Application of the Fast Far-Field Approximation to the Computation of UHF Pathloss over Irregular Terrain

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***Abstract*—The recent availability of fast numerical methods has rendered the integral-equation approach suitable for practical application to radio planning and site optimization for UHF mobile radio systems. In this paper, we describe a conceptually simple scheme for the efficient computation of UHF radial propagation loss over irregular terrain, which is based on the fast far-field approximation. The method is substantially faster than conventional integral-equation (IE) solution techniques. The technique is improved by incorporating the Green's function perturbation method and we outline a way in which the formulation can be made more exact. Computational issues such as terrain profile truncation and the effect of small-scale roughness are addressed. The method has been applied to gently undulating terrain and compared to published experimental results in the 900-MHz band. It has also been successfully applied to more hilly terrain and to surfaces with buildings added.**

***Index Terms*— Mobile communication, propagation, terrain factors.**

I. Introduction

The computation of UHF propagation loss is of central importance to the planning of wireless communications systems. Coverage analysis and site-optimization tools require efficient and accurate propagation algorithms. If such tools are to operate with the minimum of supervision it is important that field-computation algorithms are reliable and robust. In this regard, the deterministic (as opposed to empirical) approach to propagation modeling has clear advantages.

One particularly important problem, the subject of this paper, is the computation of UHF land–mobile radio radial-propagation loss over irregular terrain. This is a well-known problem and many solutions have been proposed. In this paper, an integral-equation formulation is adopted: a formulation, which can, in principle, be applied to other typical problems arising in wireless communications engineering where surface scattering is the predominant physical phenomenon. These will not be considered; however, many of the key ideas are manifested in the treatment of the terrain-propagation problem.

Other deterministic methods which have been applied to terrain propagation include the geometric theory of diffraction (GTD) and also methods deriving from the parabolic approximation in both integral and differential form. These methods are well known. GTD is well suited to high-frequency asymptotic problems, however, the grazing incidence associated with terrain propagation (resulting in the delocalization of interaction regions), the large number of vertices in a typical terrain profile, and the problem of multiple transition-zone diffraction makes its application to terrain propagation rather difficult to justify. The main advantage of the parabolic equation (PE) is its ability to handle tropospheric refractive index variations. The form of the parabolic approximation (in the context of integral equations) involves the assumption of forward propagation, the extraction of a phase term $\exp(j\beta x)$ (assuming two-dimensional propagation in the x direction) from both the propagating field and the Green's function for the problem together with certain assumptions about the derivatives of the reduced field. These and indeed further simplifications arise in a more natural way in recent fast-solution strategies for surface integral-equation (IE) formulation. Finite-difference and spectral-domain solutions of the PE require careful handling of the impedance boundary used to satisfy the radiation condition.

We adopt an exact IE formulation as a starting point. The application of IE methods to antennas and in microwave engineering is well known. In those disciplines, people typically seek solution methodologies that have wide applicability. Our task is to exploit the specific nature of the terrain-propagation problem to obtain a fast algorithm for the solution of the integral equation. The IE formulation lends itself particularly well to this specialization because it is manifestly physical—we can separate specific interactions between parts of the surface and remove them or approximate them as we see fit. This approach is largely precluded when we opt for a differential equation methodology.

The method proposed in this paper provides massive computational savings when compared to previous attempts to apply surface integral equations to terrain-propagation modeling [1], [2].

Section II outlines the IE formulation to the terrain scattering problem and identifies the main restriction of conventional solutions, namely the prohibitively large computational burden encountered. Section III introduces the fast far-field algorithm (FAFFA) [3] and further implementational considerations are discussed in Section IV. Most notable of these is the incorporation of the Green's function perturbation method (GFPM) [4] to expedite the FAFFA scheme even further. A further po-

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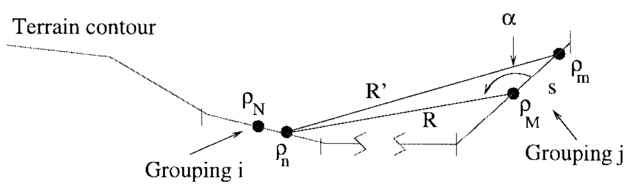

Fig. 1. Groupings in FAFFA scheme.

tential improvement to the FAFFA formulation is sketched in Section V. Section VI considers some further issues pertaining to the UHF propagation problem, namely the truncation of the terrain profile and the effect of small-scale random roughness. We close with numerical examples which demonstrate the accuracy and efficiency of the methods outlined.

II. Problem Formulation

A certain degree of abstraction is required in order to tackle any scattering problem of this scale. Our terrain model assumes invariance in one dimension reducing the problem to two dimensions. The terrain is assumed to be perfectly electrically conducting (PEC), an assumption really only justifiable at the grazing incidences that concern us. The surface is considered to be composed of linear segments connecting sampled terrain points. A later section outlines how the introduction of small scale roughness need not significantly affect the application of the FAFFA algorithm. We stress also that extension to three dimensions is perfectly feasible as is the treatment of dielectric surfaces.

The terrain model described above allows us to use (assuming TM^z polarization with time dependence of $\exp(j\omega t)$ assumed and suppressed) the two-dimensional (2-D) electric field integral equation (EFIE) for a PEC surface [5].

A simple numerical solution of the EFIE proceeds by modeling the unknown surface current J in terms of D pulse-basis functions of length Δs centered on the D collocation points $\rho_1 \cdots \rho_D$. This leads to the following $D \times D$ matrix equation [5]

$$ZJ = V \tag{1}$$

where

$$Z_{mn} = \frac{\beta\eta}{4} H_0^{(2)}(\beta|\rho_m - \rho_n|)\Delta s \tag{2}$$

$$Z_{mm} \simeq \Delta s \frac{\beta\eta}{4}\left(1 - j\frac{2}{\pi}\ln(K\Delta s)\right) \tag{3}$$

$$V_m = E^i(\rho_m) \tag{4}$$

where B is the wavenumber and K is a constant equal to $\frac{1.781\beta}{4e}$. J is a vector, whose unknown entries are the coefficients of the pulse-basis functions and V is a vector whose entries are the incident electric field at the D collocation points. The incident field is defined as the field that would be present in the absence of the scatterer. This formulation allows for interaction between *all* sections of the scatterer, regardless of intervisibilty conditions, and so provides for precise calculation of *all* multiple scattering effects, a provision, which is facilitated by the fact that J is assumed to radiate in free-space, as evidenced by the presence of the free-space Green's function $H_0^{(2)}$ in the matrix entries.

The necessity to accurately model the quickly varying J, coupled with the huge scale of the UHF propagation problem (many tens of thousands of wavelengths) means that the matrix Z is of extremely high order and, generally, cannot be explicitly stored. However, a solution is feasible if we employ iterative solution methods that do not explicitly store Z. Instead, (1) is solved by recursively updating an estimate of J until some convergence criterion is satisified. Probably the most physical iterative scheme is the "forward/backward" [6] or "method of ordered interactions" [7] scheme, which successively incorporates effects due to the appropriate forward and backward scattering events at each iteration. Reference [1] uses a simplified version that allows for forward-scattered energy only, once again, sufficient for the specific problem addressed in this paper. We stress, however, that the fast methods we describe below are general and can be applied to *any* iterative scheme and the assumption of forward scattering is by no means fundamental. Iterative schemes, while computationally tractable, are extremely time consuming if applied in their basic form. The computational burden arises from the numerical calculation of scattering integrals

$$\sum_{n=1}^{D} Z_{mn}J_n. \tag{5}$$

Specifically, a forward-scattering scheme approximates Z as being lower triangular and writes for $m = 1 \ldots N$

$$Z_{mm}J_m = V_m - \sum_{n<m} Z_{mn}J_n. \tag{6}$$

The "point to point" interactions inherent in such a scheme means that the computational complexity is $O(D^2)$. An efficient implementation of such a scheme must address the issue of expediting these summations in some simple and accurate fashion. It is to this issue that we turn our attention.

III. Fast Far-Field Algorithm

There exist a number of efficient iterative schemes such as the adaptive integral method [8], matrix decomposition algorithm [9], and various forms of the fast multipole method [10], [11] to which the fast far-field algorithm is related. All succeed by a process of grouping points together and a twostep approximation of the point to point interactions inherent in an iterative scheme. This two-step scheme involves first the calculation of fields scattered to the group centres and then the dissemination of this scattering information to other points in each group. Indeed, beyond the scope of this paper, but discussed in [12], is the idea that the success of the "well-informed" basis sets of [13]–[15] can be interpreted in a manner very similar to the discussion below.

The FAFFA proceeds by grouping together large numbers of collocation points, each group having a designated group center. Also defined for each group j is a "near-field" NF_j, usually consisting of the group itself and neighboring groups. Other groups are considered to lie in j's far field FF_j.

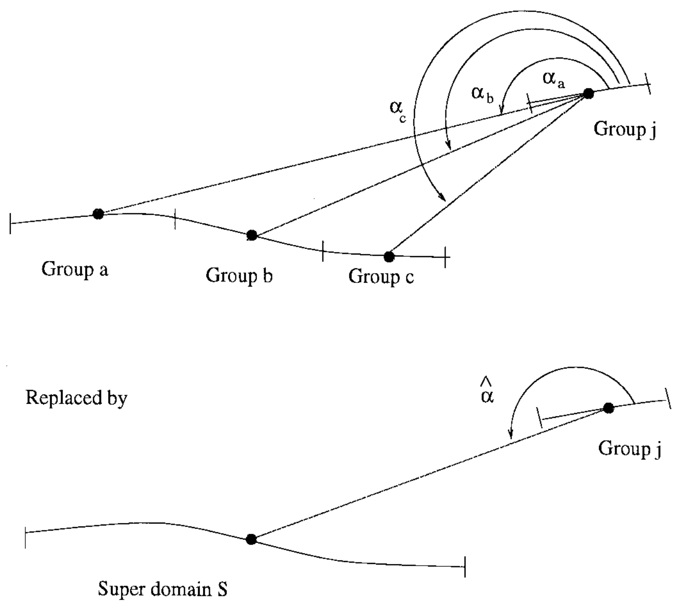

Fig. 2. Amalgamation of integration domains.

Now, for a point ρ_m in a group j with center ρ_M (see Fig. 1), we write

$$\sum_{n=1}^{N} Z_{mn}J_n = \sum_{i\in \mathrm{FF}_j} F_{Mm}^i \sum_{n\in i} Z_{Mn}J_n + \sum_{i\in \mathrm{NF}_j}\sum_{n\in i} Z_{mn}J_n \tag{7}$$

where F_{Mm}^i are constants to be derived later. The first i summation is over groups deemed to be in j's far field, while the second summation is over j's near field. The ability to *reuse* the calculation $\sum_{n\in i} Z_{Mn}J_n$ to efficiently approximate the field scattered from a far-field group i to each ρ_m in j indicates the key computational saving of this scheme. The necessity to calculate exactly the near-field interactions is not a great restriction, especially when one considers that this is but a small percentage of the overall burden.

The far-field approximation to the Hankel function

$$H_0^{(2)}(x) \simeq \sqrt{\frac{2}{\pi x}} e^{-j(x-\frac{\pi}{4})}, \quad x\to\infty \tag{8}$$

will be our starting point in deriving the "shifting functions" F_{Mm}^i.

Referring to Fig. 1, we apply the cosine rule to the quantities R, R', α, and s to write (assuming R' is sufficiently large to enable us to use the above far-field approximation)

$$Z_{mn} = Z_{Mn}A_{nMm}e^{-j\phi_{nMm}} \tag{9}$$

where

$$A_{nMm} = \left(1+\frac{s^2-2Rs\cos\alpha}{R^2}\right)^{-\frac{1}{4}} \tag{10}$$

$$\phi_{nMm} = \beta R\left(\left(1+\frac{s^2-2Rs\cos\alpha}{R^2}\right)^{\frac{1}{2}}-1\right). \tag{11}$$

Both A and ϕ depend on n through their dependence on R and α and so we cannot make the identification

$$F_{Mm}^i = A_{nMm}e^{-j\phi_{nMm}} \tag{12}$$

as we demand that F_{Mm}^i be independent of n.

However, if we define

$$A_{Mm}^i = A_{NMm} \tag{13}$$

$$\phi_{Mm}^i = \phi_{NMm} \tag{14}$$

where ρ_N is the center of group i, we can use (7) with

$$F_{Mm}^i = A_{Mm}^i e^{-j\phi_{Mm}^i}. \tag{15}$$

These "shifting functions" differ slightly from those presented by Lu and Chew [3] in being more accurate though at higher computational cost. Lu's and Chew's functions have no $\hat{R}$ dependence and both variants coincide as the group separation $\hat{R}\to\infty$. This removal of the $\hat{R}$ dependence enables Lu and Chew to achieve considerably more "recycling," an idea expanded and improved in the specific instance of UHF-terrain scattering by the tabulated interaction method (TIM) [12]. We will use the shifting functions of (13) and (14) because they are suggestive of further refinements that can be made—an idea pursued in Section V.

IV. Implementation Considerations

The FAFFA scheme of the last section offers considerable computational savings, but further speed ups can be had by implementing the three ideas outlined in this section.

To facilitate the introduction of these ideas it would be perhaps beneficial to introduce some terminology. Each grouping in the FAFFA scheme has a dual role—that of radiating fields toward other groups and receiving fields scattered from other groups. We will refer to a group performing the former task as being an *integration domain* and one performing the latter task as being an *observation domain*. Hence, when one group is being an observation domain, all others are behaving as its integration domains and so on.

- **Integration-Domain Amalgamation**—the first implementation consideration, as suggested in [3], is to note that when calculating fields scattered to a given observation domain j; a further efficiency can be had by amalgamating integration domains that share a similar angular relationship with j. For example, consider the three integration domains a, b, c of Fig. 2, each scattering fields to observation domain j. If α_a, α_b, and α_c are close in value, we replace the three integration domains with one superdomain S and write

$$\sum_{i\in a,b,c} F_{Mm}^i \sum_{n\in i} Z_{Mn}J_n \simeq F_{Mm}^S \sum_{n\in S} Z_{Mn}J_n \tag{16}$$

where F_{Mm}^S is the value of F calculated with respect to the center of the superdomain S. A simple geometrical rule can govern this amalgamation procedure; that is, amalgamate the integration domains a, b, c if their angular relationship with j satisfies

$$|\cos\alpha_i - \cos\hat{\alpha}| < \epsilon \tag{17}$$

for $i \in a, b, c$ where $\hat{\alpha}$ is the angle subtended by the center of the resultant superdomain and ϵ is a prechosen threshhold constant. The utility of this idea can be easily seen. If there are P points in each of the groupings

a, b, c, j, then the computation on the left-hand side of (16) will necessitate $6P-3$ complex multiplications, $3P$ to calculate the fields scattered to the center of j and $3(P-1)$ to shift these results to the other points in the group. A similar logic dictates that the right-hand side will only require $4P-1$ complex multiplications. As evidenced by the numerical results of Section VII this procedure can produce exceptionally large integration domains in the case of propagation over terrain where the gently undulating nature of the terrain profile leads to slowly varying angular relationships between groups, a situation readily exploited by this concept.

- **Large Integration Steps**—the second implementation consideration results from the physical nature of the propagation mechanism. The grazing incidence coupled with the slowly undulating nature of the surface results, in a forward-scattering context, in the integrand occurring in the EFIE being very slowly varying. Accordingly, the numerical integration of these integrals as denoted by (5) can use a suitably larger step size. An asymptotic approximation can be employed to efficiently calculate integrals describing backscattered radiation. More details on these ideas can be found in [16]. It is hard to qualify exactly the extent of speed up obtained by such an approximation, reliant as it is on the terrain being gently undulating and the incidence being grazing, but the numerical examples cited in the results section used integration steps as large as 10λ.
- **FAFFA/GFPM Hybrid**—given the nature of the efficiencies introduced by the FAFFA one would expect that making the group sizes as large as possible would optimize the computational savings. Indeed, results obtained using the natural basis set [13]–[15] would seem to indicate that very large groupings are feasible for the types of problems that interest us. However, this ignores the necessity to calculate exactly the near-field contributions, which, of course, includes a group's self interaction. This procedure's computational intensity grows quadratically with the group size and, thus, imposes limits on the optimum group size. The third implementational consideration addresses this important issue, by introducing the Green's function perturbation method to efficiently calculate a group's self interaction and, hence, freeing us to make the group sizes as large as is possible. The GFPM basically approximates the Z matrix associated with a scattering problem by one that is Toeplitz or cyclical in structure. It does this by approximating the Euclidean distance arising in the argument of the Hankel function with the arclength distance instead. Specifically, we approximate

$$|\rho-\rho'| \simeq (c-c') \tag{18}$$

where $(c-c')$ is the arclength distance *along the terrain profile* between the points ρ and ρ'. The IE, which results from this approximation, is convolutional in form and can be efficiently solved using fast Fourier transform techniques. Applying this to the specific problem of rapid calculation of a group's self-interaction is a simple matter. A new scattering problem is postulated with a new "incident field" consisting of the original incident field on the group plus the field scattered from its integration domains. This problem is then solved using the GFPM. While the GFPM is exact for a flat plate, we stress that it can deal accurately with small-scale roughness, providing the slope variations are not too large [17], [18]. Thus, it is ideally suited to calculate the self interactions of groups occurring in the present context, where we expect that locally the terrain would have gentle slope variations. Indeed, results presented in Section VII use the GFPM to calculate the self interaction of groups up to 600 λ in length that span several linear segments. The addition of small-scale roughness to these linear segments is feasible, too, as evidenced in [19].

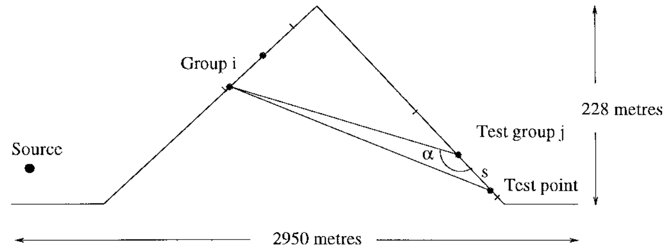

Fig. 3. Geometry for wedge example.

V. IMPROVED FAFFA SCHEME

The FAFFA scheme disseminates information about fields scattered to a group j from a group i by means of two shifting functions ϕ and A. Of these, ϕ is the more important and depends on the quantites α, s, and R. The introduction of the (in some sense) "average" quantities $\hat{\alpha}$ and $\hat{R}$ enables us write

$$\phi(s,\hat{\alpha},\hat{R}) = \phi^i_{Mm} \tag{19}$$

and facilitates "recycling" of scattered field information as explained in Section III.

As stated earlier, this expression is suggestive of a potentially more accurate formulation, as discussed below. Specifically, we introduce an improved estimate of ϕ

$$\phi(s,\alpha,R) \simeq \phi\left(s,\hat{\alpha},\hat{R}\right) + (\alpha-\hat{\alpha})\frac{\partial\phi}{\partial\alpha}\bigg|_{\alpha=\hat{\alpha}}. \tag{20}$$

With this improved ϕ estimate we get

$$\sum_{n\in i} Z_{mn}J_n \simeq A^i_{Mm}\exp\left(-j\phi^i_{Mm}\right) \times \sum_{n\in i} Z_{Mn}J_n \exp\left(-j(\alpha-\hat{\alpha})\frac{\partial\phi}{\partial\alpha}\bigg|_{\alpha=\hat{\alpha}}\right) \tag{21}$$

$$= A^i_{Mm}\exp\left(-j\phi^i_{Mm}\right)\sum_{n\in i} Z_{Mn}J_n f(s,\alpha). \tag{22}$$

Unfortunately, the summation on the right-hand side of (22) cannot be reused as it takes a different value for each s.

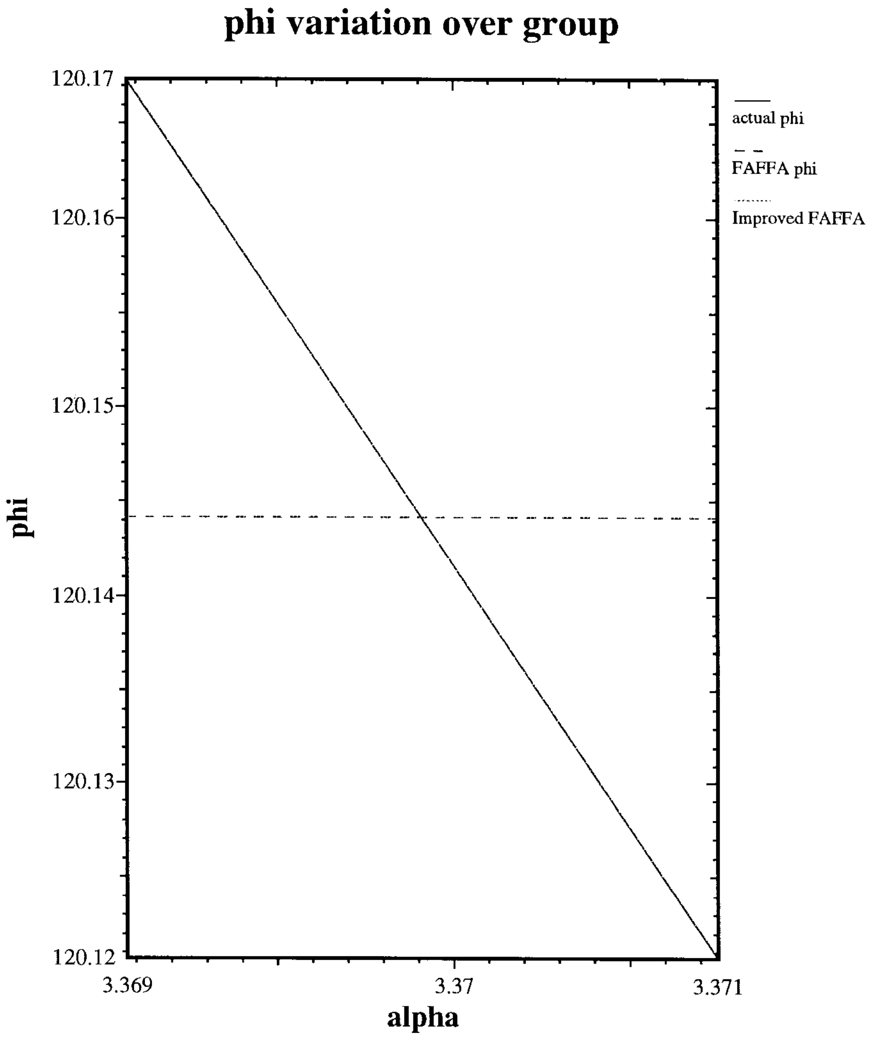

Fig. 4. Φ variation in wedge example.

Remembering that it is the potential reuse of calculations that offers the FAFFA its computational efficiency prompts us to find an expression that will facilitate it. With this in mind, we write

$$f(s,\alpha) = f(0,\alpha) + s\frac{\partial f}{\partial s}\bigg|_{s=0} + \frac{s^2}{2}\frac{\partial^2 f}{\partial s^2}\bigg|_{s=0} + \cdots. \tag{23}$$

Inserting this into (22) yields

$$\sum_{n\in i} Z_{mn}J_n \simeq A^i_{Mm}\exp\left(-j\phi^i_{Mm}\right)\Bigg(\sum_{n\in i} Z_{Mn}J_n f(0,\alpha) + s\sum_{n\in i} Z_{Mn}J_n\frac{\partial f}{\partial s}\bigg|_{s=0} + \frac{s^2}{2}\sum_{n\in i} Z_{Mn}J_n\frac{\partial^2 f}{\partial s^2}\bigg|_{s=0} + \cdots\Bigg). \tag{24}$$

Noting that

$$f(0,\alpha) = 1$$

indicates that the first term of (24) is the usual FAFFA summation. The other summations provide corrections to the basic formulation at little extra computational expense, as it can be shown that for each sum

$$\sum_{n\in i} Z_{Mn}J_n\frac{\partial^k f}{\partial s^k}\bigg|_{s=0} = K_k\sum_{n\in i} Z_{Mn}J_n(\alpha-\hat{\alpha})^k \tag{25}$$

and so the sums are broadly similar, their terms only varying by a real multiplicative factor. Obviously, we must be careful in applying this approach as $|f(s,\alpha)| = 1$ and premature truncation of the series (23) can result in error. However, for most applications, only a modest number of terms are necessary.

To illustrate the concepts outlined in this section, consider the example of wave propagation over a 2-D wedge structure, as shown in Fig. 3. A source is placed 10.4 m over the left-most point, radiating at 970 MHz. The surface was divided into groups of length 35λ. For a fixed test point in group j, we move through the points in group i causing α to vary. Fig. 4 compares the value of the following:

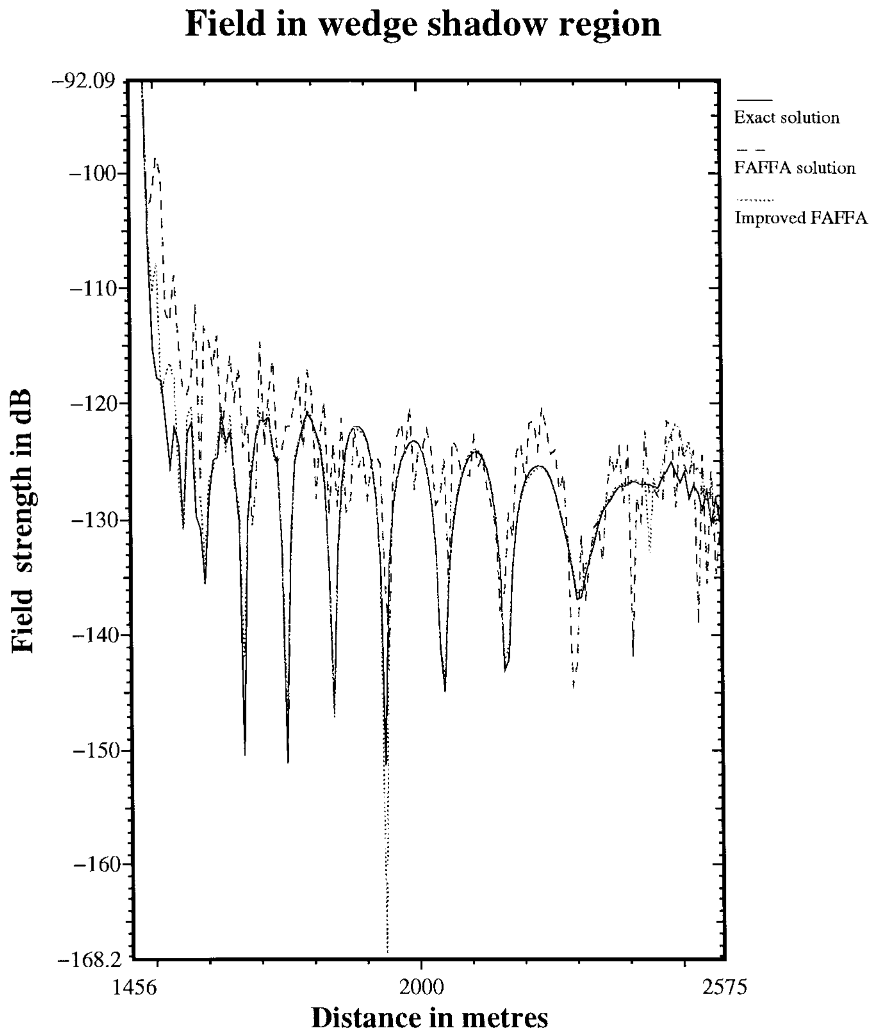

Fig. 5. Field strength over wedge.

- the exact phase relationship $\phi(s, \alpha, \hat{R})$;
- the FAFFA approximate phase relationship $\phi(s, \hat{\alpha}, \hat{R})$;
- the improved FAFFA phase relationship $\phi(s, \hat{\alpha}, \hat{R}) + (\alpha - \hat{\alpha})\frac{\partial \phi}{\partial \alpha}|_{\alpha=\hat{\alpha}}$.

Our new linear estimate of ϕ agrees within graphical accuracy with the actual value of ϕ and provides a significant improvement over the usual FAFFA estimate given by the constant line in Fig. 4. The improvement in field calculation is displayed in Fig. 5, which shows the fields calculated 2.4 m above the surface for a region in deep shadow and also plots a reference forward-scattering solution. The improved FAFFA, which retained four terms of (23) offers a significantly better solution than the basic FAFFA, at only a slightly higher computational cost as illustrated below.

Solution scheme	**Computation time**
FAFFA	140
Improved FAFFA	166

VI. Profile Truncation and Small-Scale Roughness

Before presenting numerical results based on the ideas discussed it would be useful to address some of the issues that our terrain model raises, specifically the issue of how to decide what portions of terrain interact with each other and the treatment of the small-scale roughness that lies along the terrain profile and is inevitably random in nature.

Obviously, we cannot deal with an infinitely long terrain profile and must choose some way of truncating it. The forward-scattering assumption achieves this very naturally, we truncate the profile just under the antenna and "march" the solution forward, each surface point being allowed interact only with points between it and the antenna. A potential problem with this approach is the possibility of the truncation point acting as a line source with energy being diffracted around the truncation point and "under" the terrain profile. While we have yet to encounter this in our calculations, we acknowledge its potential manifestation. It can be simply addressed by extending the truncation point further backwards from the antenna, thus reducing this diffraction effect. A similar procedure can be used to prevent erroneous diffraction effects at the other truncation point in a "forward/backward" scheme.

An obvious route to improving the terrain model is the addition of some small-scale random roughness to the ter-

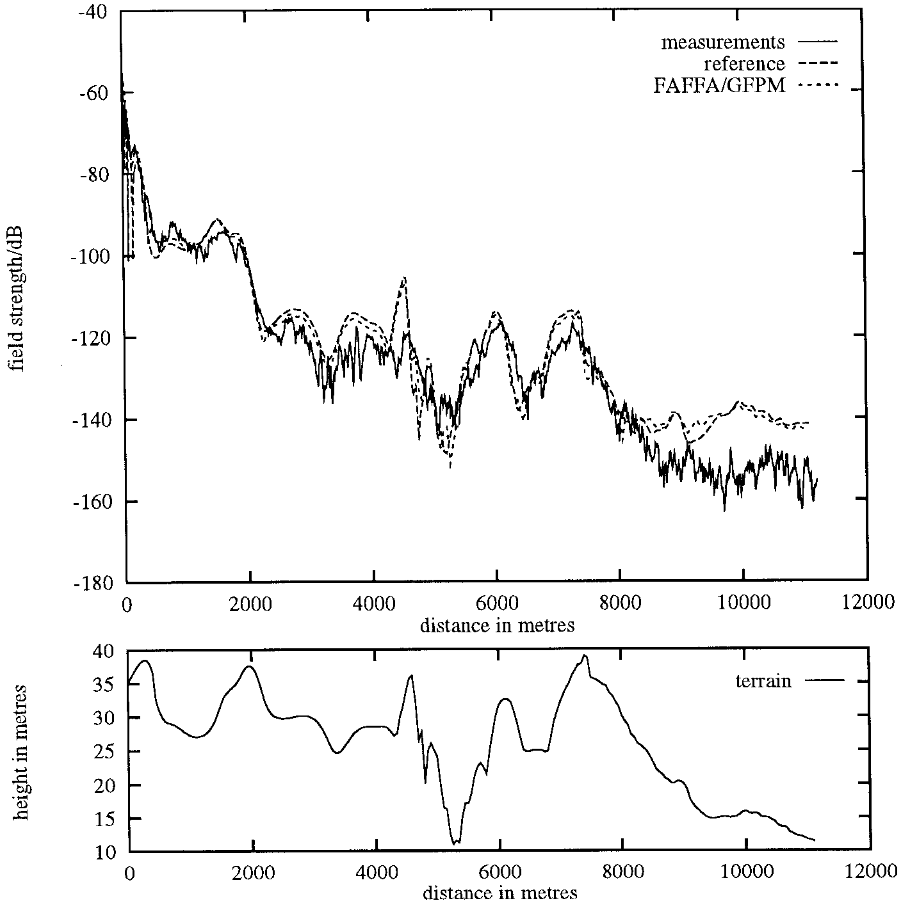

Fig. 6. Fields over profile Hjorringvej.

rain profile. The effects of this roughness can be significant (especially at off-grazing incidence) and it is an important consideration. One potential treatment is to the examine the statistics generated by a large number of computations, each using a particular realization of surface roughness. The techniques outlined in this paper can efficiently deal with such a surface realization at no extra computational cost. While the specifics of the group to group interaction will change, the field being scattered more diffusely for example, the implementation of the basic FAFFA algorithm will remain broadly the same. We can still compute group centres and the geometrical factors $\hat{R}$ and $\hat{\alpha}$ (which for groups far away will not alter dramatically). What will change is the value of the recycled summation

$$\sum_{n\in i} Z_{Mn} J_n$$

reflecting the more diffuse nature of the scattering. The near-field scattering will be significantly different, but as our near-field calculations are done exactly or via the GFPM, which can handle moderate surface roughness, this need not concern us. We will not pursue the issue of random roughness further here, but instead refer the interested reader to [19].

VII. Results and Discussion

The techniques discussed in this paper have been applied with success to UHF propagation problems. The first example is a terrain profile taken from northern Denmark. The antenna was placed 10.4 m above the left-most point and radiated at 970 MHz. Fig. 6 shows the following.

- The terrain profile Hjorringvej.
- The measured field strength.
- The field strength calculated using a slow forward-scattering reference solution.
- The field strength calculated using the methods described in this paper. We used groups of 200 m ($\simeq 600\lambda$) in length. The far field was restricted to a each groups self interaction and this self interaction was calculated using the GFPM. An ϵ tolerance was set at 10^{-4} and this produced very large integration domains up to 9 km in length. We also exploited the slowly varying nature of the integrand to use large integration steps (some 10 λ). Note the excellent agreement between the predicted and measured results. The discrepancy between the measured data and predicted results over the last kilometer is due to the presence of a small urban area not included in our terrain model.

The table below emphasises the computational savings available with the FAFFA/GFPM hybrid. Times quoted are

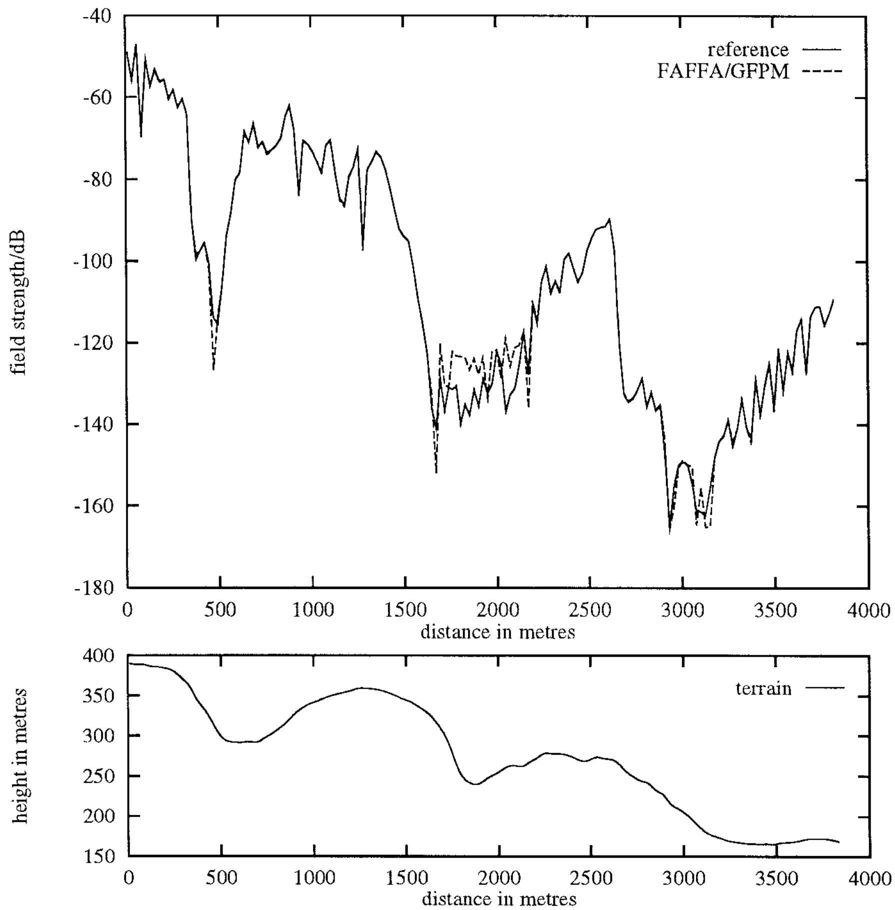

Fig. 7. Fields over mountainous profile.

in seconds and the computations were performed on an IBM Power PC.

Solution scheme	**Hjorringvej**	**Mountainous**
Reference	100857	12800
FAFFA/GFPM	50	426

The FAFFA/GFPM hybrid obviously offers significant computational savings over conventional IE-solution techniques. However, for gently undulating terrain, as is the case in this example, the FAFFA/GFPM's performance can be significantly bettered by both the natural basis set and the TIM, which can produce results of equal accuracy in around 2 s on the same machine.

Our second example involves a mountainous terrain profile displaying steeper slopes than our first example. Unfortunately, we have no measured data for this example, but we would expect a strong agreement between the IE solutions and experimental results as noted in the last example. Once again, an antenna radiated at 970 MHz, this time 52 m over the left-most point. Fig. 7 shows the following.

- The terrain profile.
- The field strength calculated using a slow forward-scattering reference solution.
- The field strength calculated using the methods described in this paper. We used groups of 5 m ($\simeq 15\lambda$) in length. Once again, the far field was restricted to a each groups self interaction and this self interaction was calculated using the GFPM, though the smaller nature of the groups restricted the speed ups attainable using the GFPM. An ϵ tolerance procedure produced integration domains up to 20 m in length.

Once again, the FAFFA/GFPM offers significant computational savings though the efficiencies achieved are not as dramatic as in the last example. The reason for this is essentially the more mountainous nature of the terrain profile which produces significantly greater angular relationships between groups. Thus, it is necessary to form accordingly smaller groups to maintain accuracy. The steep slopes also restrict our ability to use large integration steps. Despite this, the FAFFA/GFPM still produces good results in relatively quick time and in this case outperforms the natural basis set, which does not perform optimally in a mountainous environment. We mention again the TIM, which, with recent improvements, can produce a solution to this problem in around 10 s. However, the TIM in it's present form cannot deal with more general scattering environments, something the FAFFA can do quite easily.

To illustrate this, in Fig. 8, we have randomly added some buildings to the terrain profile of Example 1 and used IE solutions to predict the resultant field strength. Group sizes were 50 m along the flat areas, but were reduced to $\simeq 5\lambda$

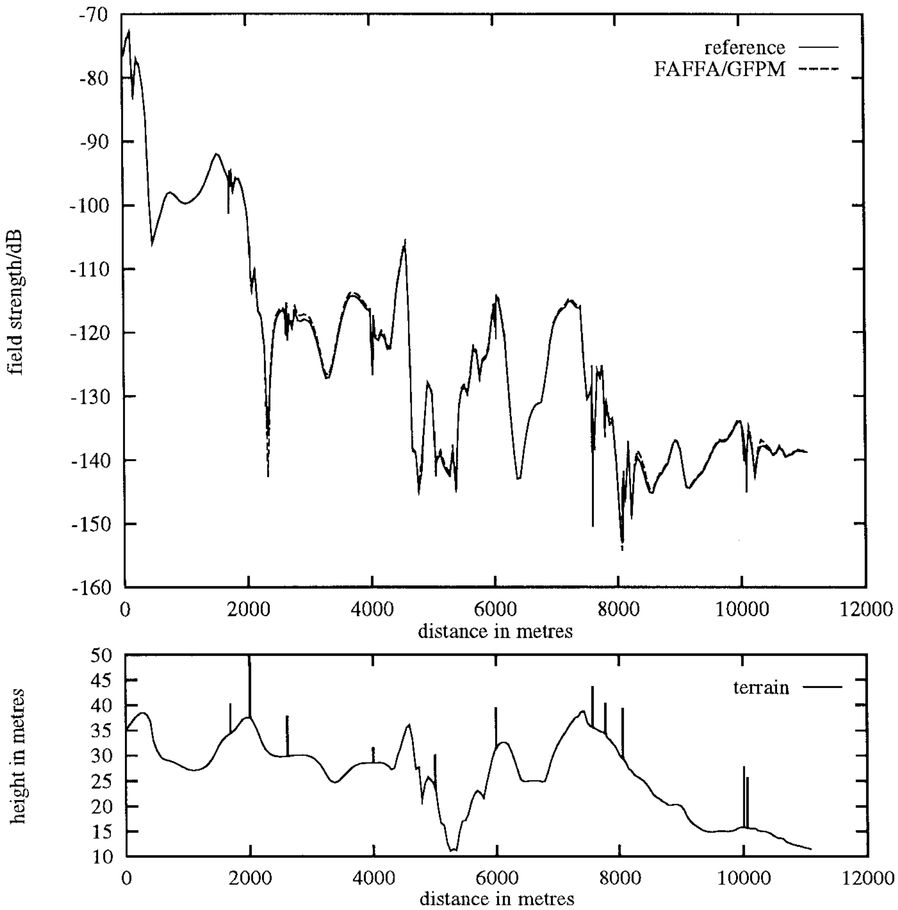

Fig. 8. Buildings added to Hjorringvej profile.

along the buildings. This is necessary due to the sharp angles encountered in these vicinties. Employing the improved FAFFA scheme of Section V produces accurate answers when compared against the reference solution, accurately predicting the local effects of the buildings.

VIII. CONCLUSION

A fast solution method has been described which renders tractable the solution of large scale UHF propagation problems using an IE formulation. The method proceeds by grouping points together and replaces the point to point interactions of standard iterative solutions with group to group interactions. Implementational considerations have been discussed, most notably the incorporation of the Green's function perturbation method to calculate each group's self interaction. An improved version of the "shifting function" ϕ was introduced, which can improve the techniques performance for more challenging problems such as scattering from a wedge. The issues of profile truncation and small-scale roughness effects were addressed and numerical results presented which showed excellent agreement with published measured data.

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REFERENCES

[1] J. T. Hviid, J. Bach Anderson, J. Toftgärd, and J. Bøjer, "Terrain based propagation model for rural area—An integral equation approach," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 41–46, Jan. 1995.
[2] J. T. Johnson, R. T. Shin, J. C. Eidson, L. Tsang, and J. A. Kong, "A method of moments model for VHF propagation," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 115–125, Jan. 1997.
[3] C. C. Lu and W. C. Chew, "Fast far-field approximation for calculating the RCS of large objects," *Microwave Opt. Tech. Lett.*, vol. 8, no. 5, pp. 238–241, 1995.
[4] D. Moroney and P. J. Cullen,"The Green's function perturbation method for the solution of electromagnetic problems," *Progress Electromagn. Res. PIER*, vol. 15, pp. 221–252, 1997.
[5] C. A. Balanis, *Advanced Engineering Electromagnetics*. New York: Wiley, 1989.
[6] D. Holliday, L. L. DeRaad, Jr., and G. J. St.-Cyr, "Forward backward: A new method for computing low grazing angle scattering," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 722–729, 1996.
[7] D. A. Kapp and G. Brown,"A new numerical method for rough surface scattering calculations," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 711–721, 1996.
[8] E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, "AIM: Adaptive integral method for solving large-scale electromagnetic scattering and radiation problems," *Radio Sci.*, vol. 31, pp. 1225–1251, 1996.
[9] E. Michielssen and A. Boag, "A multilevel matrix decomposition algorithm for analysing scattering from large structures," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 1086–1093, Aug. 1996.
[10] V. Rokhlin, "Rapid solution of integral equations of scattering theory in two dimensions," *J. Comput. Phys.*, vol. 86, pp. 414–439, 1990.
[11] R. L. Wagner and W. C. Chew, "A ray-propagation fast multipole algorithm," *Microwave Opt. Tech. Lett.*, vol. 7, no. 10, pp. 435–438, 1994.
[12] P. Cullen and C. Brennan, "Efficient techniques for the computation of UHF grazing incidence scattering," in *Int. Conf. Electromagn. Adv. Applicat.*, Torino, Italy, Sept. 1997, pp. 351–355.

[13] R. M. James, "A contribution to scattering calculation for small wavelengths—The high frequency panel method," *IEEE Trans. Antennas Propagat.*, vol. 38, pp. 1625–1630, Oct. 1990.

[14] K. Aberegg and A. Peterson, "Integral equation asymptotic phase method to two-dimensional scattering," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 534–537, May 1995

[15] D. Moroney and P. J. Cullen,"A fast integral equation approach to UHF coverage estimation," in *Mobile Personal Commun.: Proc. 2nd Joint Cost 227/231 Workshop Mobile Personal Commun.*, Florence, Italy, Apr. 1995, pp. 20–21.

[16] D. Moroney, "Computational methods for the calculation of electromagnetic scattering from large-scale perfect electrical conductors," Ph.D. dissertation, Univ. Dublin, Trinity College, Dublin, Ireland, 1995.

[17] P. McSharry, D. Moroney, and P. J. Cullen, "Wave scattering by a two-dimensional pressure release surface based on a perturbation of the Green's function," *Jnt. J. Acoust. Soc. Amer.*, vol. 98, no. 3, pp. 1699–1716, Sept. 1995.

[18] P. McSharry, P. J. Cullen, and D. Moroney, "Wave scattering by a two-dimensional bandlimited fractal surface based on a perturbation of the Green's function," *Jnt. J. Appl. Phys.*, vol. 78, pp. 6940–6948, Dec. 1995.

[19] C. Brennan, P. J. Cullen, and L. Rossi, "A high speed adaptive methodology for calculating UHF propagation loss over terrain," in *8th IEEE Int. Symp. Personal, Indoor, Mobile Radio Commun.*, Helsinki, Finland, Sept. 1997, pp. 629–634.

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