Calculation of the Surface-Wave Excitation in Multilayered Structures

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***Abstract*— A novel rigorous analysis of the surface waves excited by a Hertzian dipole embedded in a multilayered structure is presented. A transmission-line resonator equivalent circuit is used to calculate the surface wave's electromagnetic field components. It is shown that the power carried by the surface waves is related to the energy stored in the resonator. An analytical method for the calculation of the stored energy is given. A simple algorithm iterating over the layers of the structure is derived to analytically calculate the surface wave's electromagnetic field components and the power carried by surface waves. The need of numerical integration or calculation of residues is omitted. This benefits a reduction in computation time and an improvement in accuracy and versatility of computer-aided design (CAD) programs. The presented method has been implemented in a microwave CAD program. Numerical results for planar antennas are presented.**

***Index Terms*—Nonhomogeneous media, surface waves.**

I. Introduction

MICROSTRIP and multilayered planar antennas excite surface waves that propagate along the dielectric substrate. The power carried by surface waves contributes significantly to the loss of the antenna. Moreover, parasitic radiation from the substrate edges occurs in the presence of surface waves, deteriorating the radiation pattern and affecting the input impedance. Therefore, the calculation of the surface-wave excitation is a key function in microwave computer-aided design (CAD) programs.

Asymptotic methods accounting for the saddle point and the singularities of Green's function in the spectral domain can be applied to calculate the electromagnetic far field both in the free-space and in the vicinity of the substrate. The "immittance approach" introduced by Itoh [1] has proven to be a standard method for the calculation of Green's function of a layered structure in the spectral domain. A matrix algorithm that is well suited for an implementation in CAD programs has been derived for that task [2]. The electromagnetic field components in the free-space can be obtained by evaluating Green's function at the saddle point. The location of the saddle point is related to the observation point's spherical coordinates [3]. An implementation in microwave CAD programs is, therefore, straightforward.

However, to compute the electromagnetic field components of the surface waves, the poles and associated residues of Green's function in the spectral domain must be calculated. While sophisticated root-finding algorithms are available for the computation of the poles, the calculation of the residues is more complicated. Two approaches to this task have been known from the literature. Analytical methods calculate the residues manually [4]. This method benefits high computation speed and high accuracy. However, it leads to considerable analytical labor in case of multilayered structures. As another drawback, the substrate configuration must be known in advance, which means a loss in generality when implemented in CAD programs. Numerical methods, on the other hand, use algorithms to numerically take the derivative. While not restricted to a special structure of the substrate, these algorithms are known to be numerically ill conditioned, resulting in low accuracy and low computational stability [5].

In this paper, a novel method is presented. It is based on a network theorem that relates the stored energy in a lossless resonator at resonance frequency to the residue of the resonator's immittance function. A transmission line resonator equivalent circuit representing the layered structure is derived. A discretized transmission line model with arbitrary dispersion characteristics is used to calculate the energy stored in the resonator. This allows for the analytical calculation of the residue of Green's function in the spectral domain. A simple algorithm iterating over the layers of the structure is derived to analytically calculate the surface-wave's electromagnetic components and the power carried by surface waves. Considerable analytic work can be saved for these types of problems. As an example, a microstrip structure is investigated and the results are compared with the literature.

A lossless structure is assumed in the following derivation of the method. The resulting equations for the microstrip configuration (cf., Section VII) turned out to be valid for a lossy structure also when a complex permittivity was assumed. The general case, however, is left for further investigations.

II. Calculation of Electromagnetic Field Components in the Spectral Domain

The novel method presented in this paper is based on the calculation of the electromagnetic field components in the spectral domain. In the following, a multilayered structure consisting of n homogenous, lossless, and isotropic layers of thickness h_i, $i = 1 \cdots n$ with the relative permittivity ϵ_i and the relative permeability μ_i is considered. We will use a Cartesian coordinate system x, y, z with its z axis perpendicular to the layers, a cylindrical coordinate system in the space domain ρ, φ, z, and a cylindrical coordinate system in the spectral domain k_t, Φ, z. An x-oriented electric point

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source

$$\mathbf{J} = \delta(x)\delta(y)\delta(z)I_0 l\mathbf{e}_x \tag{1}$$

or a magnetic point source

$$\mathbf{M} = \delta(x)\delta(y)\delta(z)K_0 l\mathbf{e}_x \tag{2}$$

is assumed to be located at the interface of two layers. The symbol δ denotes Dirac's delta function. We apply a two-dimensional Fourier transform

$$\tilde{\Psi}(k_x, k_y, z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi(x, y, z)e^{-j(k_x x + k_y y)}\, dx\, dy \tag{3}$$

to Maxwell's equations with the time dependency $\exp(j\omega t)$ omitted and use a cylindrical coordinate system in the spectral domain

$$k_x = k_t \cos\Phi, \tag{4}$$
$$k_y = k_t \sin\Phi. \tag{5}$$

This allows for a decomposition of the electromagnetic field into ϵ and μ components yielding two ordinary differential equations

$$\frac{d\tilde{E}^{(i)}_{\epsilon,\mu}}{dz} + \gamma_i Z^{(i)}_{\epsilon,\mu}\tilde{H}^{(i)}_{\epsilon,\mu} = -\tilde{M}^{(i)}_{\epsilon,\mu} \tag{6}$$
$$\frac{d\tilde{H}^{(i)}_{\epsilon,\mu}}{dz} + \gamma_i Y^{(i)}_{\epsilon,\mu}\tilde{E}^{(i)}_{\epsilon,\mu} = -\tilde{J}^{(i)}_{\epsilon,\mu} \tag{7}$$

with

$$Z^{(i)}_{\epsilon} = \frac{1}{Y^{(i)}_{\epsilon}} = \frac{\gamma_i Z_0}{j\epsilon_i k_0} \tag{8}$$
$$Z^{(i)}_{\mu} = \frac{1}{Y^{(i)}_{\mu}} = \frac{j\mu_i k_0 Z_0}{\gamma_i} \tag{9}$$

and

$$\gamma_i = \sqrt{k_t^2 - \epsilon_i\mu_i k_0^2}. \tag{10}$$

The tilde $(\sim)$ denotes quantities in the Fourier domain. The index i denotes quantities in the ith layer of the structure. The ϵ and μ components of the electromagnetic field quantities are related to the Cartesian components by

$$\begin{pmatrix}\tilde{E}_x\\ \tilde{E}_y\end{pmatrix} = \begin{pmatrix}\cos\Phi & -\sin\Phi\\ \sin\Phi & \cos\Phi\end{pmatrix}\begin{pmatrix}\tilde{E}_\epsilon\\ \tilde{E}_\mu\end{pmatrix} \tag{11}$$
$$\begin{pmatrix}\tilde{J}_x\\ \tilde{J}_y\end{pmatrix} = \begin{pmatrix}\cos\Phi & -\sin\Phi\\ \sin\Phi & \cos\Phi\end{pmatrix}\begin{pmatrix}\tilde{J}_\epsilon\\ \tilde{J}_\mu\end{pmatrix} \tag{12}$$
$$\begin{pmatrix}\tilde{H}_x\\ \tilde{H}_y\end{pmatrix} = \begin{pmatrix}-\sin\Phi & -\cos\Phi\\ \cos\Phi & -\sin\Phi\end{pmatrix}\begin{pmatrix}\tilde{H}_\epsilon\\ \tilde{H}_\mu\end{pmatrix} \tag{13}$$
$$\begin{pmatrix}\tilde{M}_x\\ \tilde{M}_y\end{pmatrix} = \begin{pmatrix}-\sin\Phi & -\cos\Phi\\ \cos\Phi & -\sin\Phi\end{pmatrix}\begin{pmatrix}\tilde{M}_\epsilon\\ \tilde{M}_\mu\end{pmatrix}. \tag{14}$$

Fig. 1 shows the relation between x, y and ϵ, μ components. This notation is consistent with the one used in [2]. It can be transformed to Itoh's u and v notation [1] by a change in the sign of some of the components. The advantage of our method lies in the fact that (6)–(10) are decoupled for ϵ and μ components, thus saving code when implemented in a CAD program. However, this can only be achieved by a transformation that relates the ϵ and μ components to the x and y components differently, depending on the type of field (electric or magnetic). We do not need to consider the z components $\tilde{H}_z$ and $\tilde{E}_z$ here since they can be calculated from the ϵ and μ components using the divergence property of the electromagnetic field quantities. As can be seen from (21)–(24), the ϵ components of the electromagnetic field correspond to the ρ components of the surface wave, whereas the μ components correspond to the φ components. The (6) and (7) hold within each layer of the substrate. At the interfaces of the layers, the ϵ and μ components of the electromagnetic field must be continuous [6]. We substitute

$$\begin{aligned}\tilde{E}^{(i)}_{\epsilon} &\to v\\ \tilde{H}^{(i)}_{\epsilon} &\to i\\ \gamma^{(i)} &\to \gamma\\ Z^{(i)}_{\epsilon} &\to Z_W\\ \cos\Phi I_0 l &\to i_0\\ -\sin\Phi U_0 l &\to v_0\end{aligned} \tag{15}$$

for ϵ components and

$$\begin{aligned}\tilde{E}^{(i)}_{\mu} &\to v\\ \tilde{H}^{(i)}_{\mu} &\to i\\ \gamma^{(i)} &\to \gamma\\ Z^{(i)}_{\mu} &\to Z_W\\ -\sin\Phi I_0 l &\to i_0\\ -\cos\Phi U_0 l &\to v_0\end{aligned} \tag{16}$$

for μ components. From (6) and (7), we get

$$\frac{dv}{dz} + \gamma Z_W\, i = -v_0\delta(z - z_0) \tag{17}$$
$$\frac{di}{dz} + \gamma\frac{1}{Z_W}v = -i_0\delta(z - z_0) \tag{18}$$

which are the transmission line equations. This permits the construction of a transmission line equivalent circuit resonator describing the layered structure [1]. Each section of the resonator corresponds to a layer of the structure. The solution of (17) and (18) for one section can be written as

$$i = i_+ + i_- = i_{0+}e^{-\gamma z} + i_{0-}e^{\gamma z} \tag{19}$$
$$v = v_+ + v_- = Z_W i_{0+}e^{-\gamma z} - Z_W i_{0-}e^{\gamma z}. \tag{20}$$

The index i selecting the layers has been omitted. Together with the continuity condition at the interfaces of the layers, the voltages and currents along each section of the resonator can be calculated from the current through the voltage source (which represents a magnetic point source) or from the voltage across the current source (which represents an electric point source). A matrix algorithm to calculate i_{0+}, i_{0-}, v_{0+}, and v_{0-} is particularly useful for an implementation of this task in CAD programs [7]. Backsubstituting the electromagnetic field components yields Green's function in the spectral domain for any given z.

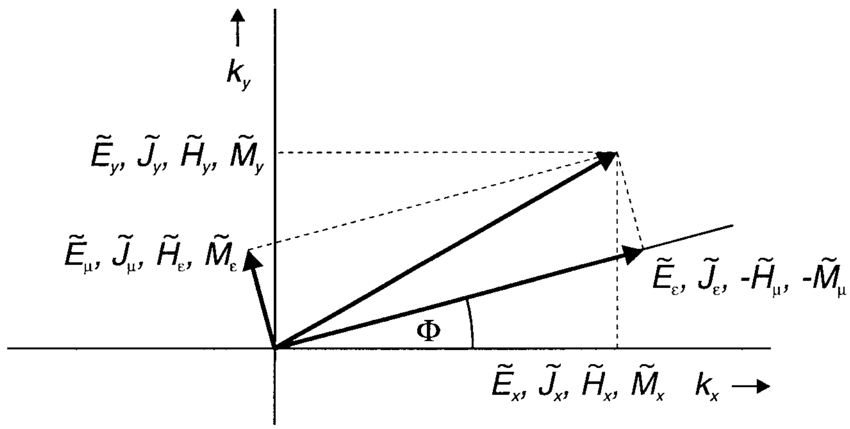

Fig. 1. Relation between x, y and ϵ, μ components.

III. Calculation of Surface-Wave Modes

While numerical techniques must be applied in the near-field region to obtain Green's function in the space domain, asymptotic techniques can be used in the far-field region. Applying the modified saddle-point technique shows that the electromagnetic field components of the surface waves are related to the residues of the poles of Green's function in the spectral domain [8] by

$$E_\rho = A \operatorname{Res}\left(\frac{\tilde{E}_\epsilon}{\operatorname{trig}(\Phi)} \middle| k_{t_\infty}\right)\operatorname{trig}(\varphi) \tag{21}$$

$$E_\varphi = A \operatorname{Res}\left(\frac{\tilde{E}_\mu}{\operatorname{trig}(\Phi)} \middle| k_{t_\infty}\right)\operatorname{trig}(\varphi) \tag{22}$$

$$H_\rho = A \operatorname{Res}\left(\frac{\tilde{H}_\epsilon}{\operatorname{trig}(\Phi)} \middle| k_{t_\infty}\right)\operatorname{trig}(\varphi) \tag{23}$$

$$H_\varphi = -A \operatorname{Res}\left(\frac{\tilde{H}_\mu}{\operatorname{trig}(\Phi)} \middle| k_{t_\infty}\right)\operatorname{trig}(\varphi) \tag{24}$$

with

$$A = \sqrt{\frac{k_{t_\infty}}{2\pi\rho}}\, e^{-j\frac{\pi}{4}} e^{-jk_{t_\infty}\rho} \tag{25}$$

and

$$\operatorname{trig}(\cdots) = \pm\begin{cases}\sin(\cdots)\\ \cos(\cdots).\end{cases} \tag{26}$$

In (21)–(24), the "trig" function within one equation is the same. It is determined from (11)–(14). The index ∞ denotes the pole of the Green's function in the spectral domain and $\operatorname{Res}(f(z) \mid z_\infty)$ denotes the residue of the complex function f at its pole z_∞. We use $\frac{df}{dz}|_\infty$ to denote $\frac{df}{dz}$ evaluated at $z = z_\infty$. For a z-oriented source (which is not considered here) similar equations can be derived. Generally, k_{t_∞} is not unique, i.e., more than one surface-wave mode exists. Note that a pole of a Green's function corresponds to a resonance of the resonator equivalent circuit. A root-finding algorithm has to be applied to compute k_{t_∞} [9]. k_{t_∞} is real for propagating surface-wave modes. Therefore, by (10), γ_{i_∞} can be either real, corresponding to an evanescent wave in layer i, or purely imaginary, corresponding to a propagating wave in that layer.

Since the electromagnetic field components of the surface waves can be calculated from the above relations using (19) and (20) at any given z, it is sufficient to consider the electromagnetic field components in the plane of the source $z = 0$. Thus, to calculate the surface waves, we can view the transmission line resonator equivalent circuit as a one-port operated at a resonance frequency.

In the next section, we will give a network theorem, which permits the calculation of the residue of the immittance function of a lossless one-port from the energy stored in the one-port. This is particularly useful in our case, since the energy stored in the transmission line resonator can be calculated analytically, as will be shown in Section V.

IV. Energy Stored in a Lossless One-Port

In the following, a novel network theorem is derived relating the energy stored in a lossless one-port operated at resonance frequency to the residue of the one-port's immittance function. A proof of the theorem will be given using a canonical realization of the one-port.

We consider a lossless one-port, which is characterized by its impedance function $Z(\omega)$. We assume that $Z(\omega)$ has a simple pole at $\omega = \omega_1$, i.e., the one-port has a parallel resonance at ω_1. Likewise, the dual case of a one-port operated at a series resonance frequency can be considered by replacing all quantities by their dual counterparts in the following.

For any lossless one-port, a partial-fraction expansion of its impedance

$$Z(\omega) = \frac{1}{j\omega C_0} + \sum_{\nu=1}^{r} \frac{\frac{j\omega}{C_\nu}}{\frac{1}{L_\nu C_\nu} - \omega^2} + j\omega L_\infty. \tag{27}$$

is possible [10]. The function $Z(\omega)$ has r simple poles at the resonance frequencies $\omega_\nu = \frac{1}{\sqrt{L_\nu C_\nu}}$. In case of a distributed one-port, $r \to \infty$. Fig. 2 shows the corresponding *first Foster realization* of (27). Using

$$\frac{A\omega}{\omega_\nu^2 - \omega^2} = \frac{\frac{A}{2}}{\omega_\nu - \omega} + \frac{-\frac{A}{2}}{\omega_\nu + \omega} \tag{28}$$

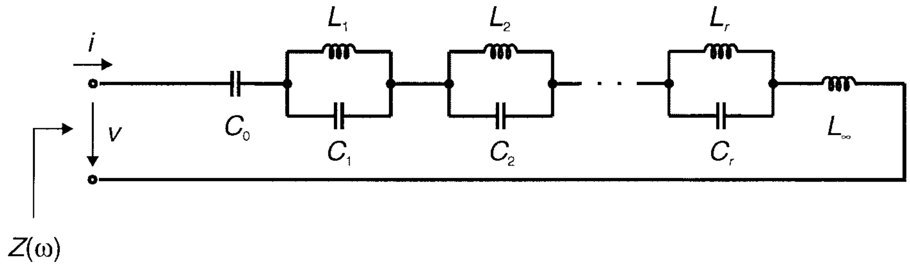

Fig. 2. First foster realization of the impedance function $Z(\omega)$.

we can write (27) as

$$Z(\omega) = \frac{1}{j\omega C_0} + \sum_{\nu=1}^{r} \left(\frac{\frac{j}{2C_\nu}}{\omega_\nu - \omega} - \frac{\frac{j}{2C_\nu}}{\omega_\nu + \omega} \right) + j\omega L_\infty \quad (29)$$

which is the Laurant expansion of $Z(\omega)$. It follows from (29) that the residue of $Z(\omega)$ at the pole ω_1 is given by

$$\mathrm{Res}(Z \mid \omega_1) = \frac{1}{2jC_1}. \quad (30)$$

We now assume that the one-port is operated at its parallel resonance frequency ω_1. A voltage $v \neq 0$ then appears across the port while the current flowing into it is zero. Therefore, the voltage across C_0 and L_∞ is zero. Also, the voltage across the parallel resonance circuits $L_\nu C_\nu$ is zero, except for $\nu = 1$. Hence, the energy stored in the one-port is concentrated in the resonant circuit $L_1 C_1$. This energy is given by

$$W = \frac{1}{2} C_1 |v|^2 \quad (31)$$

where v denotes the complex instantaneous voltage across the one-port. For convenience, we write ω_∞ instead of ω_1 in the following to denote the resonance frequency. Using (30), we can conclude that the following relationship between the energy W_∞ stored in a lossless one-port operated at a parallel resonance frequency ω_∞, the voltage v_∞ across the one-port in this case and the residue of the impedance function $Z(\omega)$ of the one-port at $\omega = \omega_\infty$ holds true:

$$\mathrm{Res}(Z \mid \omega_\infty) = \frac{|v_\infty|^2}{j4W_\infty}. \quad (32)$$

A similar derivation shows that

$$\mathrm{Res}(Y \mid \omega_\infty) = \frac{|i_\infty|^2}{j4W_\infty} \quad (33)$$

holds true in case of a lossless one-port operated at its series resonance frequency ω_∞. The admittance function of the one-port is denoted by $Y(\omega)$ and i_∞ denotes the complex instantaneous current flowing into the one-port.

We will now show that the residue of the impedance of a lossless one-port operated at a parallel resonance frequency is related to the derivation of the one-port's admittance function $Y(\omega)$ with respect to the frequency.

From the definition of the residue of a function [11], it follows that

$$\mathrm{Res}(Z \mid \omega_\infty) = \lim_{\omega \to \omega_\infty} (\omega - \omega_\infty) Z(\omega) = \lim_{\omega \to \omega_\infty} \frac{\omega - \omega_\infty}{Y(\omega)}. \quad (34)$$

Since

$$Y(\omega_\infty) = 0 \quad (35)$$

we can write

$$\mathrm{Res}(Z \mid \omega_\infty) = \lim_{\omega \to \omega_\infty} \frac{\omega - \omega_\infty}{Y(\omega) - Y(\omega_\infty)} \quad (36)$$

and, thus,

$$\mathrm{Res}(Z \mid \omega_\infty) = \frac{1}{\left.\frac{dY}{d\omega}\right|_\infty}. \quad (37)$$

Together with (32), it follows that the energy stored in a lossless one-port operated at a parallel resonance frequency can be calculated if the derivative of the one-port's admittance function with respect to the frequency is given. A similar derivation shows that the energy stored in a lossless one-port operated at a series resonance frequency can be calculated if the derivative of the one-port's impedance function with respect to the frequency is given.

As we will see later, the resulting equations for the residues of Green's function of a layered structure is independent of ω. This means that it would have been possible to define "$Z(k_t)$" instead of "$Z(\omega)$" for a lossless one-port. ω would then have been completely eliminated in the derivation of the method. However, the authors felt that the calculations are far more readable with a notation using ω.

V. Energy Stored in the Transmission Line Resonator Equivalent Circuit

To calculate the energy stored in the transmission line resonator, we use a distributed equivalent circuit model for each section of the resonator.

In the last section, we have shown that the energy stored in a lossless one-port operated at a parallel (series) resonance frequency can be calculated if the derivative of the one-port's admittance (impedance) function with respect to the frequency is given. Thus, we have to use a transmission line model that allows for an independent adjustment of the transmission line's characteristic impedance Z_W, the propagation constant γ and the dispersion characteristics $\frac{dZ_W}{d\omega}$ and $\frac{d\gamma}{d\omega}$. Furthermore, the equivalent circuit must support evanescent waves. This is accomplished by the equivalent circuit shown in Fig. 3, which represents a section of the transmission line with differential length dz. It results from a combination of the equivalent

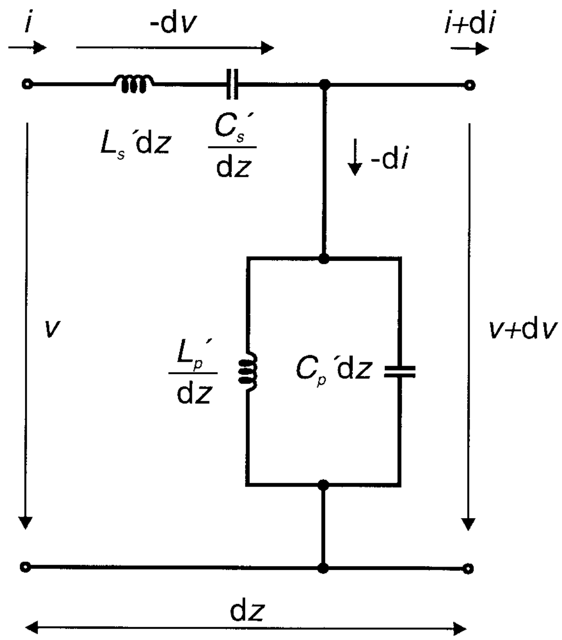

Fig. 3. Discretized transmission line model.

circuits supporting TM and TE waves in hollow waveguides [6].

We first show how the network elements $L_s' dz$, $\frac{C_s'}{dz}$, $L_p' dz$, and $\frac{L_p'}{dz}$ have to be chosen in order to match the characteristic impedance Z_W, the propagation coefficient γ, and the dispersion characteristics $\frac{dZ_W}{d\omega}$ and $\frac{d\gamma}{d\omega}$ of a given transmission line. We use (17) and (18) and apply Kirchhoff's current law and Kirchhoff's voltage law to the circuit shown in Fig. 3. This yields

$$\gamma Z_W = j\omega L_s' + \frac{1}{j\omega C_s'} \tag{38}$$

$$\frac{\gamma}{Z_W} = j\omega C_p' + \frac{1}{j\omega L_p'} \tag{39}$$

and by taking the derivative we obtain the circuit elements

$$L_s' = -\frac{j}{2}\left(\frac{\gamma Z_W}{\omega} + \frac{d(\gamma Z_W)}{d\omega}\right) \tag{40}$$

$$C_s' = -\frac{2j}{\omega\left(\gamma Z_W - \omega\frac{d(\gamma Z_W)}{d\omega}\right)} \tag{41}$$

$$C_p' = -\frac{j}{2}\left(\frac{\gamma}{\omega Z_W} + \frac{d\left(\frac{\gamma}{Z_W}\right)}{d\omega}\right) \tag{42}$$

$$L_p' = -\frac{2j}{\omega\left(\frac{\gamma}{Z_W} - \omega\frac{d\left(\frac{\gamma}{Z_W}\right)}{d\omega}\right)}. \tag{43}$$

The instantaneous energy stored in the lumped elements $L_s' dz$, $\frac{C_s'}{dz}$, $L_p' dz$, and $\frac{L_p'}{dz}$ of the equivalent circuit is given by

$$dW_{L_s} = \frac{1}{2} L_s' \operatorname{Re}(ie^{j\omega t})^2 dz \tag{44}$$

$$dW_{C_s} = \frac{1}{2}\frac{1}{\omega^2 C_s'} \operatorname{Re}(ie^{j\omega t})^2 dz \tag{45}$$

$$dW_{C_p} = \frac{1}{2} C_p' \operatorname{Re}(ve^{j\omega t})^2 dz \tag{46}$$

$$dW_{L_p} = \frac{1}{2}\frac{1}{\omega^2 L_p'} \operatorname{Re}(ve^{j\omega t})^2 dz. \tag{47}$$

From (19) and (20), it follows for a propagating wave

$$\operatorname{Re}(ie^{j\omega t})^2 = (\operatorname{Re}(i_+e^{j\omega t}) + \operatorname{Re}(i_-e^{j\omega t}))^2 \tag{48}$$

$$\operatorname{Re}(ve^{j\omega t})^2 = |Z_W|^2(\operatorname{Re}(i_+e^{j\omega t}) - \operatorname{Re}(i_-e^{j\omega t}))^2 \tag{49}$$

$$\operatorname{Im}(ie^{j\omega t})^2 = (\operatorname{Im}(i_+e^{j\omega t}) + \operatorname{Im}(i_-e^{j\omega t}))^2 \tag{50}$$

$$\operatorname{Im}(ve^{j\omega t})^2 = |Z_W|^2(\operatorname{Im}(i_+e^{j\omega t}) - \operatorname{Im}(i_-e^{j\omega t}))^2 \tag{51}$$

and for an evanescent wave

$$\operatorname{Re}(ie^{j\omega t})^2 = (\operatorname{Re}(i_+e^{j\omega t}) + \operatorname{Re}(i_-e^{j\omega t}))^2 \tag{52}$$

$$\operatorname{Re}(ve^{j\omega t})^2 = |Z_W|^2(\operatorname{Im}(i_+e^{j\omega t}) - \operatorname{Im}(i_-e^{j\omega t}))^2 \tag{53}$$

$$\operatorname{Im}(ie^{j\omega t})^2 = (\operatorname{Im}(i_+e^{j\omega t}) + \operatorname{Im}(i_-e^{j\omega t}))^2 \tag{54}$$

$$\operatorname{Im}(ve^{j\omega t})^2 = |Z_W|^2(\operatorname{Re}(i_+e^{j\omega t}) - \operatorname{Re}(i_-e^{j\omega t}))^2. \tag{55}$$

We use a polar form of the currents

$$i_{0+} = |i_{0+}|e^{j\varphi_+} \tag{56}$$

and

$$i_{0-} = |i_{0-}|e^{j\varphi_-} \tag{57}$$

in the following. By (19) and (20) (for propagating waves) the total energy stored in a section of length dz of the transmission line is given by

$$\begin{aligned} dW_p = \Bigg(&\frac{1}{2}\big(|i_{0+}|^2\cos(2\varphi_+ - 2\beta z + 2\omega t) \\ &+ |i_{0-}|^2\cos(2\varphi_- + 2\beta z + 2\omega t)\big)\frac{\beta\operatorname{Re}(Z_W)}{\omega} \\ &+ \frac{1}{2}\big(|i_{0+}|^2 + |i_{0-}|^2\big)\operatorname{Re}(Z_W)\frac{d\beta}{d\omega} \\ &+ |i_{0+}||i_{0-}|\cos(\varphi_+ - \varphi_- - 2\beta z)\beta\frac{d\operatorname{Re}(Z_W)}{d\omega}\Bigg)dz \end{aligned} \tag{58}$$

and for evanescent waves, the energy is given by

$$\begin{aligned} dW_e = \Bigg(&\frac{1}{2}|i_{0+}|^2 e^{-2\alpha z}\cos(2\varphi_+ + 2\omega t)\frac{\alpha\operatorname{Im}(Z_W)}{\omega} \\ &+ \frac{1}{2}|i_{0-}|^2 e^{2\alpha z}\cos(2\varphi_- + 2\omega t)\frac{\alpha\operatorname{Im}(Z_W)}{\omega} \\ &+ \frac{1}{2}\big(|i_{0+}|^2 e^{-2\alpha z} + |i_{0-}|^2 e^{2\alpha z}\big)\alpha\frac{d\operatorname{Im}(Z_W)}{d\omega} \\ &+ |i_{0+}||i_{0-}|\cos(\varphi_+ - \varphi_-)\operatorname{Im}(Z_W)\frac{d\alpha}{d\omega}\Bigg)dz. \end{aligned} \tag{59}$$

We have chosen to express the stored energy in terms of the current along the transmission line. However, this is not mandatory; alternatively, we could use the voltage across the transmission line, which leads to similar expressions. It can be seen from (58) and (59) that the stored energy is composed of two terms. One term harmonically oscillates with the frequency 2ω, while the other term is constant. This is also true for the total energy stored in the resonator since the total energy is calculated by integrating dW over dz. However, in the resonance case, the total stored energy must be stationary. Therefore, we can neglect the oscillating term and use the average energy

$$d\bar{W}_p = \left(\frac{1}{2}(|i_{0+}|^2 + |i_{0-}|^2)\,\mathrm{Re}(Z_W)\frac{d\beta}{d\omega} + |i_{0+}||i_{0-}|\cos(\varphi_+ - \varphi_- - 2\beta z)\beta\frac{d\mathrm{Re}(Z_W)}{d\omega}\right)dz$$

$$d\bar{W}_e = \left(\frac{1}{2}(|i_{0+}|^2 e^{-2\alpha z} + |i_{0-}|^2 e^{2\alpha z})\alpha\frac{d\mathrm{Im}(Z_W)}{d\omega} + |i_{0+}||i_{0-}|\cos(\varphi_+ - \varphi_-)\,\mathrm{Im}(Z_W)\frac{d\alpha}{d\omega}\right)dz.$$

The $-$ denotes averaged quantities. We substitute Z_ϵ and Z_μ for the characteristic impedance Z_ω and evaluate the above equations at the pole $\omega = \omega_\infty$, $k_t = k_{t\infty}$, $\gamma_0 = \gamma_{0\infty}$. This yields

$$d\bar{W}_p = -\left(\frac{1}{2}(|i_{0+}|^2 + |i_{0-}|^2) \pm |i_{0+}||i_{0-}|\cos(\varphi_+ - \varphi_- - 2\beta_\infty z)\right) \cdot jZ_{\epsilon,\mu}\frac{\gamma_{0\infty}}{\gamma_\infty}\frac{d\gamma_0}{d\omega}\bigg|_\infty dz \tag{60}$$

$$d\bar{W}_e = -\left(\pm\frac{1}{2}(|i_{0+}|^2 e^{-2\alpha_\infty z} + |i_{0-}|^2 e^{2\alpha_\infty z}) + |i_{0+}||i_{0-}|\cos(\varphi_+ - \varphi_-)\right) \cdot jZ_{\epsilon,\mu}\frac{\gamma_{0\infty}}{\gamma_\infty}\frac{d\gamma_0}{d\omega}\bigg|_\infty dz \tag{61}$$

with

$$\gamma_{0\infty} = \sqrt{k_{t\infty}^2} - k_0^2. \tag{62}$$

The upper sign is valid for ϵ surface-wave modes, while the lower sign is valid for μ surface-wave modes. Integrating $d\bar{W}$ over the length h of the considered section of the resonator yields

$$\bar{W}_p = -h\left(\frac{1}{2}(|i_{0+}|^2 + |i_{0-}|^2) \pm |i_{0+}||i_{0-}|\cos(\varphi_+ - \varphi_- - \beta_\infty h)\,\mathrm{sinc}(\beta_\infty h)\right) \cdot jZ_{\epsilon,\mu}\frac{\gamma_{0\infty}}{\gamma_\infty}\frac{d\gamma_0}{d\omega}\bigg|_\infty \tag{63}$$

$$\bar{W}_e = -h\left(\pm\frac{1}{2}(|i_{0+}|^2\,\mathrm{ex}(-2\alpha_\infty h) + |i_{0-}|^2\,\mathrm{ex}(2\alpha_\infty h)) + |i_{0+}||i_{0-}|\cos(\varphi_+ - \varphi_-)\right) \cdot jZ_{\epsilon,\mu}\frac{\gamma_{0\infty}}{\gamma_\infty}\frac{d\gamma_0}{d\omega}\bigg|_\infty. \tag{64}$$

The sinc-function is given by

$$\mathrm{sinc}(x) = \begin{cases}\frac{\sin x}{x}, & \text{for } x \neq 0\\ 1, & \text{for } x = 0\end{cases} \tag{65}$$

and the ex-function is given by

$$\mathrm{ex}(x) = \begin{cases}\frac{e^x - 1}{x}, & \text{for } x \neq 0\\ 1, & \text{for } x = 0.\end{cases} \tag{66}$$

Using the immittance approach derived in Section II, the currents i_{0+} and i_{0+} in each section of the resonator can be calculated. The summation over the energy stored in each section of the resonator yields the total energy

$$W_\infty = \sum_{i=1}^{n} W_{a,e_i}. \tag{67}$$

From (10), (32), and (33), we get

$$\begin{aligned}\mathrm{Res}(Z \mid k_{t\infty}) &= \mathrm{Res}(Z \mid \omega_\infty)\frac{dk_t}{d\omega}\bigg|_\infty \\ &= \mathrm{Res}(Z \mid \omega_\infty)\frac{dk_t}{d\gamma_0}\bigg|_\infty \frac{d\gamma_0}{d\omega}\bigg|_\infty \\ &= \mathrm{Res}(Z \mid \omega_\infty)\frac{\gamma_0}{k_t}\frac{d\gamma_0}{d\omega}\bigg|_\infty \\ &= \frac{|v_\infty|^2}{j4W_\infty}\frac{\gamma_{0\infty}}{k_{t\infty}}\frac{d\gamma_0}{d\omega}\bigg|_\infty\end{aligned} \tag{68}$$

$$\mathrm{Res}(Y \mid k_{t\infty}) = \frac{|i_\infty|^2}{j4W_\infty}\frac{\gamma_{0\infty}}{k_{t\infty}}\frac{d\gamma_0}{d\omega}\bigg|_\infty. \tag{69}$$

Note that $\mathrm{Res}(Z \mid k_{t_\infty})$ and $\mathrm{Res}(Y \mid k_{t_\infty})$ are independent of ω since $W_\infty \sim \frac{d\gamma_0}{d\omega}|_\infty$. Once the residues of Green's function in the plane $z = 0$ have been calculated, the electric and magnetic field components at any given z can be calculated using (19)–(20).

Using the divergence property

$$\mathrm{div}\,\mathbf{E} = 0 \tag{70}$$

$$\mathrm{div}\,\mathbf{H} = 0 \tag{71}$$

the z components of the electromagnetic field can be calculated. This permits the calculation of Poynting's vector, which is defined as

$$\mathrm{Re}(\mathbf{T}) = \frac{1}{2}\mathrm{Re}(\mathbf{E} \times \mathbf{H}^*). \tag{72}$$

The total power transported in a surface-wave mode results from the integration of Poynting's vector over the surface of a cylinder with a radius $\rho \to \infty$ and $z = -\infty \cdots \infty$, cf., [4]

$$P_s = \lim_{z\to\infty}\lim_{\rho\to\infty}\int_{\varphi=0}^{2\pi}\int_{z'=-z}^{z}\mathrm{Re}(T_\rho(\rho,\varphi,z'))\,\rho dz'\,d\varphi. \tag{73}$$

Note that $T_\rho(\rho,\varphi,z') = 0$ if $z' < -h$.

Due to (19) and (20), the z dependency of the electromagnetic field of the surface wave is of the form

$$A\exp(-\gamma_i z) + B\exp(\gamma_i z) \tag{74}$$

cf., [12]. Therefore, the integration over z can be carried out analytically.

VI. ALGORITHM

As a result, the following algorithm, which calculates the electromagnetic field components of the surface wave excited by a Hertzian dipole embedded in a multilayered structure, can be given as follows.

1) Calculate Green's function in the spectral domain using the immittance approach.

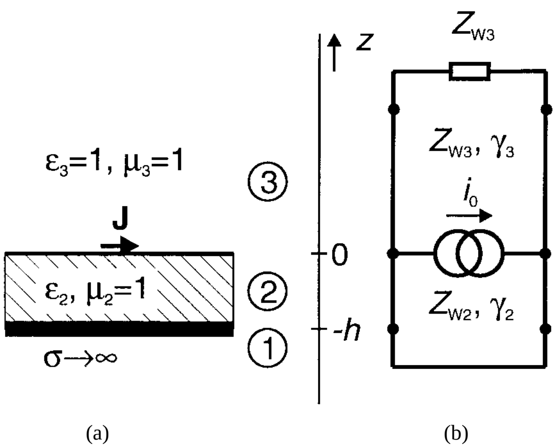

Fig. 4. (a) Microstrip substrate. (b) Corresponding transmission line equivalent circuit resonator.

2) Calculate the propagation constant of a surface-wave mode using a root-finding algorithm.
3) Iterating over the layers of the structure; calculate the energy stored in the resonator equivalent circuit using (63)–(64) as a function of the voltage across the current source or the current through the voltage source.
4) Calculate the residue of Green's function in the spectral domain using the energy residue theorem (68).
5) Calculate the field components using (21)–(24).
6) Calculate the power carried by the surface wave using (73).
7) Repeat the above steps iterating over the propagating surface-wave modes.

This algorithm has been implemented in the microwave CAD program analysis and optimization of planar antennas (AOPA).

VII. Example: Microstrip Configuration

To validate the method derived in this paper, a microstrip configuration as depicted in Fig. 4 is considered as an example. While the presented algorithm is not restricted to a special substrate configuration, a moderate size expression results in case of a microstrip configuration, which is suitable for a treatment in this paper. Furthermore, the results can be compared with the literature.

The microstrip substrate, which forms layer two, is characterized by its relative permittivity ϵ_2 and its thickness h. The backside metallization forming layer one is characterized by its conductivity $\sigma \to \infty$. The substrate is covered by the free-space with $\epsilon_3 = 1$, $\mu_3 = 1$ (layer three). In Fig. 4(b), the corresponding transmission line resonator equivalent circuit is shown. We obtain

$$Z = \frac{Z_{W\,2} Z_{W\,3} \sinh \gamma_2 h}{Z_{W\,2} \sinh \gamma_2 h + Z_{W\,3} \cosh \gamma_2 h} \tag{75}$$

where Z is the input impedance of the resonator equivalent circuit seen by the current source. The pole of Z can be found from

$$Z_{W\,2} \sinh \gamma_2 h + Z_{W\,3} \cosh \gamma_2 h = 0 \tag{76}$$

which is the denominator of Z. We restrict ourselves to the first pole, which corresponds to the TM_0 surface-wave mode [4]. Note that by (10), $\gamma_{2\infty} = j\beta_{2\infty}$ is purely imaginary, whereas $\gamma_{3\infty} = \alpha_{3\infty}$ is real. This means, a propagating wave exists in Section II of the resonator, whereas an evanescent wave exists in Section III. Backsubstitution yields the spectral domain representation of the electric field in the plane $z = 0$

$$\tilde{E}_{\epsilon,\mu} = \frac{Z_{\epsilon,\mu}^{(2)} Z_{\epsilon,\mu}^{(3)} \sinh \gamma_{2\infty} h}{Z_{\epsilon,\mu}^{(2)} \sinh \gamma_{2\infty} h + Z_{\epsilon,\mu}^{(3)} \cosh \gamma_{2\infty} h} I_0 l \cdot \begin{cases} -\sin \Phi, & \text{for } \epsilon \text{ components} \\ \cos \Phi, & \text{for } \mu \text{ components.} \end{cases} \tag{77}$$

Using the immittance approach, we obtain for the current in Section II

$$i_{2+} = i_{2-} = \frac{-v_\infty}{2 Z_{W\,2} \sinh \gamma_{2\infty} h} \tag{78}$$

whereas for the current in Section III, we obtain

$$i_{3+} = \frac{1}{Z_{W\,3}} v_\infty \tag{79}$$

$$i_{3-} = 0. \tag{80}$$

v_∞ is the voltage across the current source of the equivalent circuit shown in Fig. 4 in case of resonance. In the case of our example, we obtain for the energy stored in Section II of the resonator

$$\bar{W}_p = -\frac{d\gamma_3}{d\omega}\bigg|_\infty \frac{|v_\infty|^2 \epsilon_2 \gamma_{3\infty} k_0 (2\gamma_{2\infty} h + \sinh 2\gamma_{2\infty} h)}{8 Z_0 \gamma_{2\infty}^3 \sinh^2 \gamma_{2\infty} h}. \tag{81}$$

and for the energy stored in Section III of the resonator

$$\bar{W}_e = -\frac{d\gamma_3}{d\omega}\bigg|_\infty \frac{|v_\infty|^2 k_0}{4 Z_0 \gamma_{3\infty}}. \tag{82}$$

Z_0 is the characteristic impedance of the free-space. With (68), after some lengthy calculations using complex trigonometric identities, we get the result

$$\text{Res}(Z \mid k_{t\infty}) = \frac{Z_0}{jk_0} \frac{\gamma_{3\infty}}{k_{t\infty}} \frac{\gamma_{3\infty}\gamma_{2\infty}}{\frac{\gamma_{3\infty}}{\gamma_{2\infty}}(\epsilon_2 \gamma_{3\infty} h + 1) - \frac{\gamma_{2\infty}}{\gamma_{3\infty}}\left(\frac{\gamma_{3\infty} h}{\epsilon_2} + 1\right)}. \tag{83}$$

The same result is obtained by calculating the residue directly form (75). Using (21), we obtain

$$E_{\rho 0} = -\cos\varphi I_0 l \sqrt{\frac{k_{t\infty}}{2\pi\rho}}\, e^{-j\frac{\pi}{4}} e^{-jk_{t\infty}\rho} \frac{Z_0}{jk_0} \frac{\gamma_{3\infty}}{k_{t\infty}} \cdot \frac{\gamma_{3\infty}\gamma_{2\infty}}{\frac{\gamma_{3\infty}}{\gamma_{2\infty}}(\epsilon_2 \gamma_{3\infty} h + 1) - \frac{\gamma_{2\infty}}{\gamma_{3\infty}}\left(\frac{\gamma_{3\infty} h}{\epsilon_2} + 1\right)} \tag{84}$$

which is the ρ component of the TM_0 surface-wave mode's electric field in the plane $z = 0$. The results comply with those reported in [4].

VIII. Conclusion

In this paper, a novel method for the analytic calculation of the surface-wave excitation in multilayered structures has been presented. The immittance approach has been used to set up a transmission line equivalent circuit of the stratified medium. A surface-wave mode corresponds to a resonance of the equivalent circuit. A novel network theorem has been derived that relates the energy stored in a lossless one-port operated at the resonance frequency to the residue of its immittance function. A discretized transmission line model with arbitrary dispersion characteristics is used to calculate the energy stored in the resonator. This allows for an analytic calculation of the surface wave's field components. The new method circumvents the need of a numerical calculation of residues, which is an ill-conditioned problem. A simple algorithm iterating over the layers of the structure has been given, which is well suited for an implementation in microwave CAD programs. Considerable analytic labor is saved in case of multilayered structures. Furthermore, the structure of the substrate need not be known in advance, adding versatility to CAD programs. The method has been applied to a microstrip structure and the results have been compared with the literature.

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