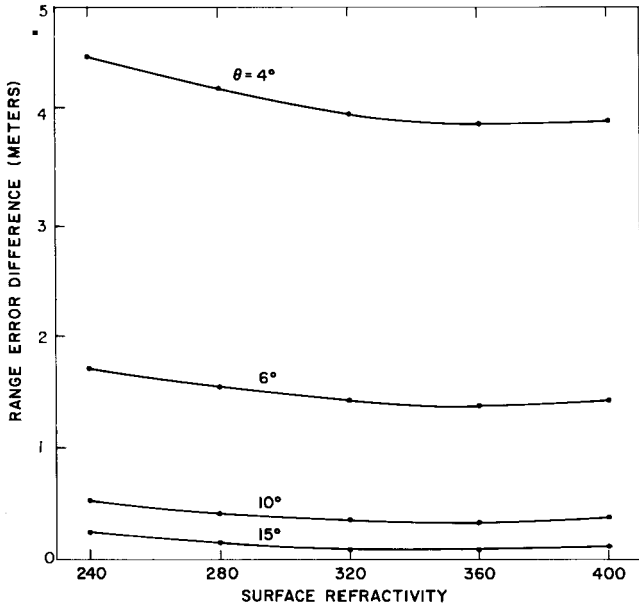

Fig. 2. Differences between approximate and more accurate expressions for range error as a function of surface refractivity for constant elevation angle.

Fig. 3. Range error as a function of altitude for selected angles and surface refractivities.

Equation (3) yields range errors from the earth's surface through the total troposphere that are within 0.2 m of those computed by Millman [13]. The next step is to obtain an expres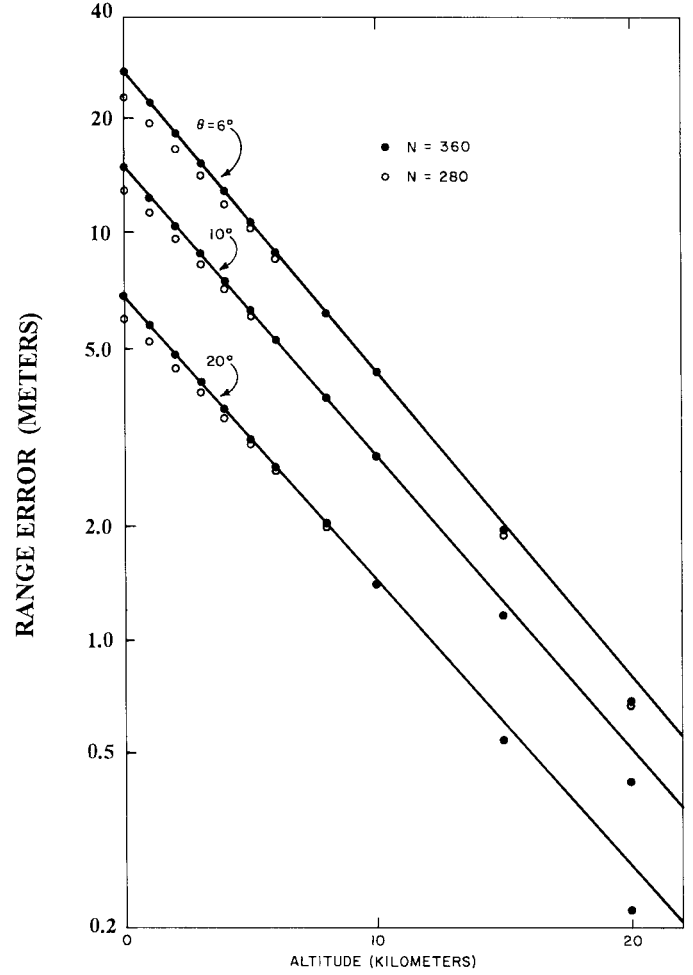sion for range error for an aircraft to the satellite. Range errors for a path from sea level to an aircraft at altitude h have been computed by Millman [14]. From these results, it is possible to estimate the range error for the path between an aircraft and a satellite. For example, if the total range error from sea level to a satellite is ΔR_{e0} and the range error from sea level to an aircraft at altitude h is $\Delta R'_{e}$, then the range error ΔR_{eh} for an aircraft at altitude h to the satellite is approximately $\Delta R_{eh} = \Delta R_{e0} - \Delta R'_{e}$. From Fig. 3 where Millman's [14] range errors are plotted as a function of altitude for several elevation angles and surface refractivities of $N = 280$ and $N = 360$, it can be seen that the height dependence of range error is almost exponential at low altitudes but falls off more rapidly at higher altitudes. In Appendix B it is shown that the range error from an aircraft at altitude h through the total troposphere to a satellite can be approximated by

$$\Delta R_{eh} = \Delta R_{eo} \exp \left\{ - \left[(0.0002N_S + 0.070)h + \left(\frac{0.83}{N_S} - 0.0017 \right) h^2 \right] \right\}.$$

(4)

Equation (4) yields range errors that agree with Millman's values to within ±0.2 m. It has now been shown that range

errors in excellent agreement with those obtained based on the CRPL reference atmosphere 1958 can be computed quite easily from (4) for elevation angles of 5° and above. It has been assumed that the surface refractivity is known. The accepted expression for the calculation of refractivity from temperature, pressure, and water vapor pressure is

$$N(h) = 77.6P(h)/T(h) + 3.73 \times 10^{-5}e(h)/[T(h)]^2$$

where

$P(h)$ pressure in millibars at altitude h;
$T(h)$ temperature in degrees Kelvin at altitude h;
$e(h)$ water vapor pressure in millibars at altitude h.

$P(h) = $ pressure in millibars at altitude h,

$T(h) = $ temperature in degrees Kelvin at altitude h,

$e(h) = $ water vapor pressure in millibars at altitude h.

For an aircraft at altitude h, the refractivity is related to the surface refractivity by

$$N(h) = N_s \exp(-h/H)$$

where H is a scale height of about 7 km [4].
Therefore

$$N(h) = N_s \exp(-.143h)$$

where h is the height above sea level in kilometers.

If the surface refractivity is not known, it must be estimated. A statistical analysis of our surface refractivity data showed that the average global surface refractivity is 324.8 N-units and that the standard deviation of this sample is 30.1 N-units. A further analysis of the data showed that N_s is correlated with site latitude and day of year. The following regression line of N_s as a function of these parameters can therefore be derived:

$$N_s = 369.0 - |L|[.9244 - .1936\sin^2(\pi d_{\mathrm{mod}}/365)$$
$$+ .0596\cos^2(\pi d_{\mathrm{mod}}/365)] \qquad (5)$$

where

N_s surface refractivity;
d day of year;
L latitude
     if $L \geq 0$, then $d_{\mathrm{mod}} = d$
     if $L < 0$, then $d_{\mathrm{mod}} = \mathrm{mod}((d = 182), 365)$.

This expression describes the worldwide variation of N_s and has a standard error of 17.0 N-units, approximately 5% of the mean N_s value.

For illustration, monthly surface refractivities are tabulated in Table I. for the northern hemisphere. It is seen that the surface refractivity ranges from a low of about 280 near the pole to 369 N-units at the equator. The corresponding range errors through the troposphere for an elevation angle of 5° range from 23.8 to 27.1 m. Thus, there is a difference of over 3 m, depending on the latitude of the GPS receiver. Typical range errors as a function of elevation angles, height of the receiver above sea level, elevation angle to the satellite, and surface refractivity are shown in Table II. Note the decrease in range error with increasing height and elevation angle.

TABLE I
SURFACE REFRACTIVITIES AS A FUNCTION OF LATITUDE AND MONTH

M\L	0	10	20	30	40	50	60	70	80	90
JAN	369	359	349	340	330	320	310	300	291	281
FEB	369	360	350	341	331	322	312	303	293	284
MAR	369	360	351	342	333	324	315	306	297	288
APR	369	361	352	344	336	328	319	311	303	294
MAY	369	361	354	346	338	330	323	315	307	300
JUN	369	362	354	347	340	332	325	318	310	303
JUL	369	362	354	347	340	332	325	318	310	303
AUG	369	361	354	346	338	331	323	315	308	300
SEP	369	361	353	344	336	328	320	311	303	295
OCT	369	360	351	342	334	325	316	307	298	290
NOV	369	360	350	341	331	322	312	303	294	284
DEC	369	359	350	340	330	320	310	300	291	281

TABLE II
RANGE-ERROR CORRECTIONS AS A FUNCTION OF ELEVATION
ANGLE, HEIGHT ABOVE SEA LEVEL, AND SURFACE REFRACTIVITY

N_s =280

h / θ	5	10	20	60	90
0	23.78	12.70	6.59	2.63	2.29
5	12.34	6.58	3.42	1.37	1.19
10	6.08	3.24	1.68	0.67	0.58
20	1.27	0.68	0.35	0.14	0.12

N_s =320

h / θ	5	10	20	60	90
0	25.35	13.40	6.95	2.77	2.40
5	12.77	6.75	3.50	1.39	1.21
10	6.21	3.28	1.70	0.68	0.59
20	1.34	0.71	0.37	0.14	0.12

N_s =360

h / θ	5	10	20	60	90
0	26.78	14.10	7.30	2.91	2.52
5	13.06	6.88	3.56	1.42	1.23
10	6.24	3.29	1.70	0.68	0.59
20	1.35	0.71	0.37	0.15	0.13

IV. DISCUSSION OF ERRORS

It is extremely difficult to determine the accuracy with which tropospheric range error can be estimated since it is not practicable to measure range error directly. Although range error can be accurately calculated by means of ray tracing techniques, if the refractivity along the path is known, it is not always possible to ascertain atmospheric refractivity since it is variously affected by winds, clouds, and precipitation. In addition, the assumption that the atmosphere is horizontally stratified is not always valid. It is particularly difficult to accurately measure refractivity at the higher altitudes. There is no uniformity of sensors used in radiosondes; although temperature and pressure sensors generally show fair agreement, humidity sensors are erratic [4]. Refractometers are more capable of accurately measuring refractivities, but they are relatively expensive and complex to operate. Also it should be emphasized that the correlations of range error were with surface refractivity while the GPS altitude corresponds to height above sea level. Fortunately, there are not too many locations for which this difference is significant. With these

TABLE III
UNCERTAINTY IN RANGE ERROR DUE TO
UNCERTAINTY IN SURFACE REFRACTIVITY (m/N-Unit)

θ \ N_s	240	260	280	300	320	340	360	380	400
4	0.067	0.057	0.054	0.051	0.048	0.045	0.042	0.040	0.037
5	0.044	0.043	0.041	0.039	0.037	0.036	0.034	0.032	0.030
6	0.035	0.034	0.033	0.032	0.031	0.030	0.028	0.027	0.026
7	0.029	0.028	0.027	0.027	0.026	0.025	0.024	0.0230	0.023
8	0.025	0.024	0.024	0.023	0.023	0.022	0.021	0.021	0.020
9	0.022	0.021	0.021	0.020	0.020	0.020	0.019	0.019	0.018
10	0.019	0.019	0.019	0.018	0.018	0.017	0.017	0.017	0.016
12	0.016	0.016	0.015	0.015	0.015	0.015	0.014	0.014	0.014
14	0.013	0.013	0.013	0.013	0.012	0.012	0.012	0.012	0.012
16	0.012	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.010
18	0.010	0.010	0.010	0.010	0.010	0.010	0.009	0.009	0.009
20	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
25	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
30	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
40	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
50	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
60	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
70	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
80	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
90	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003

TABLE IV
STANDARD DEVIATION OF RANGE-ERROR CORRECTIONS
AS A FUNCTION OF INFORMATION AVAILABLE

θ	A	B	C	D	Avg ΔR_e
5	0.90	1.56	1.80	2.06	25.54
6	0.79	1.33	1.53	1.74	21.70
7	0.70	1.16	1.32	1.50	18.84
8	0.62	1.02	1.17	1.32	16.64
9	0.57	0.91	1.04	1.18	14.90
10	0.51	0.82	0.94	1.06	13.47
12	0.43	0.69	0.78	0.88	11.34
14	0.37	0.59	0.67	0.76	9.81
16	0.33	0.52	0.59	0.66	8.63
18	0.29	0.46	0.52	0.59	7.71
20	0.26	0.42	0.48	0.53	6.98
25	0.21	0.34	0.38	0.43	5.67
30	0.18	0.28	0.32	0.37	4.82
40	0.13	0.21	0.24	0.27	3.75
50	0.11	0.18	0.21	0.23	3.14
60	.010	0.16	0.18	0.20	2.77
70	0.09	0.15	0.16	0.19	2.56
80	0.09	0.14	0.16	0.18	2.44
90	0.09	0.14	0.16	0.18	2.41
$\sigma_{\Delta Re}/_{\Delta Re}$	0.037	0.061	0.070	0.080	

A: N_s known
B: N_s unknown; latitude, altitude and day of year known
C: N_s unknown; altitude and day of year known
D: No information

limitations in mind, we proceed to examine the range-error corrections.

The standard errors in range error, calculated from the standard errors in surface refractivity by using (4), are listed in Table III as a function of surface refractivity N_s and elevation angle θ. These tabulations represent only the uncertainties that are due to the statistical variation in surface refractivity. Even when the surface refractivity is known, the model has an inherent error, which must also be included. Unfortunately, the accuracy with which range error can be corrected on the basis of the CRPL reference atmosphere 1958 is extremely difficult to determine. Norton [11] has calculated range errors for the CRPL standard sample and plotted the standard deviation of the normalized range error $\Delta R_e/R_e$ for cases for which the surface refractivity is both known and unknown. If it is assumed that the standard deviation of the range error for the CRPL reference atmosphere 1958 is the same as for the CRPL standard sample, then the accuracy of the range-error correction for a horizontally stratified atmosphere can be estimated. Errors due to atmospheric anomalies such as a nonstratified atmosphere, clouds, and precipitation must also be taken into account; these are estimated to be about 2% of the total range error.

The standard errors of range-error corrections are listed in Table IV as a function of elevation angle and amount of information available. It is seen that if N_s is known, the standard error is approximately 3.7% of the total value of the range-error correction. If N_s is not known, but the latitude, height above sea level, and day of year are known, then the standard error increases to slightly more than 6%. If the latitude of the observer is unknown, the standard error is about 7%. If only the global surface refractivity of 324.8 N units is used, then the standard error increases to about 8% of the range-error correction.

For the GPS, for the purpose of estimating the surface refractivity, only the day of year and latitude are given. Since the surface refractivity is a function of the height of the

surface above sea level, an error will arise for locations such as Denver, CO, which is approximately 1625 m above sea level. Equation (8) would estimate the surface refractivities for Denver to be about 330 and 340 N-units for the 15th (winter) and 195th (summer) days of the year; more accurate values are 250 and 267 N-units, respectively. As a result, the corresponding range errors for an elevation angle of 5° would be overestimated by about 10% of the total range error. This type of error could be avoided if both longitude and latitude were used, thus making it possible to identify those locations that are significantly above sea level and including them in the algorithm.

V. CONCLUSIONS

Tropospheric range-error corrections based on a global average surface refractivity of 324.8 N units have a standard error of about 8% of the total refractive range error. Since the range error from the surface to the satellite varies from about 25 m for an elevation angle of 5° to about 2 m for angles close to zenith, standard errors of about 2–0.2 m would be expected if all systems throughout the world used corrections based on elevation angle only. It has been shown statistically that range-error corrections based on a regression model that is a function of site latitude, height above sea level, day of year, and elevation angle show a standard error of approximately 6% of the total range error. Thus, a 25% improvement in the accuracy of the range-error correction can be obtained by simply using information that can readily be obtained for any user location. If the height of the surface at which the user is located is not used, then the error in estimating the surface refractivity for regions that are at a high elevation can produce a significant error in the surface refractivity estimate,

which will in turn produce a corresponding overestimation of the range error. However the range-error correction can be further improved if the surface refractivity at the user location is known; the standard error of the range-error correction for this case is only about 3.7% of the total correction.

It must be emphasized that our results are global and applicable for any location. If the user is confined to a region within which the climatology does not vary appreciably from point to point, then more accurate range-error corrections can be derived from refractivity statistics for that region.

APPENDIX A
CORRECTION OF RANGE ERROR FROM SEA LEVEL

As seen in Fig. 1, the range-error difference Δ has an elevation angle dependence that is approximately linear on a log–log scale. Therefore, Δ should have the form

$$\ln \, \Delta(\theta, N_s \text{ constant}) = \ln \, c_1 + c_2 \, \ln \, \theta$$

or

$$\Delta(\theta, N_s \text{ constant}) = c_1 \theta^{c_2}.$$

With $N = 240$

for $\theta = 6°$: $\ln \, 1.8 = \ln \, c_1 + c_2 \, \ln \, 6°$

for $\theta = 10°$: $\ln \, 0.55 - \ln \, c_1 + c_2 \, \ln \, 10°$.

Solving for c_1 and c_2 yields

$$c_1 = 115$$
$$c_2 = -2.30.$$

Therefore

$$\Delta(\theta, N_s = 240) = 115 \, \theta^{-2.30}.$$

In Fig. 2, the range-error difference was seen to have a refractivity dependence that is slightly quadratic with a minimum at about 360. Therefore, $\Delta(N_s, \theta \text{ constant})$ should have the form

$$\Delta(N_s, \theta \text{ constant}) = c_3(360 - 280)^2 + c_4.$$

With $\theta = 6°$

for $N_s = 280$: $1.62 = c_3(360 - 280)^2 + c_4$

for $N_s = 360$: $1.43 = c_4$.

Therefore

$$c_3 = 2.86 \times 10^{-5}$$
$$\Delta(N_s, \theta = 6°) = 2.86 \times 10^{-5}(N_s - 360)^2 + 4.7.$$

To combine these results into a single expression noting that Δ has a stronger θ-dependence than N-dependence, let it be required that

$$c_5[\Delta(N_s, \theta = 6°)] = 115, \quad \text{for } N = 240$$
$$c_5[2.86 \times 10^{-5}(240 - 360)^2 + 1.43] = 115.$$
$$c_5 = 62.5.$$

Then

$$\Delta(\theta, N_s) = [0.00178(N_s - 360)^2 + 89.4]\theta^{-2.30}.$$

APPENDIX B
CORRECTION OF RANGE ERROR FROM AN AIRCRAFT

Range errors plotted versus altitude on semilog paper (Fig. 3) are seen to be almost linear, which indicates an exponential altitude dependence. To compensate for the nonlinearity at the higher altitudes, an additional term is added. The expression for range error as a function of height is then

$$\ln \, \Delta R_{eh} = \ln \, \Delta R_{e0} + c_1 h + c_2 h^2.$$

The term c_1 is determined from values of ΔR_{eh} at $h = 0$ and $h = 2$ km; c_2 is determined from values of R_{eh} at $h = 0$ and $h = 20$ km. Thus

$$c_1 = [\ln \, \Delta R_{e2} - \ln \, \Delta R_{e0}]/2$$
$$c_2 = [\ln \, \Delta R_{e20} - \ln \, \Delta R_{e0}]/400.$$

These have been computed for $N_s = 280$ and $N_s = 360$ at five elevation angles as follows:

	$Ns = 280$		$Ns = 360$	
	$C1$	$C2$	$C1$	$C2$
6	-0.130	-0.0012	-0.143	-0.0007
10	-0.126	-0.0012	-0.143	-0.0006
15	-0.123	-0.0014	-0.141	-0.0006
20	-0.122	-0.0014	-0.140	-0.0004
30	-0.121	-0.0012	-0.139	$-0.0009.$

On the basis of the above results, the constants

for $N_s = 280$ are taken as: $c_1 = -0.126, c_2$
$$= -0.0013$$
for $N_s = 360$ are taken as: $c_1 = -0.142, c_2 = -0.0006.$

Since these constants vary slightly with N_s, a linear dependence is assumed as follows:

$$280a_1 + a_0 = -0.126$$
$$360a_1 + a_0 = -0.142$$
$$a_0 = -0.07, a_1 = -0.0002$$
$$b_1/280 + b_0 = -0.0013$$
$$b_1/360 + b_0 = -0.0006$$
$$b_0 = -0.0017, b_1 = -0.83$$
$$c_1 = -0.07 - 0.0002N_s$$
$$c_2 = -0.83 + 0.0017N_s$$

$a_0, a_1, b_0,$ and b_1 are regression coefficients

$$\Delta R_{eh} = \Delta R_{eo} \exp - \left[(0.0002N_S + 0.070)h + \left(\frac{0.83}{N_S} - 0.0017 \right) h^2 \right]$$

where h is in kilometers.

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