

# Slot Antenna in a Resistive Screen

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**Abstract**— Loss resistance and efficiency are calculated for a slot in a resistive screen and for the complementary dipole. Double numerical integration of the magnetic near field is used. Strip dipole loss resistance is three to five times that of the complementary slot for short elements.

**Index Terms**— Slot antennas.

## I. INTRODUCTION

RESISTIVE loss in slots or dipoles is of importance for only a few applications; these include high-power slots in a thin film screen or dipoles made of thin film and electrically short matched or aperiodic slots. There are also applications involving reduction of radar cross section. Unfortunately, Babinet's principle does not apply to lossy screens. Senior [1], [2] showed that Babinet's principle could be extended to either a resistive screen and a complementary magnetic conductive screen or two resistive screens of different surface resistances. In the former case, the magnetic conductor can only be approximated (infinite permeability), while in the latter, the electric and magnetic fields for the two screens differ. Thus, there is no principle that relates the loss resistance of a slot in a resistive screen to the loss resistance of the complementary resistive strip dipole. Fante [3] computed an upper bound on the change in impedance in an infinite slot using a spectral integral. However, the accuracy of the two-term expansion is unknown, but more importantly, the current patterns are much different between an infinite slot and a short or resonant slot. The compensation theorem [4] has been used by Wait [5] to formulate the problem of antennas over earth with various shapes of finite ground planes. This formulation is basically the same as that used directly to calculate loss effects in transmission lines and waveguides.

## II. THE RADIAL ATTENUATION COEFFICIENT

Slot loss power  $P_\ell$  is simply given by the integral of tangential magnetic field squared over the screen

$$P_\ell = 2R_s \iint |H|^2 dA \quad (1)$$

where  $R_s$  is the surface resistance in  $\Omega/\square$ , and the factor of two represents loss on both sides of the ground plane. This loss must, of course, be finite and, thus, the magnetic fields must have an attenuation coefficient  $\alpha$ . Slots radiate spherical waves

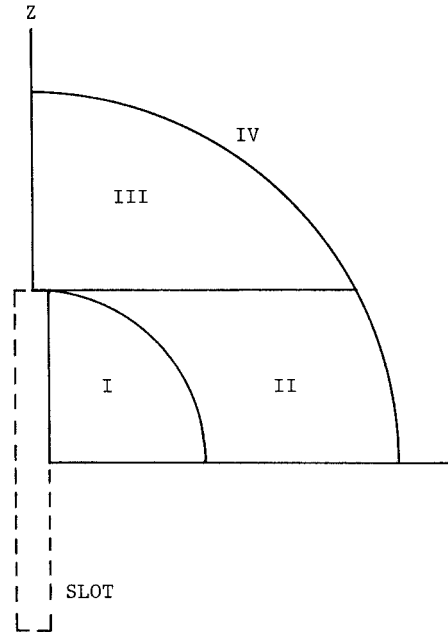


Fig. 1. Integration regions.

from the ends and center and the far-field radial behavior is  $\exp(-jkr)/r$ . Loss is incorporated by replacing  $k$  by  $k + j\alpha$ , thus making the integral finite.

It is instructive to see how  $\alpha$  is determined for transmission lines and waveguides. There

$$P_{\text{out}} = P_{\text{in}} \exp(-2\alpha z) \quad (2)$$

with  $P_{\text{in}} = P_{\text{out}} + P_\ell$ . Under the common assumption that the loss is sufficiently small and that the tangential magnetic field is unchanged,  $P_\ell$  is easily calculated. Then  $\alpha z = P_\ell/2P_{\text{out}}$ . A similar procedure has been used here for the resistive slot screen

$$P_{\text{rad}} = (P_{\text{rad}} + P_\ell) R_0 \int_{R_0}^{\infty} \exp(-2\alpha R)/R^2 dR \quad (3)$$

where  $R_0$  denotes the radius of a circle larger than the slot. The integral is expressed as an exponential integral  $E_1(x)$  with the result

$$\frac{1}{1 + P_\ell/P_{\text{rad}}} = \exp(-2\alpha R_0) - 2\alpha R_0 E_1(2\alpha R_0). \quad (4)$$

Because  $P_\ell$  depends upon  $\alpha$ , this equation must be solved iteratively or by a root finder. First, however, it must be

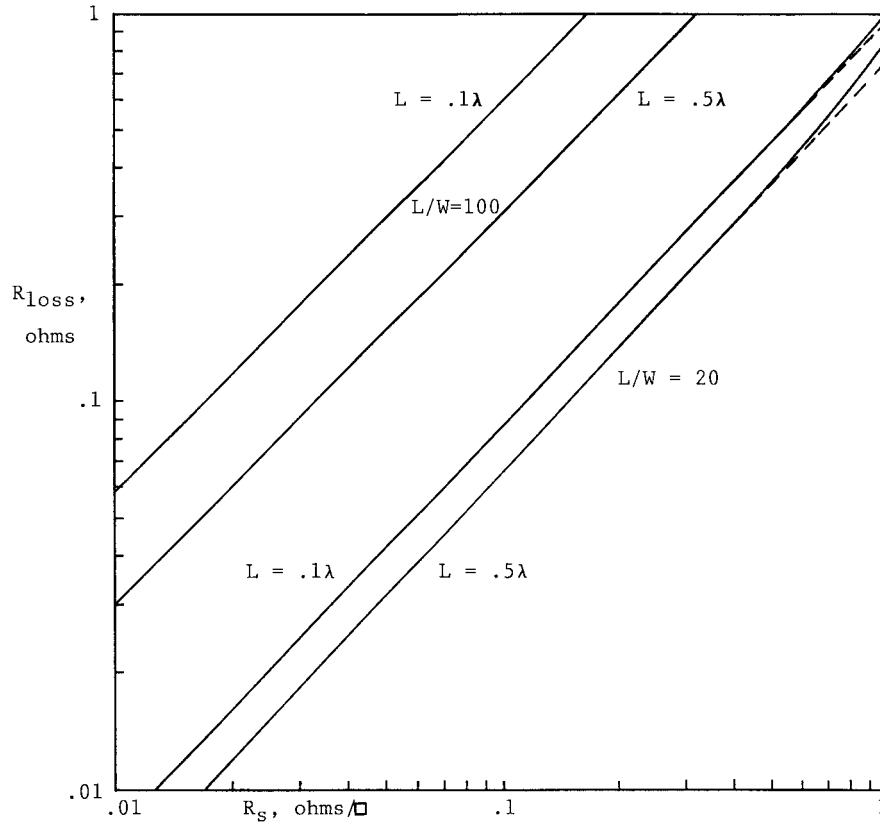


Fig. 2. Slot loss versus surface resistance.

determined whether  $\alpha R_0$  is a constant. For a transmission line,  $\alpha$  is constant; for a slot in a resistive sheet, this needs to be shown. To do this, the slot magnetic far field was integrated with a Wegstein rooter used to find  $\alpha$  for various values of  $R_0/\lambda$  and  $R_s$ . The value of  $\alpha R_0$  was constant for a given  $R_s$  for slot lengths of  $0.5\lambda$  and  $0.1\lambda$  for  $R_0$  from two to four. Further, the quantity  $\alpha R_0 R_{\text{rad}}/R_s$  is approximately constant, for slot lengths from  $0.1$  to  $0.5\lambda$ , and for  $R_s$  from  $0.01$  to  $1 \Omega/\square$ . The stage is then set to use this value of  $\alpha R_0$  along with a calculation of  $R_\ell$  that utilizes the exact magnetic near-fields around the slot.

### III. NUMERICAL INTEGRATION TO GET LOSS

The numerical integration of the magnitude squared of tangential magnetic field was divided into four regions as sketched in Fig. 1. The regions cover one quadrant. As the near-field [6] is given in cylindrical coordinates, the integration was performed in  $\rho, z$ . The attenuation coefficient  $\alpha$  was introduced into each of the three spherical wave terms of the near field. As the major effect of  $\alpha$  is for large  $R$ , use of this  $\alpha$  for smaller  $R$  should be quite good. Slot length and width are  $L$  and  $W$ . For region I the  $\rho$  and  $z$  limits are

$$\begin{aligned} \rho \text{ limits} & \quad W/2 \text{ to } \sqrt{L^2/4 - z^2} \\ z \text{ limits} & \quad 0 \text{ to } \sqrt{L^2/4 - W^2/4}. \end{aligned}$$

Double Simpson integration of  $40 \times 80$  points was used. For

region II, limits were

$$\begin{aligned} \rho \text{ limits} & \quad \sqrt{L^2/4 - z^2} \text{ to } \sqrt{R_0^2 - z^2} \\ z \text{ limits} & \quad 0 \text{ to } \sqrt{L^2/4 - W^2/4}. \end{aligned}$$

Double Simpson of  $40 \times 200$  was used. For region III, limits were

$$\begin{aligned} \rho \text{ limits} & \quad 0 \text{ to } \sqrt{R_0^2 - z^2} \\ z \text{ limits} & \quad \sqrt{L^2/4 - W^2/4} \text{ to } R_0. \end{aligned}$$

Double Simpson of  $200 \times 300$  was used. Finally, for region IV polar coordinates and far field were used. The radial part integrates directly to an exponential integral, while the  $\theta$  part was integrated by a seventh-order Romberg. Note that the pattern integral over a volume produces radiation resistance; the integral here is over a plane, hence, must be done numerically.

The value of  $\alpha R_0$  used in each calculation was determined for the specific  $L/\lambda$  and  $R_s$  from the rooter procedure of Section II.

Since the magnetic far-field decays as  $1/r$ , a finite ground plane should have small effect on efficiency as long as  $R > 2\lambda$ .

### IV. RESULTS

Fig. 2 shows  $R_\ell$  versus  $R_s$  for two slot lengths and for slot length/width ratios of 20 and 100. As expected, narrow

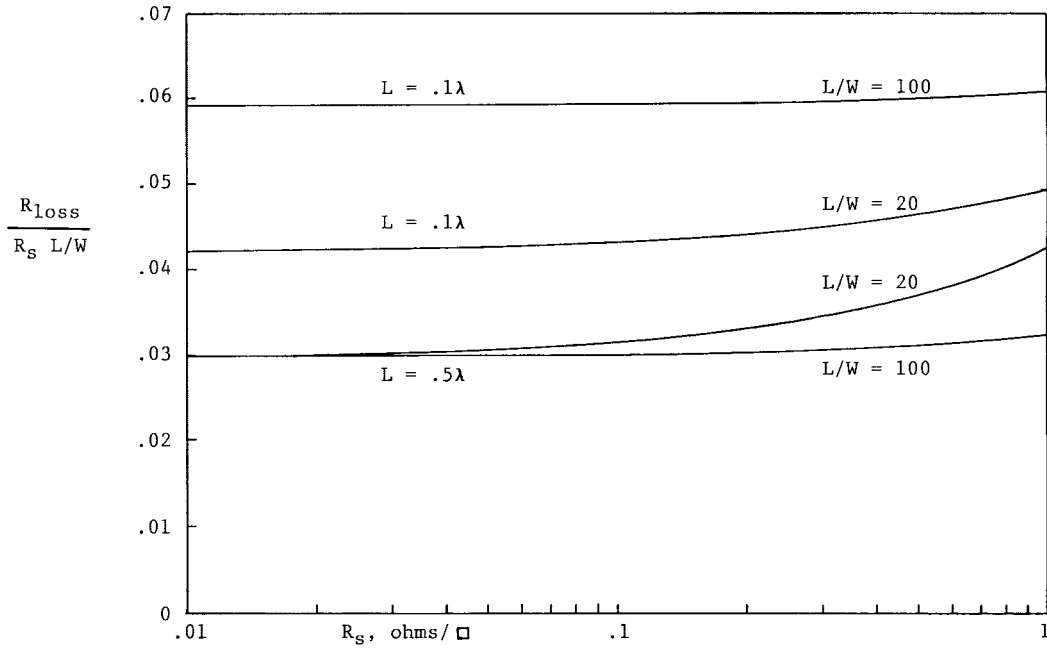


Fig. 3. Normalized loss resistance versus surface resistance.

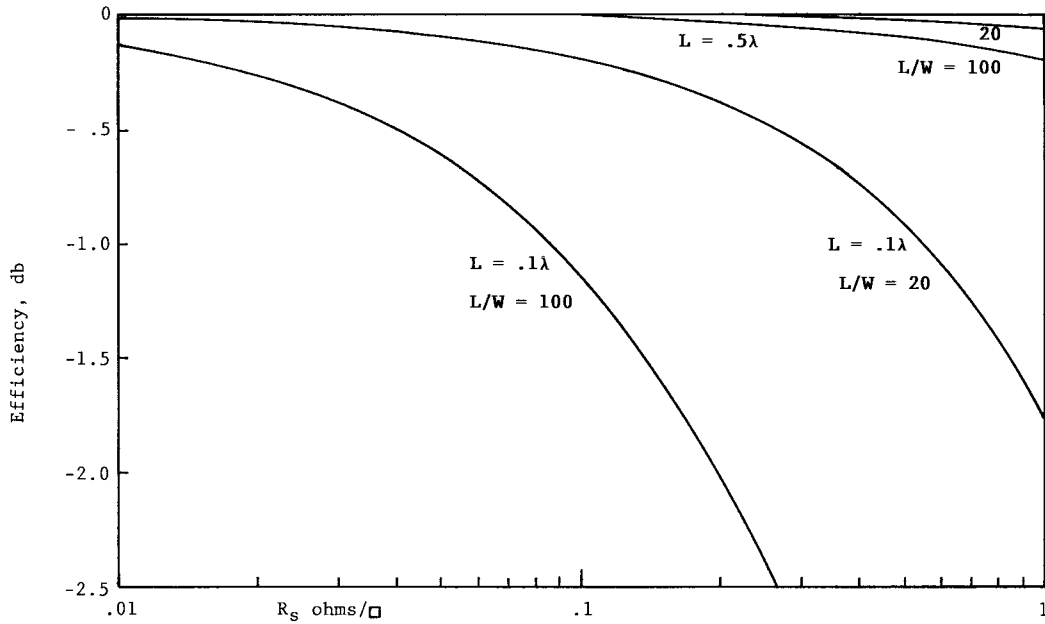


Fig. 4. Slot efficiency versus surface resistance.

slots like narrow dipoles have higher loss.  $R_\ell$  is linearly proportional to  $R_s$  until the loss starts affecting the field distribution. This occurs around  $R_s = 1 \Omega/\square$ ; moment-method calculations should be used for these and higher values of  $R_s$ . Similar results are given in Fig. 3, where the normalized loss resistance  $R_\ell/R_s L/W$  is plotted versus  $R_s$ . Significant departures from constant values again occur around  $R_s = 1$ .

Finally, slot efficiency is shown in Fig. 4 versus  $R_s$  for slot lengths of  $0.1$  and  $0.5\lambda$ . Short slot-loss resistance is similar to that for a  $\lambda/2$  slot, but the lower radiation resistance delivers a lower efficiency.

A comparison with the efficiency of the complementary strip dipole is of interest. Current for the dipole is approximately sinusoidal along its length and constant across its width. Practical strip dipoles are sufficiently thick that edge current singularities are not important. The dipole loss resistance is

$$R_\ell = \frac{LR_s}{2W} \left[ \frac{1 - \text{sinc } kL}{2\sin^2 kL/2} \right]. \quad (6)$$

Values of slot and dipole efficiency for  $L/W = 100$  are given

in the table below for  $R_s = 0.1 \Omega/\square$

$L/\lambda$	slot efficiency	dipole efficiency
0.5	0.9959	0.9969
0.1	0.9713	0.9221

Dipole currents are constrained to the strip conductor, while the slot currents have a much larger conductor area in which to spread out. But even for  $R_s = 0.1$ , a high-surface resistance, the dipole efficiency is quite good. Slot efficiency is excellent.

## V. CONCLUSIONS

Slot loss resistance has been calculated; values are useful for screen surface resistances up to roughly  $1 \Omega/\square$ . The loss resistance varies directly with surface resistance in this range and for short elements is one-third to one-fifth of the loss resistance of the complementary strip dipole.

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