A Versatile Leaky-Wave Antenna Based on Stub-Loaded Rectangular Waveguide: Part II—Effects of Flanges and Finite Stub Length

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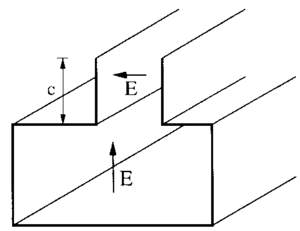
***Abstract*—A new leaky-wave antenna was presented and discussed in Part I that possesses many desirable performance features. The antenna structure is also simple in form, consisting of a rectangular waveguide with an offcenter stub on its top wall. Part I presented the principle of operation and the basic theory in which the stub guide was idealized to be infinitely long (or high) in order to stress the essential features. In practice, however, the stub guide has a finite length (or height) and it may be terminated by a flange or a ground plane or simply remain in the form of a pair of baffles. The modifications in the theory and the associated performance considerations under these practical circumstances are treated in the present paper—Part II.**

***Index Terms*—Leaky-wave antennas.**

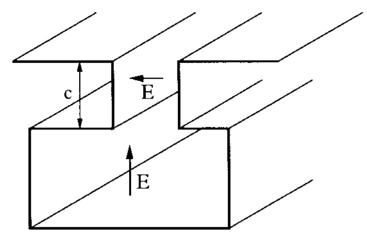
I. Introduction

IN Part I [1] of this three-part series of papers, we describe this new type of leaky-wave antenna, explain its principle of operation, present a theoretical derivation of the performance behavior, and, finally, include a variety of numerical results to illustrate the performance properties both as a leaky-waveguide structure and as an antenna. The dispersion relation and the numerical results presented in Part I are valid strictly for a stub guide of infinite length (or height). That idealization permits us to work with simpler analytical expressions and to perform faster calculations and yet obtain good numerical results that provide insight for design purposes.

A practical stub-loaded rectangular waveguide antenna structure must of course have a stub guide of *finite* length or height. In addition, the stub guide may be terminated by a *flange* or a ground plane or simply remain in the form of a pair of *baffles*. Those structures are illustrated in Fig. 1, where the stub-guide length is given as c. In a practical application, the antenna design must incorporate the numerical changes introduced by the finiteness of the stub guide and by the nature of the stub ends. The modifications in the theory required to take into account the finite stub length and the stub end in the form of baffles or flanges, are contained in this paper—Part II.



(a)



(b)

Fig. 1. Two examples of stub-loaded rectangular waveguide leaky-wave antennas with stubs of finite height c. The radiating open ends in these cases are (a) a pair of baffles and (b) a flange.

The theory described in Part I is based on a novel transverse equivalent network for which all the network elements are in simple closed form and which permits numerical values for the antenna performance to be obtained rapidly and inexpensively. When taking into account the modifications introduced by the finite stub length and by the nature of the stub ends, the transverse equivalent network approach possesses an additional important advantage. The network can be modified in a very simple and systematic way by simply changing the complex impedance element terminating the stub transmission line (now of finite length) according to the nature of the open radiating end.

Section II of this paper begins with the modifications in the transverse equivalent network that are necessary to take into account the finite rather than infinite length of the stub guide. The change in the dispersion relation is also derived, but it follows readily. In the modified network, and in the

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modified dispersion relation, the nature of the stub ends is, so far, expressed simply as a terminal complex admittance Y_R where subscript R signifies "radiating." Analytical expressions for the terminal admittance are derived next for two cases: when the stub guide ends in a pair of baffles and when it is terminated in a flange (actually a ground plane). It was found experimentally (reported in Part III) that the flange does not have to be wide in order to perform similarly to an infinite ground plane. The expression for the baffle ends is rigorous when the baffles are of zero thickness and the one for the presence of a ground plane is approximate but quite accurate and in simple form.

We then present (in Section III) some numerical results for two important considerations. The first of these is the minimum stub length necessary to eliminate the vertically polarized electric field at the radiating open end so that we are left with essentially pure horizontal polarization in the radiated beam. The required stub length will depend on the stub-guide width but, in general, a stub length less than a half wavelength is quite sufficient.

The second of these considerations involves some new effects introduced by the finite stub length. When the leakage rate is small, the only effect is that due to the mild standing wave produced by the mismatch at the radiating open end. As the stub-guide length is varied, therefore, the values of β/k_o and α/k_o undergo an almost periodic modification. When the leakage rate is large *and* for larger values of stub length, an additional set of leaky modes (modified "channel-guide" modes) are present, which couple to the antenna leaky mode and produce some interesting exotic interactions. The general properties of these new effects have been described in detail in a separate paper [2], but some aspects that are directly relevant, in particular, *how to avoid* such effects in a practical antenna design, are discussed in Section III.

II. Modifications in the Theory

A. Changes in the Transverse Equivalent Network and in the Dispersion Relation When the Stub Length Is Finite

The transverse equivalent network in Part I, Fig. 2(b) and the dispersion relation (10) of Part I that follows from it have assumed that the stub length is infinite. When the stub length becomes finite, a *radiating open end* is produced. The *first* step is to modify the transverse equivalent network in Fig. 2(b) of Part I to take into account the finite length c of the stub. Then, we have only to make the length of the (vertical) transmission line representing the stub guide equal to c and to terminate that line by the admittance $Y_R(= G_R + jB_R)$ where the subscript R signifies "radiating." The remainder of the network is unchanged. This modified transverse equivalent network is shown in Fig. 2, together with the cross sections appropriate to the radiating open end in the form of (a) a pair of baffles and (b) a flange or ground plane.

From this modified network we may readily obtain the appropriately modified dispersion relation, which is the *second* step. As in Part I, we choose a reference plane at T_r just below the turns in the transformer and sum to zero the admittances

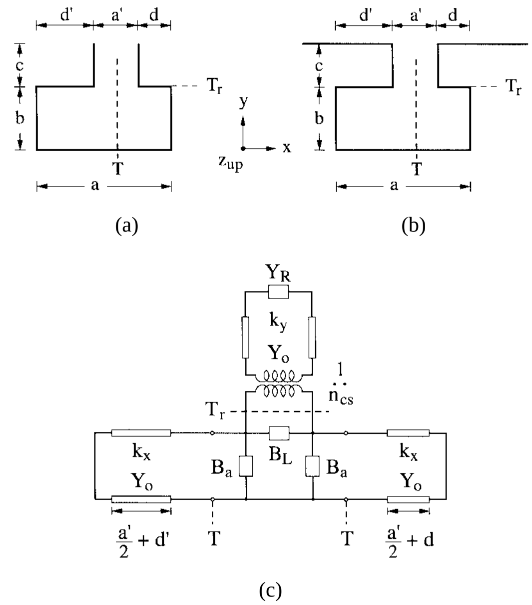

Fig. 2. Cross sections of the stub-loaded leaky-wave antenna when the radiating open end is (a) a pair of baffles and (b) a flange. (c) The transverse equivalent network for these structures is shown where the radiating element Y_R is different for each, but the rest of the network is the same.

seen looking up and down from T_r. We then find for $Y_{\rm up}$

$$Y_{\rm up} = \frac{1}{n_{\rm cs}^2} Y_{iR} \tag{1}$$

where

$$\frac{Y_{iR}}{Y_o} = \frac{j + \left(\frac{G_R}{Y_o} + j\frac{B_R}{Y_o}\right)\cot k_y c}{\cot k_y c + j\left(\frac{G_R}{Y_o} + j\frac{B_R}{Y_o}\right)}. \tag{2}$$

In Part I, where the stub guide was of infinite length, quantity Y_{iR} was simply Y_o, the characteristic admittance of the transmission line representing the stub guide. For the admittance at T_r looking down, we see the same network as in the infinite stub-guide case, so that $Y_{\rm down}$ is identical to that given by [1, eqs. (8) and (9)].

Setting $Y_{\rm up} + Y_{\rm down} = 0$ and normalizing to Y_o, we obtain the modified dispersion relation applicable when the stub guide is of finite length

$$\frac{1}{n_{\rm cs}^2}\frac{Y_{iR}}{Y_o} + j\frac{B_L}{Y_o} + j \times \frac{\left[\frac{B_a}{Y_o} - \cot k_x\left(\frac{a'}{2} + d\right)\right]\left[\frac{B_a}{Y_o} - \cot k_x\left(\frac{a'}{2} + d'\right)\right]}{2\frac{B_a}{Y_o} - \left[\cot k_x\left(\frac{a'}{2} + d\right) + \cot k_x\left(\frac{a'}{2} + d'\right)\right]} = 0. \tag{3}$$

Elements B_a/Y_o, $n_{\rm cs}$, and B_L/Y_o are given by the closed-form expressions (4)–(7) of Part I. Both wavenumbers k_x and k_y now appear in (3), but we recognize that they are equal to each other and that they are related to k_z by

$$k_z = \beta - j\alpha = \sqrt{k_0^2 - k_x^2}. \tag{4}$$

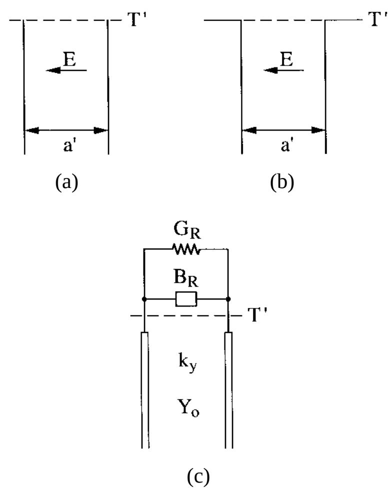

Fig. 3. The structure of the radiating open end in the form of (a) a pair of baffles or (b) a flange (or infinite ground plane). (c) The equivalent network for either open end has the same form.

B. Equivalent Networks for the Radiating Open Ends

The expression for Y_{iR}/Y_o in dispersion relation (3) is given above by (2) and the G_R/Y_o and B_R/Y_o terms in (2) will be different depending on the precise geometry of the radiating open end of the stub guide. Expressions for G_R/Y_o and B_R/Y_o for two cases are presented now. These cases apply when the stub guide terminates in either a pair of baffles or in a ground plane (or flanges).

In the Form of a Pair of Baffles A sketch of the discontinuity corresponding to this radiating open end on the stub guide and our equivalent network for it are given in Fig. 3(a) and (c), respectively. Fortunately, rigorous expressions for the elements G_R and B_R in Fig. 3 are available from the *Waveguide Handbook* [3, Sec. 4.6a, pp. 179–183], although we need to adapt those results to our notation. The following correspondences are required:

$$b \to a', \quad x \to \frac{k_y a'}{2\pi}, \quad \lambda' \to 2\pi/k_y, \quad \theta \to 2k_y\delta$$

where

$$2k_y\delta = \frac{k_y a'}{\pi}\ln\left[\frac{e}{\gamma}\frac{4\pi}{k_y a'}\right] - 2S_1\left(\frac{k_y a'}{2\pi};0,0\right) \tag{5}$$

and where

$$S_1\left(\frac{k_y a'}{2\pi};0,0\right) = \sum_{n=1}^{\infty}\left[\sin^{-1}\left(\frac{k_y a'}{2\pi}\frac{1}{n}\right) - \left(\frac{k_y a'}{2\pi}\frac{1}{n}\right)\right] \tag{6}$$

with $e = 2.718$ and $\gamma = 1.781$.

The normalized terminal admittance of the radiating open end at reference plane T' is given by

$$\frac{Y_R}{Y_o} = \frac{G_R}{Y_o} + j\frac{B_R}{Y_o} = \frac{\sin h\frac{k_y a'}{2} + j\sin 2k_y\delta}{\cos h\frac{k_y a'}{2} + \cos 2k_y\delta} \tag{7}$$

where the expression for $2k_y\delta$ appears in (5) and (6).

In the Form of a Flange or a Ground Plane When the stub guide is terminated in a flange or a ground plane rather than in the form of a pair of baffles as shown in Fig. 3(a), the discontinuity structure becomes the one seen in Fig. 3(b). The equivalent network representation for it is given by Fig. 3(c). The task at hand is to obtain an accurate theoretical expression for the discontinuity structure involving the flange. Measurements indicate that the flange does not need to be wide for its effect to be similar to that of a large ground plane; for this reason, a theoretical result valid for a ground plane should be a good approximation for all but the narrowest flanges.

We are fortunate that expressions for G_R/Y_o and B_R/Y_o for a parallel-plate stub guide terminated in a ground plane, and excited by a TEM mode at an angle, may be found in the *Waveguide Handbook* [3, Sec. 4.7a, pp. 183–185]. The expressions are approximate but accurate and two sets of expressions are given. The more accurate set corresponding to their (1a) and (2a) is

$$\frac{G_R}{Y_o} = \int_o^{k_y a'} J_o(x)\,dx - J_1(k_y a') \tag{8}$$

$$\frac{B_R}{Y_o} = -\int_o^{k_y a'} N_o(x)\,dx - N_1(k_y a') + \frac{2}{\pi}\frac{1}{k_y a'} \tag{9}$$

where the J and N quantities are Bessel's functions of the first and second kinds and where the following changes in notation were made: $k \to k_y$, and $b \to a'$. It should be added that the minus sign appearing before the integral in (9) for B_R/Y_o is missing in the *Waveguide Handbook* due to a typesetter's error.

The simpler set, which is very accurate for $k_y a' < 1$, corresponds to (1b) and (2b) there and becomes

$$\frac{G_R}{Y_o} = \frac{k_y a'}{2} \tag{10}$$

$$\frac{B_R}{Y_o} = k_y\frac{a'}{\pi}\ln\left[\frac{e}{\gamma}\frac{\pi}{k_y a'}\right] \tag{11}$$

where $e = 2.718$ and $\gamma = 1.781$, after the appropriate substitutions in notation are made. For the values appropriate to our antenna, numbers obtained from (10) and (11) differ from those from (8) and (9) by about 10% and 8%, respectively.

Equation (10) can readily be improved by taking the next set of terms in the small-argument expansions of J_o and J_1; it then becomes

$$\frac{G_R}{Y_o} = k_y\frac{a'}{2} - \frac{(k_y a')^3}{48}. \tag{12}$$

The added cubic term in (12) reduces the above-mentioned 10% to 2%, which is a dramatic improvement in our case for such a small correction. Corresponding expansions for the N_o and N_1 terms are more slowly convergent; taking the cubic terms into account reduces the 8% mentioned above to 6%, which is only a small improvement, and probably not worth the added effort. Therefore, we recommend using (11) and (12) when simple analytical forms are desired in order to obtain rapid numerical values for β/k_o and α/k_o. When greater accuracy is needed, (8) and (9) can be employed.

III. Numerical Results

Numerical results of two types are presented here. First we determine the minimum stub length required to effectively eliminate any cross-polarized radiation and then we examine the new behavioral features introduced by varying the stub-guide length. These new features include the undesired coupling to the "channel-guide" leaky modes that could occur under appropriate conditions. By combining these two types of numerical results, we can assess how best to avoid these possible coupling effects and yet have the antenna perform in an optimum fashion.

A. The Minimum Stub Length Required

Before we examine the numerical results, we should recall why we need the stub guide at all. It is to eliminate the vertically polarized electric field at the radiating open end, so that we are left with pure horizontal polarization in the radiated field. The vertical electric field arises from the mode configuration in the main guide of width a, which is above cutoff there but is below cutoff in the narrower stub guide of width a'. That modal field, associated with the $n = 1$ mode in the stub guide, decays exponentially (vertically) away from the main guide; the amount of cross-polarized power thus depends on the decay rate of that mode and the length of the stub guide.

To determine the minimum length required in order to effectively (according to some criterion) eliminate the cross-polarized power, we first calculate the decay rate of the $n = 1$ mode in the stub guide. It is given by

$$k_{y1}^2 = k_o^2 - (\pi/a')^2 - k_z^2. \tag{13}$$

Taking $\alpha^2 \ll \beta^2$, (13) becomes

$$\frac{|k_{y1}|}{k_o} = \left[\left(\frac{\lambda_o}{2a'}\right)^2 + \left(\frac{\beta}{k_o}\right)^2 - 1\right]^{1/2}. \tag{14}$$

The power decay is then governed by

$$e^{-2|k_{y1}|c} = e^{-\frac{|k_{y1}|}{k_o}4\pi\frac{c}{\lambda_o}}. \tag{15}$$

For the frequency and the dimensions used in most of our numerical results, we have $\lambda_o = 6.0$ mm, $a' = 2.4$ mm, and $\beta/k_o \cong 0.74$. Taking those values, (14) yields a decay rate $|k_{y1}|/k_o = 1.054$, which is reasonably large. To find the amount of cross-polarized power remaining at the radiating open end, expressed in decibels down from its value at the junction between the main and stub guides (at $c = 0$), we take

$$20\log_{10} e\frac{|k_{y1}|}{k_o}2\pi\frac{c}{\lambda_o} \tag{16}$$

and we find the following decibel values corresponding to several lengths c/λ_o:

c/λ_o	0.3	0.4	0.5	0.7	1.0
dB down	17.3	23.0	28.8	40.3	57.5.

From this short table we see that length c can certainly be less than a wavelength long. The length to choose depends on the criterion with respect to the level of cross polarization. For $c/\lambda_o = 0.4$, c would be equal to a'; for many purposes, a reduction of 23.0 dB would be sufficient.

It should also be realized that a narrower stub guide produces a faster decay rate and, therefore, a shorter stub length c. We should also note from Fig. 6 of Part I, where α/k_o is plotted as a function of a'/a, that $a'/a = 0.5$ (which is the value employed in the calculation just above) is located fairly far from the peak of the curve and is, therefore, not an optimum choice. A more suitable choice for a'/a would be some smaller value, say, $a'/a = 0.3$. The decay rate $|k_{y1}|/k_o$ then becomes 1.979 and, from (16), the short table shown above would be changed to

c/λ_o	0.3	0.4	0.5	0.7	1.0
dB down	32.4	43.2	54.0	75.6	108.

We now see that $c/\lambda_o = 0.4$ would be longer than necessary for most purposes, and that $c/\lambda_o = 0.3$ or even less may be quite sufficient.

B. Variations with Stub Length

The first effect caused by the finite stub length is the mild *standing wave* that is produced because of the mismatch introduced by the radiating open end. The standing wave exists in the stub guide between the radiating open end and the junction between the stub guide and the main guide; therefore, as the stub-guide length is varied, the values of β/k_o and α/k_o would be expected to undergo an almost periodic modification. That is precisely what is found when the leakage rate is low, with β/k_o being affected very little if at all and with α/k_o enduring a cyclical variation in value.

When the leakage rate is large *and* for larger values of stub length, some exotic interactions are produced. They are due to *coupling* between the desired leaky mode and another set of leaky modes, which were initially unanticipated and are a modification of the "channel-guide" mode that was known over 30 years ago. The coupling between these leaky modes occurs at the discontinuities arising at the radiating open end and the step junction between the stub and main guides. Furthermore, the coupling effects emerge automatically from our theoretical analysis. There is also a novel feature here in that the interactions involve the coupling between two *leaky* (*complex*) modes so that the modes will not couple unless *both* their β *and* α values are the same. That requirement has probably served to make such coupling relatively rare.

Since a "channel-guide" mode leaks strongly, it would not couple to the antenna leaky mode when the latter has a very low leakage rate, since coupling would occur only if the α values (as well as the β values) were equal. We, therefore, first examine quantitatively the case of an antenna mode with a lower leakage rate. That first case is presented in Fig. 4, where the behavior of β/k_o and α/k_o are seen as a function of normalized stub length c/λ_o and where the stub guide is terminated in a pair of baffles.

It is seen in Fig. 4 that the values of both β/k_o and α/k_o vary almost periodically with length c of the stub guide, but that β/k_o changes only a little whereas α/k_o goes through a rather large range. The beamwidth of the radiation pattern is, therefore, affected significantly by the length of the stub

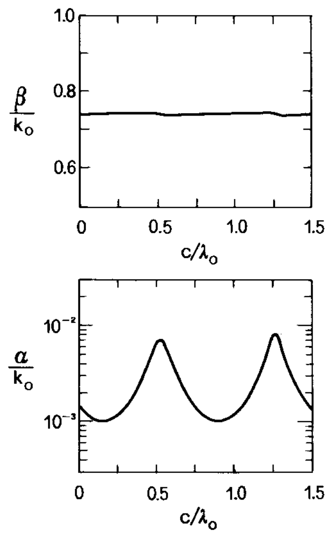

Fig. 4. Behavior of the normalized phase and leakage constants as a function of normalized stub length for the antenna leaky mode for the case of small leakage: $f = 50$ GHz; $a = 4.8$ mm; $b = 2.4$ mm; $a' = 2.4$ mm; $d = 1.0$ mm.

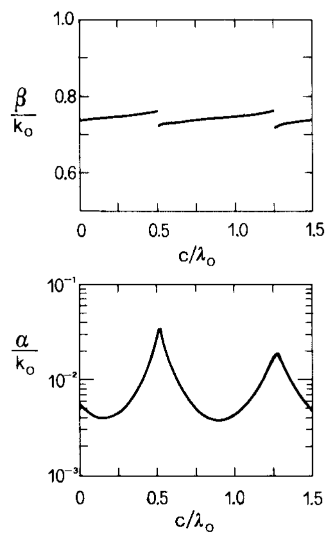

Fig. 5. Same as Fig. 4 except that the leakage rate is higher: $f = 50$ GHz; $a = 4.8$ mm; $b = 2.4$ mm; $a' = 2.4$ mm; $d = 0.8$ mm.

guide. It is interesting that the value of α/k_o for a stub guide of infinite length occurs slightly above the middle of the range of α/k_o values seen in Fig. 4. The dispersion relation [1, eq. (10)], which is valid for an infinite stub length, is, therefore, helpful for obtaining a first cut on the antenna design, but dispersion relation (3), which takes the stub-guide length into account, should be used for the final numbers.

No coupling to a "channel-guide" leaky mode is found for the c/λ_o values plotted in Fig. 4. In fact, for these dimensions, coupling effects begin to manifest themselves only when c/λ_o reaches about two or so. When the leakage rate is much greater, coupling effects arise for much smaller stub lengths. In Fig. 5, we see that these effects begin at about $c/\lambda_o = 0.5$. The ways in which β/k_o and α/k_o change are illustrated in Fig. 5, where it is seen that the variation in α/k_o is less periodic than before and that the curve for β/k_o displays gaps characteristic of directional coupler action. We have not included the curves for the "channel-guide" leaky modes in Fig. 5, but they have been presented in detail as part of a separate paper [2]. What is important here is that one should avoid values of c/λ_o corresponding to the gaps in β/k_o, due to the coupling. We have also made calculations for higher leakage rates and we have not observed any coupling effects for $c/\lambda_o < 0.5$.

We recall that the stub length c need only be long enough to prevent cross-polarized radiation, and that this length is also a function of the stub width a'. The problem associated with the coupling can, therefore, be avoided by keeping the stub length only as long as is necessary and not any longer. When the cross-polarized radiation must be kept exceptionally low, the stub-guide width should be narrowed further.

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Fabrizio Frezza (S'87–M'92–SM'95), for a photograph and biography, see this issue, p. 1040.

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Arthur A. Oliner (M'47–SM'52–F'61–LF'87), for a photograph and biography, see this issue, p. 1041.