An Efficient Technique for Eigenspace-Based Adaptive Interference Cancellation

Cheng-Chou Lee and Ju-Hong Lee, *Member, IEEE*

***Abstract*—This paper presents a technique for the computation of the interference subspace for eigenspace-based interference cancellation. Using a subarray partitioning scheme, we construct the interference subspace from the subarray interference subspaces. In the case of uniform linear arrays, the proposed technique has the advantages of reclaiming the lost degrees of freedom due to signal blocking and reduced computational burden over existing techniques. The proposed technique also possesses the capabilities to cope with the case of using nonuniform linear arrays in the environment of partially correlated signals. A computer simulation example is provided for illustration and comparison.**

***Index Terms*—Adaptive arrays.**

I. Introduction

ADAPTIVE interference cancellation is usually required for maximizing the rejection of interference regardless of the interference-to-noise power ratio (INR) when processing array data. There are several eigenspace-based (ESB) interference cancellers presented in the literature [1]–[6]. However, the methods presented in [1] and [2] are only suitable for the case without the desired signal. The method of [3] is developed under the environment of two signal sources without correlation, whereas the methods of [4] and [5] work only for the situation where a uniform linear array (ULA) is used. As to the method of [6], correlation between signal sources is not allowed.

Moreover, several major problems in implementing the ESB interference cancellation using the above methods must be considered. In addition to the loss of degrees of freedom when using a blocking matrix to eliminate the desired signal, the interference canceller of [4] suffers the expansive computing cost for performing the required generalized eigenvalue decomposition (EVD) on the correlation matrix of the blocked data vector in order to obtain the interference subspace (IS). Based on the method of [5], the directional angles of the interferers must be estimated before we can determine the required IS. On the other hand, under the case of using an array with arbitrary geometry as that considered in [6], the major problem is the difficulty of removing the component associated with the desired signal from the received data correlation matrix in order to compute the required IS. The authors of [6] resorted to an iterative process for the computation of the IS.

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The authors are with the Department of Electrical Engineering, National Taiwan University, Taipei 106, Taiwan.

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In this paper, we present an efficient technique to overcome the restrictions and drawbacks in utilizing the above existing methods. Based on the observation that a vector orthogonal to the IS of a subarray of the original array is also orthogonal to the original IS after it is appended by zero entries, we first present a subspace construction technique for the computation of the IS spanned by the received array data vector. Based on the partition of the original array into several overlapped subarrays, the IS's spanned by the subarray data vectors are first computed and then used to construct the original IS for finding the optimal weight vector. It is shown that the proposed technique can alleviate the drawbacks like loss of degrees of freedom and heavy computational burden when using existing ESB adaptive interference cancellers with a ULA. We find that the computational complexity required by using the proposed technique is much less than that required by using the existing method presented in [4]. Modifications for the proposed technique are also presented for dealing with the case of using a nonuniform linear array in the environment of partially correlated signals.

This paper is organized as follows. Section II briefly describes ESB adaptive interference cancellation (ESB-AIC). Based on a subarray partitioning scheme, a technique for constructing the IS is presented in Section III. The required computational complexity is also evaluated. Section IV presents the modifications required for performing ESB-AIC by using the proposed technique under some considered situations. A simulation example for showing the effectiveness of the proposed technique is presented in Section V. We finally conclude this paper in Section VI.

II. The ESB Adaptive Interference Cancellation (ESB-AIC)

Consider a linear array with M-sensor elements illuminated by P narrowband signal sources. The received signal at the mth sensor element can be expressed as

$$x_m(t) = \sum_{i=1}^{P} s_i(t) a_m(\theta_i) + n_m(t) \tag{1}$$

where $s_i(t)$ is the ith signal impinging on the array with direction angle θ_i; $n_m(t)$ is the received noise. Both the signal and sensor noise are assumed to be uncorrelated and zero-mean Gaussian random processes. $a_m(\theta_i)$ represents the response of the mth sensor to a signal with unit amplitude and direction angle θ_i. In vector form, (1) becomes

$$X(t) = \sum_{i=1}^{P} A(\theta_i)s_i(t) + N(t) = \boldsymbol{A}_s S(t) + N(t) \quad (2)$$

where the response vector of the ith signal $A(\theta_i) = [a_1(\theta_i),\ a_2(\theta_i),\ \cdots, a_M(\theta_i)]^T$, the noise vector $N(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T$, the signal source vector $S(t) = [s_1(t), s_2(t), \cdots, s_P(t)]^T$, and the response matrix of signal sources $\boldsymbol{A}_s = [A(\theta_1), A(\theta_2), \cdots, A(\theta_P)]$. The ensemble correlation matrix of $X(t)$ is given by

$$\boldsymbol{R} = E\{X(t)X^H(t)\} = \boldsymbol{A}_s \Psi_s \boldsymbol{A}_s^H + \Sigma_n \quad (3)$$

where $\Sigma_n = E\{N(t)N^H(t)\}$. $\Psi_s = E\{S(t)S^H(t)\}$ with its (i,j)th entry given by

$$\Psi_{ij} = \begin{cases} \pi_i, & \text{for } i = j \\ \sqrt{\pi_i \pi_j}\rho_{ij}, & \text{for } i \neq j. \end{cases} \quad (4)$$

where ρ_{ij} denotes the correlation coefficient between the ith and jth signal sources. Without loss of generality, let the first signal of the P signal sources be the desired signal and the other $J = P - 1$ be the interferers. Thus, the response matrix associated with interferers is given by

$$\boldsymbol{A}_j = [A(\theta_2), A(\theta_3), \cdots, A(\theta_p)] \quad (5)$$

The signal subspace (SS) and the interference subspace (IS) can be designated as $\Pi_s = \text{range}\{\boldsymbol{A}_s\}$ and $\Pi_j = \text{range}\ \{\boldsymbol{A}_j\}$, respectively. Moreover, let the complements of the SS and IS be represented by SSC and ISC, respectively and the basis matrices spanning the SS, IS, SSC, and ISC be denoted as $\boldsymbol{G}_s, \boldsymbol{G}_j, \boldsymbol{G}_n,$ and $\boldsymbol{G}_r$, respectively.

Consider the ESB-AIC. The optimal weight vector which minimizes the array output power with a constraint of unit gain in the direction of the desired signal, i.e., $A(\theta_1)$ and a constraint of the optimal weight vector orthogonal to the IS can be obtained by solving the following optimization problem:

$$\begin{aligned} &\text{Minimize} && W^H \boldsymbol{R} W \\ &\text{Subject to} && W^H A(\theta_1) = 1 \quad \text{and} \quad W \in \text{range}\{\boldsymbol{G}_r\}. \end{aligned} \quad (6)$$

The optimal solution for (6) is given by

$$W_o = \kappa \boldsymbol{G}_r (\boldsymbol{G}_r^H \boldsymbol{R} \boldsymbol{G}_r)^{-1} \boldsymbol{G}_r^H A(\theta_1) \quad (7)$$

where κ denotes the normalization constant. Note that the objective function $W^H \boldsymbol{R} W$ of (6) can be replaced by $W^H \Sigma_n W$ if the characteristics of the noise vector $N(t)$ are known *a priori* since the optimal weight must be such a solution that the resulting ESB adaptive interference canceller provides zero gain for all interferers and unit gain for the desired signal. Hence, the solution given by (7) becomes

$$W_o = \kappa \boldsymbol{G}_r (\boldsymbol{G}_r^H \Sigma_n \boldsymbol{G}_r)^{-1} \boldsymbol{G}_r^H A(\theta_1). \quad (8)$$

After some necessary algebraic manipulations, an equivalent expression for (8) is given by

$$W_o = \kappa(\Sigma_n^{-1} - \Sigma_n^{-1}\boldsymbol{G}_j(\boldsymbol{G}_j^H \Sigma_n^{-1} \boldsymbol{G}_j)^{-1} \boldsymbol{G}_j^H \Sigma_n^{-1}) A(\theta_1). \quad (9)$$

We note that $(\boldsymbol{G}_j^H \Sigma_n^{-1} \boldsymbol{G}_j)^{-1}$ is the inverse of a $J \times J$ matrix and $(\boldsymbol{G}_r^H \Sigma_n \boldsymbol{G}_r)^{-1}$ is the inverse of an $(M\text{-}J) \times (M\text{-}J)$ matrix. Therefore, finding the optimal weight vector from (9) requires much less computational complexity than from (8) if $M \gg J$. Moreover, (9) can be further reduced to

$$W_o = \kappa'(\boldsymbol{I}_M - \boldsymbol{G}_j(\boldsymbol{G}_j^H \boldsymbol{G}_j)^{-1} \boldsymbol{G}_j^H) A(\theta_1) \quad (10)$$

if the sensor noises are spatially white, i.e., $\Sigma_n = \pi_n \boldsymbol{I}_M$, where $\boldsymbol{I}_M$ denotes the identity matrix with size $M \times M$, π_n the noise variance, and κ' the resulting normalization constant. As shown in [5], the performance of an ESB adaptive interference canceller is usually evaluated in terms of its output signal-to-interference plus noise ratio (SINR) which is given by

$$\text{SINR} = \frac{\pi_1 |W_o^H A(\theta_1)|^2}{W_o^H(\boldsymbol{A}_j \Psi_j \boldsymbol{A}_j^H + \Sigma_n) W_o} \quad (11)$$

where Ψ_j denotes the correlation matrix of the interferers $s_i(t)$ for $i = 2, 3, \cdots, P$.

III. Computation of Interference Subspace by Overlapped Subarrays

Let the original M-element array be partitioned into K subarrays. Assume that the kth subarray has M_k $(M_k > J)$ array elements beginning with the $(M_k^- + 1)$th sensor element and ending with the M_k^+th sensor element. Hence, $M_k^+ - M_k^- = M_k$. Construct a row selecting matrix $\boldsymbol{J}_k$ as follows:

$$\boldsymbol{J}_k = [u_{M_k^-+1}, u_{M_k^-+2}, \cdots u_{M_k^+}]^T \quad (12)$$

where u_m denotes the mth column vector of $\boldsymbol{I}_M$. Then, the data vector $X_k(t)$ of size $M_k \times 1$ received by the kth subarray can be expressed as

$$X_k(t) = \boldsymbol{J}_k X(t) = \boldsymbol{A}_{sk} S(t) + N_k(t) \quad (13)$$

where $\boldsymbol{A}_{sk} = \boldsymbol{J}_k \boldsymbol{A}_s$ is the response matrix and $N_k(t) = \boldsymbol{J}_k N(t)$ is the noise vector associated with the kth subarray, respectively. Let $\boldsymbol{A}_{dk} = [A_k(\theta_1)]$ and $\boldsymbol{A}_{jk} = [A_k(\theta_2), \cdots, A_k(\theta_P)]$ represent the response matrices of the kth subarray due to the desired signal and the interferers, respectively, where $A_k(\theta_i) = \boldsymbol{J}_k A(\theta_i)$. Consider the case that the two submatrices which contain the first J rows and the last J rows of $\boldsymbol{A}_{jk}$, respectively, are full rank. (This is called the unambiguity condition.) Assume that the basis matrices of the the IS and ISC spanned by $X_k(t)$ are designated as the full rank matrices $\boldsymbol{G}_{jk}$ and $\boldsymbol{G}_{rk}$, respectively. Accordingly, we have

$$\text{range}\{\boldsymbol{G}_{jk}\} = \text{range}\{\boldsymbol{A}_{jk}\}, \quad \text{and} \quad \boldsymbol{G}_{rk}^H \boldsymbol{A}_{jk} = 0. \quad (14)$$

It follows from (14):

Lemma 1: Range $\{\boldsymbol{J}_k^T \boldsymbol{G}_{rk}\} \subseteq \text{range}\{\boldsymbol{G}_r\}$.

This lemma states that the vectors of $\boldsymbol{J}_k^T \boldsymbol{G}_{rk}$ are contained in the ISC of $X(t)$. As a result, a basis matrix which spans the same subspace as that spanned by $\boldsymbol{G}_r$ can be established by finding the $(M\text{-}J)$ linearly independent vectors from the matrices $\boldsymbol{J}_k^T \boldsymbol{G}_{rk}$ for $k = 1, 2, \cdots, K$. To develop a method for constructing this basis matrix, we present a theorem as follows.

Theorem 1: Let the partition of the original array satisfy the following conditions:

$$\boldsymbol{D}1:\quad M_1^- = 0 \quad \text{and} \quad M_1^+ = M_1$$

$$\boldsymbol{D}2:\quad M_k > J \quad \text{and} \quad \sum_{k=1}^{K}(M_k - J) = M - J$$

$$\boldsymbol{D}3:\quad M_k^- = M_{k-1}^+ - J \quad \text{and}$$
$$M_k^+ = M_k^- + M_k,\ \text{i.e.,}\ M_k^+ = M_{k-1}^+ + M_k - J.$$

Then, a full rank matrix constructed by

$$\overline{\boldsymbol{G}}_r = [\boldsymbol{J}_1^T\boldsymbol{G}_{r1}, \boldsymbol{J}_2^T\boldsymbol{G}_{r2}, \cdots \boldsymbol{J}_K^T\boldsymbol{G}_{rK}] \tag{15}$$

spans the ISC associated with the data correlation matrix $\boldsymbol{R}$ of (3).

Proof: First, let $\boldsymbol{H}_{k1}$ and $\boldsymbol{H}_{k2}$ be two row-selecting matrices, which contain the first J and the last $M_k - J$ rows of the $M_k \times M_k$ identity matrix, respectively. We then construct two matrices from $\boldsymbol{A}_{jk}$ as follows:

$$\boldsymbol{A}_{jk1} = \boldsymbol{H}_{k1}\boldsymbol{A}_{jk} \quad \text{and} \quad \boldsymbol{A}_{jk2} = \boldsymbol{H}_{k2}\boldsymbol{A}_{jk}. \tag{16}$$

Similarly, two matrices are constructed from $\boldsymbol{G}_{rk}$ as follows:

$$\boldsymbol{G}_{rk1} = \boldsymbol{H}_{k1}\boldsymbol{G}_{rk} \quad \text{and} \quad \boldsymbol{G}_{rk2} = \boldsymbol{H}_{k2}\boldsymbol{G}_{rk}. \tag{17}$$

Next, using the facts that $\boldsymbol{A}_{jk}^H\boldsymbol{G}_{rk} = 0$ and the assumption that $\boldsymbol{A}_{jk1}$ is full rank, we can easily show from (16) and (17) that

$$\boldsymbol{G}_{rk1} = -\boldsymbol{A}_{jk1}^{-H}\boldsymbol{A}_{jk2}^H\boldsymbol{G}_{rk2}. \tag{18}$$

Equation (18) reveals that each of the row vectors of $\boldsymbol{G}_{rk1}$ can be obtained from a linear combination of the row vectors of $\boldsymbol{G}_{rk2}$. This leads to that both $\boldsymbol{G}_{rk}$ and $\boldsymbol{G}_{rk2}$ have the same rank equal to $M_k - J$. Using the conditions $\boldsymbol{D}1$, $\boldsymbol{D}2$, and $\boldsymbol{D}3$, the matrix $\overline{\boldsymbol{G}}_r$ of (15) can be expressed as follows:

$$\overline{\boldsymbol{G}}_r = \begin{bmatrix} \boldsymbol{G}_{r11} & & & & \\ \boldsymbol{G}_{r12} & \boldsymbol{G}_{r21} & & \boldsymbol{0} & \\ & \boldsymbol{G}_{r22} & \ddots & & \\ & & \ddots & & \\ \boldsymbol{0} & & & \boldsymbol{G}_{r(K-1)1} & \\ & & & \boldsymbol{G}_{r(K-1)2} & \boldsymbol{G}_{rK1} \\ & & & & \boldsymbol{G}_{rK2} \end{bmatrix}. \tag{19}$$

The corresponding lower block triangular matrix of (19) has diagonal block matrices $\boldsymbol{G}_{rk2}$. It has been shown in [7] that the rank of a block triangular matrix is at least equal to the sum of the ranks of its diagonal block matrices. Since each of $\boldsymbol{G}_{rk2}$ is full rank equal to $M_k - J$, thus, the lower block triangular matrix of $\overline{\boldsymbol{G}}_r$ has rank given by $M - J$ which is the sum of $M_k - J$ for $k = 1, 2, \cdots, K$. Hence, the rank of $\overline{\boldsymbol{G}}_r$ is also equal to $M - J$. Therefore, it follows from Lemma 1 that the space spanned by the matrix $\overline{\boldsymbol{G}}_r$ is equal to the ISC spanned by $X(t)$. This completes the proof.

Similar to (16), we can construct two matrices from $\boldsymbol{G}_{jk}$ as follows:

$$\boldsymbol{G}_{jk1} = \boldsymbol{H}_{k1}\boldsymbol{G}_{jk} \quad \text{and} \quad \boldsymbol{G}_{jk2} = \boldsymbol{H}_{k2}\boldsymbol{G}_{jk}. \tag{20}$$

Moreover, let $\boldsymbol{H}_{k3}$ be a row-selecting matrix that contains the last J rows of the $M_k \times M_k$ identity matrix. One more submatrix is constructed from $\boldsymbol{G}_{jk}$ as follows:

$$\boldsymbol{G}_{jk3} = \boldsymbol{H}_{k3}\boldsymbol{G}_{jk}. \tag{21}$$

From (20) and (21), we construct a matrix as follows:

$$\boldsymbol{Q}_j = \begin{bmatrix} \boldsymbol{G}_{j13}^H & & & \boldsymbol{0} \\ -\boldsymbol{G}_{j21}^H & \boldsymbol{G}_{j23}^H & & \\ & -\boldsymbol{G}_{j31}^H & \ddots & \\ & & \ddots & \boldsymbol{G}_{j(K-1)3}^H \\ \boldsymbol{0} & & & -\boldsymbol{G}_{jK1}^H \end{bmatrix}. \tag{22}$$

Due to the unambiguity condition, all of $\boldsymbol{G}_{jk1}$ and $\boldsymbol{G}_{jk3}$ in (22) are full rank square matrices. As a result, $\boldsymbol{Q}_j$ is also full rank. Based on the above results, we present a technique for finding the IS spanned by $X(t)$ as follows.

The Interference Subspace Reconstruction (ISR) Technique

Theorem 2: Let $\boldsymbol{T}_j$ be a full rank $KJ \times J$ matrix which satisfies the relationship of $\boldsymbol{T}_j^H\boldsymbol{Q}_j = 0$. Consider that $\boldsymbol{T}_j$ is partitioned as follows:

$$\boldsymbol{T}_j = [\boldsymbol{T}_{j1}^T \quad \boldsymbol{T}_{j2}^T \quad \cdots \quad \boldsymbol{T}_{jK}^T]^T \tag{23}$$

where $\boldsymbol{T}_{jk}$ are $J \times J$ matrices for $k = 1, 2, \cdots, K$. Then a basis matrix that spans the IS associated with the original array is given by

$$\boldsymbol{G}_j = [(\boldsymbol{G}_{j1}\boldsymbol{T}_{j1})^T \quad (\boldsymbol{G}_{j22}\boldsymbol{T}_{j2})^T \quad \cdots \quad (\boldsymbol{G}_{jK2}\boldsymbol{T}_{jK})^T]^T. \tag{24}$$

Proof: From (21) and (22), we have

$$\boldsymbol{G}_{jk3}\boldsymbol{T}_{jk} = \boldsymbol{G}_{j(k+1)1}\boldsymbol{T}_{j(k+1)}. \tag{25}$$

Substituting (25) into (24), we can easily show that $\overline{\boldsymbol{G}}_r^H\boldsymbol{G}_j = 0$ where $\overline{\boldsymbol{G}}_r$ is given by (19). From Theorem 1, we note that $\overline{\boldsymbol{G}}_r$ spans the ISC associated with the original array. Therefore, the full-rank matrix $\boldsymbol{G}_j$ of (24) spans the IS associated with the original array. This completes the proof.

After finding $\boldsymbol{G}_j$, the ISC basis matrix $\boldsymbol{G}_r$ required by (7) can be computed as follows:

$$\boldsymbol{G}_r = [(-\boldsymbol{G}_1^{-H}\boldsymbol{G}_2^H)^T \quad \boldsymbol{I}_{M-J}]^T \tag{26}$$

where $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ contains the first J and the last $M - J$ rows of $\boldsymbol{G}_j$, respectively.

Next, consider the computational complexity required by using the proposed ISR technique. Let $f(M_k, J)$ denote the number of complex multiplications (CM) required for computing each of the basis matrices $\boldsymbol{G}_{jk}$ for $k = 1, 2, \cdots, K$. Due to the block-banded structure of (22), the matrix $\boldsymbol{T}_j$ can be computed with computing cost about 11 $KJ^3/3$ CM. Computing the matrix $\boldsymbol{G}_j$ of (24) costs about $J^3 + \Sigma_{k=1}^K J^2(M_k - J)$ CM. Hence, the total number of CM required by the proposed technique for computing the IS spanned by $X(t)$ is approximately given by

$$\sum_{k=1}^{K} f(M_k, J) + MJ^2 + \tfrac{11}{3}KJ^3. \tag{27}$$

IV. Modifications of the Proposed ISR Technique

In this section, we consider the modifications of the proposed ISR technique under different situations including different array configurations and signal characteristics. The received sensor noises are assumed to be spatially white with unit power. Accordingly, the corresponding optimal weight vector of an ESB adaptive interference canceller can be computed from (10).

A. ESB-AIC Using a Uniform Linear Array

Consider the case of ESB-AIC using a ULA based on the work presented in [4]. The desired signal is blocked from the received data by utilizing a suitable signal blocking matrix. Assume that there exists only partial correlation between any two of the P signals. Fig. 1 depicts the subarray partitioning scheme used for this case. Following the proposed ISR technique presented in Section III, we input the kth augmented data vector $[X_x^T(t)\ \ x_{M_k^+ +1}(t)]^T$ or $[x_{M_k^-}(t)\ \ X_k^T(t)]^T$ to the kth blocking matrix $\boldsymbol{B}_k$ and then take the correlation matrix of the output data vector. Let the correlation matrix of the kth augmented data vector be denoted as $\boldsymbol{R}_{bk}$. Then it is easy to show that the correlation matrix of the data vector at the output of $\boldsymbol{B}_k$ is given by $\boldsymbol{B}_k^H \boldsymbol{R}_{bk} \boldsymbol{B}_k$ and the corresponding noise correlation matrix is given by $\pi_n \boldsymbol{B}_k^H \boldsymbol{B}_k$. Therefore, the output data vector from $\boldsymbol{B}_k$ can be whitened by applying it to the operator $(\boldsymbol{B}_k^H \boldsymbol{B}_k)^{-1/2}$. Accordingly, the whitened data vector has correlation matrix given by $(\boldsymbol{B}_k^H \boldsymbol{B}_k)^{-1/2} \boldsymbol{B}_k^H \boldsymbol{R}_{bk} \boldsymbol{B}_k (\boldsymbol{B}_k^H \boldsymbol{B}_k)^{-1/2}$ and its EVD can be expressed as

$$\begin{aligned}(\boldsymbol{B}_k^H \boldsymbol{B}_k)^{-1/2}&(\boldsymbol{B}_k^H \boldsymbol{R}_{bk} \boldsymbol{B}_k)(\boldsymbol{B}_k^H \boldsymbol{B}_k)^{-1/2}\\ &= \boldsymbol{U}_k \Lambda_k \boldsymbol{U}_k^H + \pi_n \boldsymbol{V}_k \boldsymbol{V}_k^H\end{aligned} \tag{28}$$

where $\boldsymbol{U}_k$ and Λ_k contain the J principle eigenvectors and eigenvalues, while $\boldsymbol{V}_k$ contains the other eigenvectors with eigenvalues equal to π_n. Using (28), we can construct an $M_k \times J$ basis matrix which spans the IS associated with the received data vector $X_k(t)$ as follows:

$$\boldsymbol{G}_{jk} = (\boldsymbol{B}_k^H \boldsymbol{B}_k)^{1/2} U_k \tag{29}$$

for $k = 1, 2, \cdots, K$. Then, the IS's obtained by (29) are used to construct the original IS spanned by the received data vector $X(t)$ using the proposed ISR technique. Note from (29) that no degrees of freedom are lost in computing the $\boldsymbol{G}_{jk}$ and, hence, the resulting IS $\boldsymbol{G}_j$ of the original array still has a dimension equal to $M \times J$. It follows from (10) that the optimal weight vector will have a dimension equal to $M \times 1$. In contrast, it has been shown that the optimal weight vector obtained by using the method of [4] only has a dimension equal to $(M-1) \times 1$.

Next, consider the case of coherent signal sources. We can employ the spatial smoothing scheme presented in [9] on each subarray and, hence, the correlation matrix $\boldsymbol{R}_{bk}$ must be replaced by the spatially averaged correlation matrix in order to restore the rank of $\boldsymbol{R}_{bk}$. Assume that some I of the P signal sources are coherent, where I is not greater than P. We can take the kth augmented data vector formed by $[X_k^T(t)\ \ x_{M_k^+ +1}(t)\ \ x_{M_k^+ +2}(t) \cdots x_{M_k^+ +1}(t)]^T$ or $[x_{M_k^- - I+1}(t)\ \ x_{M_k^- - I+2}(t) \cdots x_{M_k^-}(t)\ \ X_k^T(t)]^T$ and partition it into I subvectors with M_k sensor elements overlapped for two consecutive subvectors. Then, the average of the I correlation matrices associated with these I subvectors is of size $(M_k+1) \times (M_k+1)$ and used to replace the $\boldsymbol{R}_{bk}$. It is clear that we do not lose the degrees of freedom according to the proposed technique. However, the method of [4] will lose I more degrees of freedom when dealing with I coherent signal sources.

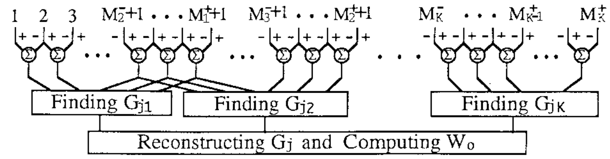


Fig. 1. The diagram of the proposed method under ULA.

Finally, the computational complexity required by using the proposed technique is evaluated. Utilizing the method of [8] to perform the EVD on (28) requires $12M_k^3$ CM. Additional $2M_k^3$ CM are required in order to whiten the data vector at the output of $\boldsymbol{B}_k$. The cost required for computing (29) is about $M_k^2 J$ CM. Thus, the number of CM required for finding the IS $\boldsymbol{G}_j$ spanned by $X(t)$ is approximately given by $14\ \Sigma_{k=1}^K\ M_k^3 + J\ \Sigma_{k=1}^K\ M_k^2 + MJ^2 + \frac{11}{3}KJ^3$. MJ^2 more CM are required to obtain the optimal weight vector from (10) after obtaining the IS $\boldsymbol{G}_j$. Therefore, the total number of CM required for performing the ESB-AIC using the proposed technique is about

$$14\sum_{k=1}^{K} M_k^3 + J\sum_{k=1}^{K} M_k^2 + 2MJ^2 + \tfrac{11}{3}KJ^3. \tag{30}$$

If $M \gg M_k \gg J$, then the last three terms of (30) can be neglected. Hence, (30) is approximately equal to $14\ \Sigma_{k=1}^K\ M_k^3$. In contrast, the computational burden required by the method of [4] for obtaining the optimal weight vector is about $14M^3$ CM, which is much more than (30).

B. ESB-AIC Using a Nonuniform Linear Array

In this situation, we consider two cases where the desired signal and interferers are uncorrelated and partially correlated, respectively.

1) Desired Signal Uncorrelated with Interferers: Let $\tilde{X}_k(t)$ denote the data vector received by the P consecutive sensor elements starting from the $(M_k^+ + 1)$th (or the $(M_k^- - P + 1)$th) sensor element to the $(M_k^+ + P)$th (or the M_k^-th) sensor element. The corresponding response matrices for the desired signal, interferers, and all incident signal sources are designated as $\tilde{\boldsymbol{A}}_{dk}$, $\tilde{\boldsymbol{A}}_{jk}$, and $\tilde{\boldsymbol{A}}_{sk}$, respectively. Hence, the cross-correlation matrix between $X_k(t)$ and $\tilde{X}_k(t)$ is given by

$$E\{X_k(t)\tilde{X}_k^H(t)\} = \boldsymbol{A}_{dk}\Psi_d \tilde{\boldsymbol{A}}_{dk}^H + \boldsymbol{A}_{jk}\Psi_j \tilde{\boldsymbol{A}}_{jk}^H \tag{31}$$

where Ψ_d and Ψ_j are the full rank correlation matrices associated with the desired signal and interferers, respectively. Under the assumption of unambiguity condition, it is easy to show that these exists such a full-rank $P \times J$ matrix $\tilde{C}_k$ that $\tilde{C}_k{}^H \tilde{A}_{dk} = 0$ and $\tilde{C}_k^H \tilde{A}_{jk}$ is a full-rank square matrix. Multiplying both sides of (31) by $\tilde{C}_k$ yields

$$E\{X_k(t)\tilde{X}_k^H(t)\}\tilde{C}_k = A_{jk}\Psi_j \tilde{A}_{jk}^H \tilde{C}_k. \tag{32}$$

Examining (32), we note that the product of $\Psi_j \tilde{A}_{jk}^H \tilde{C}_k$ represents a square matrix with full rank. Moreover, (32) spans the IS of the kth subarray for $k = 1, 2, \cdots, K$. Then, the interference subspaces obtained by (32) are used to construct the original IS spanned by the received data vector $X(t)$ using the proposed technique.

Based on the above description, we note that the $f(M_k, J)$ becomes about $J^2 M_k$. It follows from (27) that the total number of CM for finding the basis matrix which spans the IS associated with the original array is about $\Sigma_{k=1}^K J^2 M_k + MJ^2 + 11KJ^3/3$. Moreover, MJ^2 CM are required for finding the optimal weight vector from (10). Therefore, performing the ESB-AIC based on the proposed technique requires about $3MJ^2 + 14KJ^3/3$ CM.

2) Desired Signal Partially Correlated with Interferers: First, let $\overline{H}_{k1}$ and $\overline{H}_{k2}$ be two row selecting matrices containing the first P rows and the last $M_k - P$ rows of the $M_k \times M_k$ identity matrix, respectively. Then, we construct two matrices from G_{sk} as follows:

$$\overline{G}_{sk1} = \overline{H}_{k1} G_{sk} \quad \text{and} \quad \overline{G}_{sk2} = \overline{H}_{k2} G_{sk}. \tag{33}$$

Similarly, two matrices are constructed from G_{jk} as follows:

$$\overline{G}_{jk1} = \overline{H}_{k1} G_{jk} \quad \text{and} \quad \overline{G}_{jk2} = \overline{H}_{k2} G_{jk}. \tag{34}$$

Next, we present the following theorem.

Theorem 3: If $\overline{G}_{sk1}$ is invertible, then

$$\overline{G}_{jk2} = \overline{G}_{sk2} \overline{G}_{sk1}^{-1} \overline{G}_{jk1}. \tag{35}$$

Proof: Similar to (33), the response matrix associated with the desired signal for the kth subarray can be partitioned as follows:

$$A_{dk} = [A_{dk1}^T \quad A_{dk2}^T]^T \tag{36}$$

where $\overline{A}_{dk1} = \overline{H}_{k1} A_{dk}$ and $\overline{A}_{dk2} = \overline{H}_{k2} A_{dk}$. Since the signal sources are not coherent, we note that the matrices $[A_{dk} G_{jk}]$ and G_{sk} both span the same SS associated with the kth subarray. Hence, there exists a unique transformation matrix Γ_k such that

$$\begin{bmatrix} \overline{A}_{dk1} & \overline{G}_{jk1} \\ \overline{A}_{dk2} & \overline{G}_{jk2} \end{bmatrix} = \begin{bmatrix} \overline{G}_{sk1} \\ \overline{G}_{sk2} \end{bmatrix} \Gamma_k. \tag{37}$$

It follows from (37) that

$$\Gamma_k = \overline{G}_{sk1}^{-1} [\overline{A}_{dk1} \quad \overline{G}_{jk1}]. \tag{38}$$

Substituting (38) into (37), we obtain the result as shown by (35). This completes the proof.

From Theorem 3, we note that $\overline{G}_{j12}$ can be easily found from (35) once $\overline{G}_{j11}$ is available. If we partition the original array in such a way that there are P sensor elements overlapped between any two consecutive subarrays, then $\overline{G}_{jk2}$ is equal to $\overline{G}_{j(k+1)1}$ and, hence, $\overline{G}_{j(k+1)2}$ can be easily computed using Theorem 3 for $k = 1, 2, \cdots, K$. Accordingly, a basis matrix spanning the IS associated with the original array can be constructed as follows:

$$G_j = [\overline{G}_{j11}^T \quad \overline{G}_{j12}^T \quad \overline{G}_{j22}^T \quad \cdots \quad \overline{G}_{jK2}^T]^T. \tag{39}$$

Finally, we evaluate the computational complexity required for this case. Computing each of G_{sk} for $k = 1, 2, \cdots, K$ requires $12M_k^3$ CM based on the method of [8]. Computing $\overline{G}_{jk2}$ from (35) costs about $[2P^3/3 + 2JP^2 + (M_k - P)PJ]$ CM. Therefore, the total number of CM required for performing the ESB-AIC in this case by using the proposed technique is about

$$12\sum_{k=1}^K M_k^3 + 2MJ^2 + \tfrac{8}{3}KP^3. \tag{40}$$

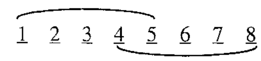


Fig. 2. The subarray partitioning configuration for simulation example.

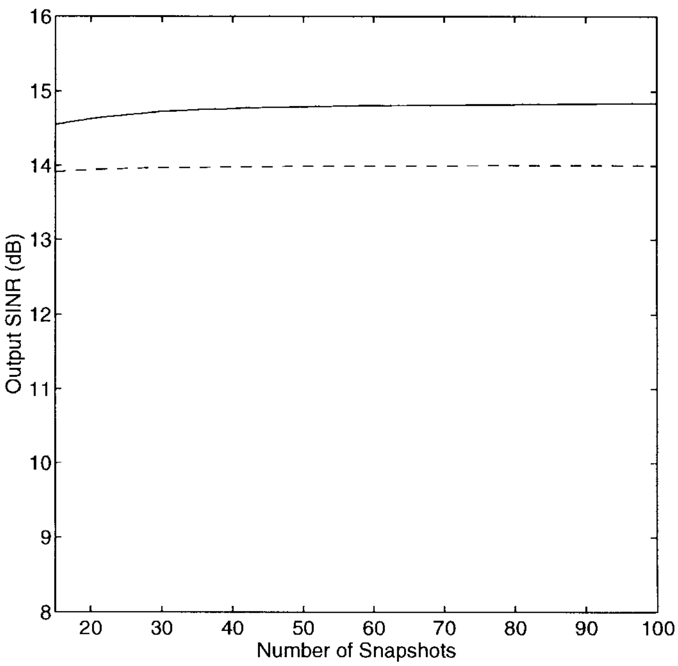
V. Simulation Example and Comparison

In this section, a simulation example for illustration and comparison is presented. For this example, each simulation result is the average of 100 independent runs with independent noise samples and independent signal samples for each run.

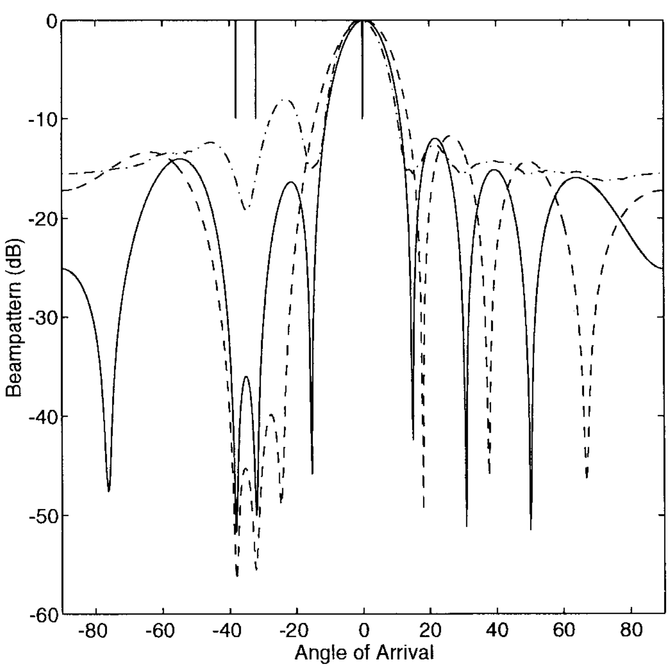
Example: Here, we evaluate the performance of the ESB-AIC using an eight-element ULA with interelement spacing equal to $\lambda/2$ (half wavelength). The received sensor noise is assumed to be spatially white with unit power. There are two interferers with INR = 10 dB are impinging on the array from $-32°$ and $-38°$ off broadside, while a desired signal with signal-to-noise ratio (SNR) = 6 dB is impinging on the array from the broadside. The correlation coefficients between these three partially correlated signal sources are given by $\rho_{12} = 0.3\exp(0.5j)$, $\rho_{13} = 0.3\exp(0.9j)$, and $\rho_{23} = 0.2\exp(0.1j)$, respectively. Fig. 2 shows the partitioned subarrays for this example. The array output SINR versus the number of snapshots is depicted in Fig. 3(a). In addition to the result of using the proposed technique, the result of using the method of [4] is also presented for comparison. It can be seen that the proposed technique is more effective than the method of [4] as expected since the degrees of freedom are not lost by using the proposed technique. The corresponding array output beampatterns obtained after 600 data snapshots are plotted in Fig. 3(b). For comparison, we also show the beampattern of using the conventional adaptive beamforming technique with a unit-gain constraint in the desired signal direction like that of [10]. Both the proposed technique and the method of [4] provide very deep nulls in the interference directions. However, the mainlobe by using the method of [4] is wider than that by using the proposed technique. Although the conventional technique of [10] can produce almost the same mainlobe as the proposed technique, it cannot effectively suppress the interference.

VI. Conclusion

This paper has presented an efficient technique for eigenspace-based (ESB) adaptive interference cancellation (AIC). To save computational complexity and avoid the loss of degrees of freedom in suppressing undesired signals, we have proposed a technique for constructing the interference



(a)



(b)

Fig. 3. The results of the *Example*. Solid line: the proposed method. Dash line: the method of [4]. Dash-dotted line: the conventional technique of [10]. (a) The output SINR versus the number of snapshots. (b) The output beampatterns.

subspace spanned by the received array data vector from the subarray subspaces. These subarray subspaces are spanned by the data vectors received by the subarrays which are obtained from the partition of the original array. In theory, it has been shown that the proposed technique can alleviate the drawbacks like loss of degrees of freedom and heavy computational burden when using existing ESB adaptive interference cancellers with a uniform linear array. Modifications for the proposed technique have also been presented for dealing with the case of using a nonuniform linear array in the environment of partially correlated signals. Computer simulations have demonstrated the effectiveness of the proposed technique.

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Cheng-Chou Lee was born in Taipei, Taiwan, on July 20, 1969. He received the B.S. degree in electrical engineering from the National Taiwan University, Taipei, Taiwan, in 1991. He is currently working toward the Ph.D. degree in the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan.

His current research interests include the adaptive signal processing, array signal processing, and the digital signal processing for wireless communication systems.

Ju-Hong Lee (S'81–M'83) was born in I-Lan, Taiwan, on December 7, 1952. He received the B.S. degree from the National Cheng-Kung University, Tainan, Taiwan, in 1975, the M.S. degree from the National Taiwan University, Taipei, Taiwan, in 1977, and the Ph.D. degree from Rensselaer Polytechnic Institute, Troy, NY, in 1984, all in electrical engineering.

From September 1980 to July 1984, he was a Research Assistant and was involved in research on multidimensional recursive digital filtering in the Department of Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute. From August 1984 to July 1986 he was a Visiting Associate Professor and then became an Associate Professor in the Department of Electrical Engineering at the National Taiwan University. Since August 1989 he has been a Professor at the same institution. He held an appointment of Visiting Professor in the Department of Computer Science and Electrical Engineering at the University of Maryland, Baltimore, during his sabbatical leave in the academic year of 1996. His current research interests include miltidimensional digital signal processing, image processing, detection and estimation theory, analysis and processing of joint vibration signals for a diagnosis of cartilage pathology, and adaptive signal processing and its applications in communications.

Dr. Lee received Outstanding Research Awards from the National Science Council in the academic years of 1988, 1989, and 1991–1994.