# Electromagnetic Excitation of a Thin Wire: A Traveling-Wave Approach

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***Abstract*—An approximate representation for the current along a perfectly conducting straight thin wire is presented. The current is approximated in terms of pulsed waves that travel along the wire with the velocity of the exterior medium. At the ends of the wire, these pulses are partially reflected, with a constant reflection coefficient and delay time. Subsequently, the traveling-wave representation for the current is used to derive an approximate expression for the electric field outside the wire that is caused by this current. For voltage excitation, this expression contains only closed-form contributions. For plane-wave excitation, the expression contains a single integral over the initial pulse that must be computed numerically. Although the expression obtained is essentially a far-field approximation, it turns out to be valid from distances of the order of a single wire length. Results for a representative choice of wire dimensions and pulse lengths are presented and discussed.**

***Index Terms*—Electromagnetic radiation, electromagnetic scattering, linear antennas.**

## I. Introduction

A CENTURY ago, Pocklington [1] introduced his integro-differential equation for the total electric current along a straight thin-wire segment. In his paper, Pocklington also presented an approximate solution to this equation. In doing so, he already introduced the idea on which the present-day theory of scattering by thin wires is based, i.e., that only the total current that flows along the wire is of interest. This has led to a variety of so-called thin-wire integral equations for this current. In their derivation, the two-dimensional integral equation for the electric current density on the surface of the wire is reduced to a one-dimensional (1-D) equation, which relates this current to the electric field intensity parallel to the wire.

In the 1930's, Hallén [2], [3, pp. 444–504] used a Green's function technique to derive an equivalent integral equation, in which the space and time differentiations that occur in Pocklington's equation are avoided. In Hallén's equation, two extra unknown time signals are introduced that can be determined by imposing boundary conditions at both ends of the wire. From this equation, closed-form expressions for the induced current along the wire can be determined iteratively. Both Pocklington's and Hallén's equation occur in two versions, the so-called "exact" and "approximate" equations. Only a few years ago [4], it has been observed that the "approximate" versions give an almost exact description of the total current. A review of these integral equations and of several approximate and numerical solutions can be found in [4] and [5].

In this paper, we present an approximate expression for the scattered electric field caused by a polarized pulsed plane wave that is incident upon the wire at arbitrary angles of incidence and polarization. In this expression, the scattered field consists of spherical waves caused by a direct current wave along the wire and by repeated reflections of such waves at the end points. At a sufficient distance from the wire, each of the constituents of the scattered electric field again reduces to a plane wave. Therefore, our expression is capable of describing multiple-scattering effects. This makes the model suitable for several "statistical" applications, such as analyzing the scattering of an electromagnetic wave by a cloud of metal wires, also referred to as chaff [6], or determining the effective electromagnetic properties of composite media [7]. Besides "statistical" applications the model also enhances the physical insight into radiation by thin-wire structures [8].

The desired expression is obtained in three steps. First, we determine an approximate expression for the current along the wire in case of a delta-gap excitation. To this end, a model is used which is inspired by a first-order approximate solution of Hallén's equation [9], [10], and earlier published results [4], [11]–[13]. In this model, the current is described by waves that travel along the wire with the velocity of the exterior medium, and are repeatedly reflected at the ends. A similar model has also been used by Gómez Martín *et al.* [11].

The model parameters are given in the frequency domain or in the time domain. In the frequency domain, the model parameters are a real-valued admittance that depends on frequency and on the coordinate along the wire, and a complex, frequency-dependent reflection coefficient. For these parameters, expressions are used which were found in Shen *et al.* [14] and in Ufimtsev [15], respectively. In the time domain, we use an even simpler model with only three constant real-valued parameters. An admittance determines the amplitude of the traveling waves, while the reflections at the end points are governed by a reflection coefficient and a delay time which corresponds to the fact that the electromagnetic wave travels in the exterior medium. The values of these parameters are determined numerically, by comparing the approximate expression with the result of solving Hallén's integral equation

Manuscript received October 1, 1997; revised May 4, 1998.

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Publisher Item Identifier S 0018-926X(98)06103-1.

by time marching as outlined in [4]. The differences between this model and the one discussed in [11] are the use of an extra parameter to describe the time delay of the waves during reflection, and the fact that the parameters are evaluated by minimizing the time-integrated squared error between the model current and an "exact" numerical result at a single observation point.

The second step is to find an expression for the radiated electric field caused by this current. This expression is obtained by applying a far-field approximation to the pertaining integral representation. Our version of this approximation consists of integrating by parts until all integrals are of $\mathcal{O}(R^{-2})$ or higher, with $R$ being the distance between the observation point and a point on the wire. These integrals are then neglected compared with the boundary terms of $\mathcal{O}(R^{-1})$. This leaves us with a representation of the radiated field in terms of spherical waves that originate either from the delta gap or from the ends of the wire.

In the third and final step, the effect of plane-wave excitation is identified as the cumulative effect of a continuous distribution of delta-gap sources. This implies that the scattered field can be constructed from a suitable linear combination of radiated delta-gap fields as described above. This application of the superposition principle results in an extra integral over the source location. For the spherical waves that arrive directly from the gap locations, this integral must be evaluated numerically. For the constituents departing from the end points, the integration can be carried out in closed form.

To assess the quality of the different models, we have considered two typical delta-gap excitations: a short pulse that excites several natural modes of the wire and a long pulse that primarily excites the dominant mode. In the frequency-domain model and for short-pulse excitation, the current is approximated with remarkable accuracy. The electric field, however, is not nearly as accurate. For long-pulse excitation, the approximation deteriorates considerably for both quantities. This is hardly surprising, because high-frequency considerations were used to obtain the expression for the reflection coefficient. When the time-domain model is used to describe short-pulse excitation, satisfactory approximations are found for the current as well as for the electric field. In fact, the expression for the electric field agrees better with the numerical results than its frequency-domain counterpart. For long-pulse excitation, the analytical and numerical results are almost indistinguishable. These results demonstrate that, with a smaller set of parameters, our new model provides a more accurate description of the radiated electric field than the conventional frequency-domain analysis. Therefore, we have only used this model to describe plane-wave excitation.

The paper is organized as follows. In Section II, the problem is formulated. In Section III, a first-order approximation of the solution of Hallén's integral equation is presented. Section IV deals with the current along the wire in case of voltage excitation. Section V presents the corresponding radiated electric field. In Section VI, superposition is applied to deal with plane-wave excitation. The conclusions, finally, are given in Section VII.

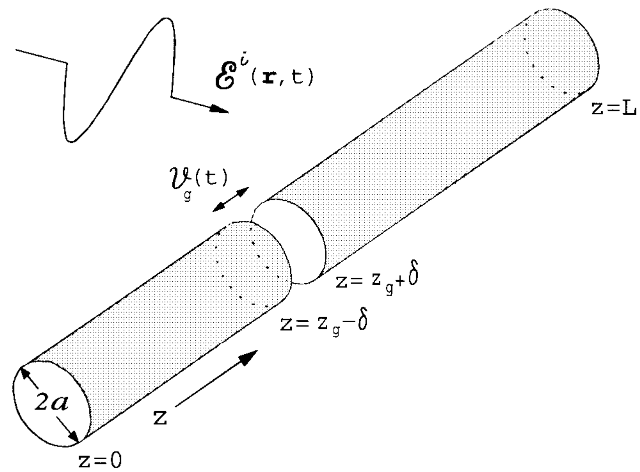


Fig. 1. Transient excitation of a straight thin-wire segment by an incident field or an impressed voltage.

## II. Formulation of the Problem

We consider a perfectly conducting straight thin-wire segment of length $L$ with a circular cross section of radius $a$ embedded in a homogeneous lossless dielectric with permittivity $\varepsilon$ and permeability $\mu$ (Fig. 1). The wave speed $c$ in the exterior medium is given by $c \overset{\text{def}}{=} 1/\sqrt{\varepsilon\mu}$. A Cartesian coordinate system is introduced with the central axis of the wire located at $\mathbf{r} = z\mathbf{u}_z$, with $0 < z < L$. The wire is excited by an incident electric field $\mathcal{E}^i(\mathbf{r}, t)$, which is a solution of Maxwell's equations in absence of the wire and/or driven by an impressed voltage $\mathcal{V}_g(t)$ across the gap $z_g - \delta < z < z_g + \delta$. The dimensions of the wire are chosen such that $\delta \ll a \ll L$. Both the incident field and the impressed voltage are identically zero before the initial instant $t = t_0$. The aim of the computation is to determine an approximate expression for the scattered electric field $\mathcal{E}^s(\mathbf{r}, t)$ around the wire for $t_0 \leq t < \infty$ when the incident field is a linearly polarized pulsed plane wave.

### A. Integral Representation for the Scattered Electric Field

In this section a closed-form integral representation for the electric field around the wire is given. When the wire is driven only by a delta-gap source, the electric field induced by the current on the wire is indicated as the radiated electric field $\mathcal{E}^r(\mathbf{r}, t)$. When the wire is excited by an incident electric field $\mathcal{E}^i(\mathbf{r}, t)$, the electric field induced by the current on the wire is indicated as the scattered electric field $\mathcal{E}^s(\mathbf{r}, t)$.

An exact description for the radiated/scattered electric field $\mathcal{E}^{r,s}(\mathbf{r}, t)$ around the wire, which holds for all $t$, is given by the integral representation [16]

$$\varepsilon\partial_t \mathcal{E}^{r,s}(\mathbf{r}, t) = \nabla\nabla\cdot\mathcal{A}(\mathbf{r}, t) - \frac{1}{c^2}\partial_t^2\mathcal{A}(\mathbf{r}, t) \tag{1}$$

where

$$\mathcal{A}(\mathbf{r}, t) = \oint_{\partial\mathcal{D}} \frac{\mathcal{J}(\mathbf{r}', t - R/c)}{4\pi R}\, dS(\mathbf{r}'). \tag{2}$$

$\partial\mathcal{D}$ denotes the closed surface of the thin-wire segment while $\mathcal{A}$ and $\mathcal{J}$ represent the vector potential and the electric current density flowing along $\partial\mathcal{D}$, respectively. $R = |\mathbf{R}|$ with $\mathbf{R} = \mathbf{r} - \mathbf{r}'$. This integral representation for the radiated/scattered electric field (1) can be reduced to a 1-D integral by using the thin-wire approximation. This thin-wire approximation can be used in the exact field representation (1) or can be used to define an approximate vector potential, which results in

$$\mathcal{A}(\mathbf{r}, t) \approx \mathbf{u}_z \int_0^L \frac{\mathcal{I}(z', t - R/c)}{4\pi R}\, dz' \tag{3}$$

where $\mathcal{I}(z, t)$ is the total current that flows along the wire.

When, in case of the approximated vector potential (3), the boundary conditions for the total current at the end points of the wire are used

$$\mathcal{I}(z_0, t) = 0, \qquad \text{for } z_0 = 0, L \tag{4}$$

both thin-wire approximations lead to the same integral representation for the radiated/scattered electric field

$$\mathcal{E}^{r,s}(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon} \int_0^L \left\{ \frac{\mathcal{Q}(z', t - R/c)\mathbf{R}}{R^3} + \frac{\partial_t \mathcal{Q}(z', t - R/c)\mathbf{R}}{R^2 c} - \frac{\partial_t \mathcal{I}(z', t - R/c)\mathbf{u}_z}{R c^2} \right\} dz' \tag{5}$$

with

$$\mathcal{Q}(z, t) = -\int_{-\infty}^{t} \frac{\partial}{\partial z} \mathcal{I}(z, \tau)\, d\tau \tag{6}$$

being the total charge per unit length along the wire. From (5) is observed that the total current $\mathcal{I}(z, t)$ along the wire completely determines the radiated/scattered electric field. In equations where the thin-wire approximation is used, like in (3) and (5), the vector $\mathbf{R}$ reduces to $\mathbf{R} = \mathbf{r} - z'\mathbf{u}_z$.

### B. Integral Equation for the Current

An almost exact description for the total current $\mathcal{I}(z, t)$ along the wire is provided by the so-called reduced form of Pocklington's thin wire integro-differential equation. Tijhuis *et al.* [4] have demonstrated that this reduced form is more accurate than the "exact" form of Pocklington's equation. Taking the observation point $z$ on the central axis of the cylinder, $0 \le z \le L$, and neglecting only the radial currents on the end faces at $z = 0, L$ results in

$$-\varepsilon \partial_t \mathcal{V}_g(t)\delta(z - z_g) - \varepsilon \partial_t \mathcal{E}_z^i(z\mathbf{u}_z, t) = \left[\partial_z^2 - \frac{1}{c^2}\partial_t^2\right] \int_0^L \frac{\mathcal{I}(z', t - R_a/c)}{4\pi R_a}\, dz' \tag{7}$$

where $R_a = \sqrt{(z - z')^2 + a^2}$ and $\delta(z)$ denotes Dirac's one-dimensional (1-D) delta function.

To obtain the corresponding Hallén form, the combination of space and time differentiations occurring in (7) is recognized as the differential operator governing the propagation of plane waves in a homogeneous, lossless dielectric. For that operator, the Green's function $\mathcal{G}(z, t)$ can be defined as the causal solution of the inhomogeneous second-order differential equation

$$\left[\partial_z^2 - \frac{1}{c^2}\partial_t^2\right]\mathcal{G}(z, t) = -\delta(z)\delta(t). \tag{8}$$

This solution is given by (e.g., Tijhuis [16])

$$\mathcal{G}(z, t) = \frac{c}{2}\,\mathcal{U}\left(t - \frac{|z|}{c}\right) \tag{9}$$

where $\mathcal{U}(t)$ denotes the Heavyside unit time-step function. Starting from (7) and (9) and using the superposition principle, we obtain by a straightforward convolution for $0 \le z \le L$ and for $t_0 \le t < \infty$:

$$\int_0^L \frac{\mathcal{I}(z', t - R_a/c)}{4\pi R_a}\, dz' = \frac{Y}{2} \int_0^L \mathcal{E}_z^i\left(z'\mathbf{u}_z, t - \frac{|z - z'|}{c}\right) dz' + \frac{Y}{2}\mathcal{V}_g\left(t - \frac{|z - z_g|}{c}\right) + \mathcal{F}_0\left(t - \frac{z}{c}\right) + \mathcal{F}_L\left(t - \frac{L - z}{c}\right) \tag{10}$$

where $Y \stackrel{\text{def}}{=} \sqrt{\varepsilon/\mu}$ denotes the wave admittance of the dielectric medium surrounding the wire. The unknown time signals $\mathcal{F}_0(t)$ and $\mathcal{F}_L(t)$ represent two independent homogeneous solutions of the 1-D wave equation (7). These signals can be determined by using the boundary conditions (4) for the total current at the end points of the wire. Equation (10) is known as the reduced form of Hallén's integral equation [2], [3, pp. 444–504].

## III. First-Order Approximate Solution of the Hallén Integral Equation

As introduced by Hallén [2], [3, pp. 444–504] and also used by Bouwkamp [10], the integral on the left-hand side of (10) can be written as

$$\int_0^L \frac{\mathcal{I}(z', t - R_a/c)}{4\pi R_a}\, dz' = \frac{\mathcal{I}(z, t - a/c)}{4\pi} \int_0^L \frac{1}{R_a}\, dz' + \int_0^L \frac{\mathcal{I}(z', t - R_a/c) - \mathcal{I}(z, t - a/c)}{4\pi R_a}\, dz'. \tag{11}$$

This integral is approximated by neglecting the second integral term in the right-hand side of (11). The remaining integral in the right-hand side of (11) can be evaluated analytically and is given as

$$\int_0^L \frac{1}{R_a}\, dz' = \Omega(z) = 2\ln\left(\frac{L}{a}\right) + \ln\left(\frac{z + \sqrt{z^2 + a^2}}{L}\right) + \ln\left(\frac{L - z + \sqrt{(L - z)^2 + a^2}}{L}\right). \tag{12}$$

When only delta-gap excitation is considered, the approximated form of Hallén's integral equation gives as result for the current $\mathcal{I}(z,t)$

$$\mathcal{I}(z,t) = \frac{2Y\pi}{\Omega(z)}\mathcal{V}_g\left(t+\frac{a}{c}-\frac{|z-z_g|}{c}\right) + \frac{4\pi}{\Omega(z)}\,\mathcal{F}_0\left(t+\frac{a}{c}-\frac{z}{c}\right) + \frac{4\pi}{\Omega(z)}\,\mathcal{F}_L\left(t+\frac{a}{c}-\frac{L-z}{c}\right) \quad (13)$$

where the unknown time signals $\mathcal{F}_0(t)$ and $\mathcal{F}_L(t)$ can be determined by applying the boundary conditions (4) for the total current at the end points of the wire. The total current $\mathcal{I}(z,t)$ is then obtained as

$$\begin{aligned}\mathcal{I}(z,t) = {} & \frac{2Y\pi}{\Omega(z)}\mathcal{V}_g\left(t+\frac{a}{c}-\frac{|z-z_g|}{c}\right) \\ & - \frac{2Y\pi}{\Omega(z)}\sum_{n=0}^{\infty}\mathcal{V}_g\left(t+\frac{a}{c}-\frac{z+z_g}{c}-\frac{2nL}{c}\right) \\ & - \frac{2Y\pi}{\Omega(z)}\sum_{n=0}^{\infty}\mathcal{V}_g\left(t+\frac{a}{c}-\frac{2L-z-z_g}{c}-\frac{2nL}{c}\right) \\ & + \frac{2Y\pi}{\Omega(z)}\sum_{n=0}^{\infty}\mathcal{V}_g\left(t+\frac{a}{c}-\frac{2L+z-z_g}{c}-\frac{2nL}{c}\right) \\ & + \frac{2Y\pi}{\Omega(z)}\sum_{n=0}^{\infty}\mathcal{V}_g\left(t+\frac{a}{c}-\frac{2L-z+z_g}{c}-\frac{2nL}{c}\right).\end{aligned} \quad (14)$$

In this equation the total current is described by a direct current wave as a result of the delta-gap source and subsequent reflections of this wave which are identified as the signals $\mathcal{F}_0(t)$ and $\mathcal{F}_L(t)$ in (13). The reflection coefficient at both ends of the wire is $\Gamma = -1$, and there is no time delay for the departure of the reflected waves. Comparing (13) and (14) shows that the reflected current waves may be regarded as the homogeneous solution of the 1-D wave equation (7).

## IV. Current for Voltage Excitation

In this section, the approximated time-domain expression for the current along a wire is introduced. This is done for the situation where the wire is excited by a delta-gap source. As mentioned in the introduction, this model is inspired by the first-order solution of Hallén's equation, Section III, as well as by earlier published results. Further, it is described how the values of the three model parameters are obtained.

In addition to the time-domain description for the current, an equivalent frequency-domain description is introduced. This is done, because suitable expressions for the parameters in such a description were available from the literature.

### *A. Approximate Expression for the Current*

The current for a delta-gap excitation, indicated with $\mathcal{I}_\delta(z,t)$ or $I_\delta(z,\omega)$, is described by waves traveling across the wire which are repeatedly reflected at the ends of the wire.

With the assumption of a suppressed time factor $\exp(-i\omega t)$ and the introduction of the wavenumber $k \overset{\text{def}}{=} \omega\sqrt{\varepsilon\mu}$, the frequency-domain description for the approximate current is given as

$$\begin{aligned}I_\delta(z,\omega) = {} & \Sigma_f(z,\omega)V_g(\omega)\exp[ik|z-z_g|] - \Sigma_f(z,\omega)\Gamma_f(\omega) \\ & \times \sum_{n=0}^{\infty}\Gamma_f^{2n}(\omega)V_g(\omega)\exp[ik(z+z_g+2nL)] \\ & - \Sigma_f(z,\omega)\Gamma_f(\omega)\sum_{n=0}^{\infty}\Gamma_f^{2n}(\omega)V_g(\omega) \\ & \times \exp[ik(2L-z-z_g+2nL)] \\ & + \Sigma_f(z,\omega)\Gamma_f^2(\omega)\sum_{n=0}^{\infty}\Gamma_f^{2n}(\omega)V_g(\omega) \\ & \times \exp[ik(2L+z-z_g+2nL)] \\ & + \Sigma_f(z,\omega)\Gamma_f^2(\omega)\sum_{n=0}^{\infty}\Gamma_f^{2n}(\omega)V_g(\omega) \\ & \times \exp[ik(2L-z+z_g+2nL)]\end{aligned} \quad (15)$$

where

| | |
|---|---|
| $V_g(\omega)$ | exciting voltage; |
| $z_g$ | position of excitation; |
| $\Sigma_f(z,\omega)$ | unknown real-valued admittance; |
| $\Gamma_f(\omega)$ | unknown complex reflection coefficient. |

The subscript "$f$" indicates that the admittance $\Sigma_f(z,\omega)$ and reflection coefficient $\Gamma_f(\omega)$ are frequency-domain model parameters. The reflection coefficient can be frequency dependent while the admittance can depend on frequency as well as on position. From the numerical results presented in [4], [11], [12], and [13], it is observed that during the reflection of the current waves at the end points of the wire, a phase shift arises. Because we want to describe this phase shift only with the reflection coefficient $\Gamma_f(\omega)$, the admittance $\Sigma_f(z,\omega)$ has to be real valued.

In the time-domain description for the current there are three model parameters; an admittance $\Sigma_t$, a reflection coefficient $\Gamma_t$ and a delay time $t_d$ which denotes the delay time during the reflections of the current waves at the end points of the wire. The time-domain description for the approximate current is given as

$$\begin{aligned}\mathcal{I}_\delta(z,t) & \\ = {} & \Sigma_t\mathcal{V}_g\left(t-\frac{|z-z_g|}{c}\right) \\ & - \Sigma_t\Gamma_t\sum_{n=0}^{\infty}\Gamma_t^{2n}\mathcal{V}_g\left(t-\frac{z+z_g}{c}-\frac{2nL}{c}-(2n+1)t_d\right) \\ & - \Sigma_t\Gamma_t\sum_{n=0}^{\infty}\Gamma_t^{2n}\mathcal{V}_g\left(t-\frac{2L-z-z_g}{c}-\frac{2nL}{c} - (2n+1)t_d\right) + \Sigma_t\Gamma_t^2\sum_{n=0}^{\infty}\Gamma_t^{2n} \\ & \mathcal{V}_g\left(t-\frac{2L+z-z_g}{c}-\frac{2nL}{c}-(2n+2)t_d\right) \\ & + \Sigma_t\Gamma_t^2\sum_{n=0}^{\infty}\Gamma_t^{2n}\mathcal{V}_g\left(t-\frac{2L-z+z_g}{c}-\frac{2nL}{c} - (2n+2)t_d\right).\end{aligned} \quad (16)$$

The three time-domain model parameter $\Sigma_t$, $\Gamma_t$, and $t_d$ have constant, e.g., position-independent, real values, which makes the time-domain description simpler than the general frequency-domain description given in (15). In fact, the frequency-domain counterpart of (16) follows from (15) by making the special choice $\Sigma_f(z,\omega) = \Sigma_t$ and $\Gamma_f(\omega) = \Gamma_t \exp(i\omega t_d)$. This choise was inspired by observation of several time-domain results computed by the procedure described in [4].

### *B. Frequency-Domain Model Parameters*

For the two frequency-domain model parameters $\Sigma_f(z,\omega)$ and $\Gamma_f(\omega)$ approximate expressions have been found in the literature. Both parameters were derived by using the thin-wire approximation, which implies that they are valid for small values of $ka$ $(ka < 0.2)$. The admittance $\Sigma_f(z,\omega)$ is obtained from Shen et al. [14], who derived an approximation for the current on a infinite wire, excited by a delta-gap source at $z_g = 0$. This approximated current is written as (17), shown at the bottom of the page, where $\gamma = 0.577\,215\,664\,9\ldots$ is Euler's constant.

The expression (17) consists of a phase factor $\exp(ik|z|)$, which represents the traveling-wave nature of the solution, and an almost real-valued admittance. For an infinite wire, the reflection terms are absent from (15), and the admittance can be identified as

$$\Sigma_f(z,\omega) = \exp(-ik|z|)\frac{I_\infty(z)}{V_g(\omega)}. \qquad (18)$$

Because a real value for $\Sigma_f(z,\omega)$ is desired, either the real part or the absolute value of the remaining terms in (17) can be taken as representative for this parameter. In our computations, we have chosen the absolute value, which leads to

$$\Sigma_f(z,\omega) = \left|\frac{I_\infty(z)}{V_g(\omega)}\right|. \qquad (19)$$

The amplitude of the wave which propagates directly from $z_g$ to $z$ without any reflection is equal to $\Sigma_f(|z - z_g|,\omega)$. Due to the fact that the amplitude of this wave depends strongly on $|z - z_g|$, it is not approximated and the dependence on position is preserved. The amplitudes of the waves which are reflected first at $z = 0$ are approximated by $\Sigma_f(z_g,\omega)$ while the amplitudes of the waves which reflect first at $z = L$ are approximated by $\Sigma_f(L - z_g,\omega)$.

The reflection coefficient $\Gamma_f(\omega)$ is obtained from Ufimtsev [15]. For the complete derivation we refer to [15], but also to Vainshtein [17]–[19]. The same expression was also used by Shamansky [12]. The approximated reflection coefficient is written as

$$\Gamma_f(\omega) = \frac{\ln\left(\frac{-1}{(\Upsilon ka)^2}\right)}{\ln\left(\frac{i2L}{\Upsilon ka^2}\right) - E(2kL)\exp(-i2kL)} \qquad (20)$$

where

$$E(y) = Ci(y) + i\Big[Si(y) - \frac{\pi}{2}\Big] = \int_\infty^y \frac{\cos(t)}{t}\,dt + i\int_0^y \frac{\sin(t)}{t}\,dt - i\frac{\pi}{2} \qquad (21)$$

with $Si(y)$ and $Ci(y)$ as the sine and cosine integrals [20] and $\Upsilon = \exp(\gamma)$.

### *C. Time-Domain Model Parameters*

The time-domain model parameters $\Sigma_t$, $\Gamma_t$, and $t_d$ are determined by comparing the approximate expression for the current at a single point with results of a numerical computation. This numerical solution is determined by solving Hallén's thin-wire integral equation with the marching-on-in-time method [4]. The values of the time-domain model parameters are obtained by minimizing the time-integrated squared error between the approximate current and the numerical solution for fixed values of the excitation and observation points $z_g$ and $z$. The pulse shape of the delta-gap source $\mathcal{V}_g(t)$ is Gaussian and is given by

$$\mathcal{V}_g(t) = \exp\left[-\left(\frac{t}{T_p} - 4\right)^2\right] \qquad (22)$$

where $T_p$ is related to the pulse duration. When the points of excitation and observation are in the range $0.1L < (z_g, z) < 0.9L$, the results of the minimization turn out to be almost independent of the particular choice of $z_g$ and $z$. The results presented in this paper where obtained for $z_g = 0.5L$ and $z = 0.3L$, respectively. The upper and lower time limits are chosen such that the pulses belonging to the first four reflections participate in the minimization process while the first pulse, which propagates from $z_g$ to $z$ without reflection, is ignored.

The minimization is a time-consuming process, which seems to make the model less suitable for fast "statistical" applications. However, when the model parameters are determined as a function of the physical dimensions of the wire, like the length $L$ and the radius $a$, it appears that the values of all three parameters are smooth functions of $\ln(L/a)$. When $\ln(L/a) > 8$ the relations are almost linear. We notice that the quantity $\ln(L/a)$ is half the position-independent term of $\Omega(z)$ (12). This means that we only need to calculate the model parameters once for some values of $\ln(L/a)$, and store these values in a table. The intermediate points can then be found by inter- or extrapolation. Of course, extrapolation is only allowed as long as the thin-wire conditions are satisfied.

From the parameter investigation, it was observed that the values of the model parameters do depend on the frequency content of the voltage excitation. Comparing the spectrum of the true and approximated currents revealed that this depends on the number of natural modes that are excited. It turns out

$$I_\infty(z,\omega) = iY\exp(ik|z|)V_g(\omega)\ln\left[1 + \frac{2\pi i}{2\ln(ka) + \gamma - \ln(k|z| + \sqrt{(kz)^2 + \exp(-2\gamma)}) - i\frac{3}{2}\pi}\right] \qquad (17)$$

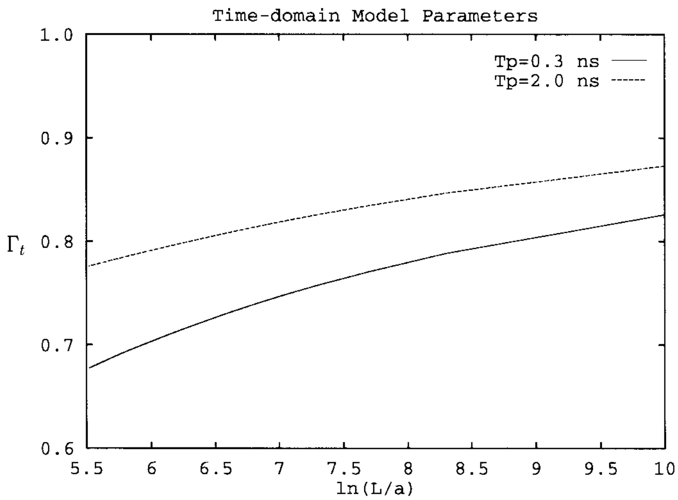


Fig. 2. Reflection coefficient $\Gamma_t$ as function of $\ln(L/a)$, with $L = 1$ m.

that there is one choice of parameters which gives an accurate description of the first natural mode, and one which favors the higher order modes. As an example, in Fig. 2, for two characteristic values of $T_p$ the reflection coefficient $\Gamma_t$ as a function of $\ln(L/a)$ is depicted.

The choice of the physical parameters $L$, $a$, and $T_p$ will be addressed in the next subsection. In this section it will also be demonstrated that with $T_p = 2.0$ ns, only the first natural mode is excited while with $T_p = 0.3$ ns, several natural modes of the wire are excited.

### D. Results

A wire with length $L = 1$ m and radius $a = 2$ mm is excited with a Gaussian delta-gap source $\mathcal{V}_g(t)$ [see (22)]. In Fig. 3 the time-domain results of the numerical and both the approximated currents, e.g., for $T_p = 0.3$ ns, are depicted. In Fig. 4 the frequency-domain spectra of the same currents are shown. In this figure the currents are depicted as a function of a relative frequency $L/\lambda$, where $\lambda$ is defined as the wavelength. From this figure it is observed that the currents have peaks when

$$L = n\frac{\lambda}{2}, \qquad \text{with } n = 1, 3, 5, \ldots, 15. \tag{23}$$

Each value of $n$ corresponds with a natural mode of the wire [3, pp. 444–504], [21], whose spatial distribution is approximately represented by a standing-wave pattern of the current. Because the wire is excited at $z_g = L/2$ only the "odd" natural modes are excited $(n = 1, 3, 5, \ldots)$. The "even" natural modes $(n = 2, 4, 6, \ldots)$ are not excited because at $z = L/2$ the natural-mode current distribution is equal to zero.

From both figures it can be concluded that the less accurate approximation of the time-domain model, in comparison with the frequency-domain model (Fig. 3), is mainly caused by the mismatch of the first natural mode of the wire $(n = 1)$ (Fig. 4). The reason that the first mode is approximated less accurate is that the time-domain model parameters are obtained by minimizing the squared error over the first four reflected waves of the current. During this early-time interval the higher

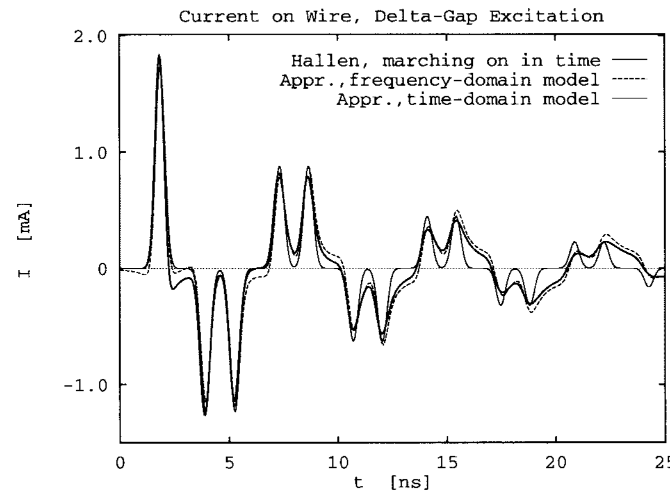


Fig. 3. Total current $\mathcal{I}(0.3L, t)$ on a wire with dimensions $L = 1$ m and $a = 2$ mm, excited by a delta gap at $z_g = L/2$ with a Gaussian pulse with $T_p = 0.3$ ns.

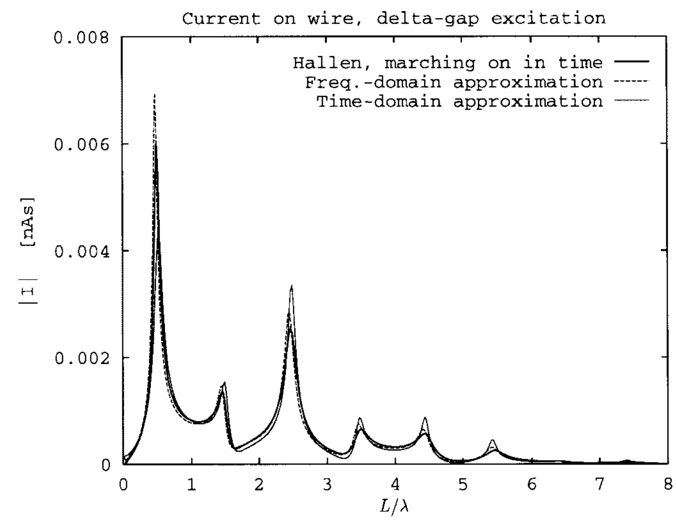


Fig. 4. Total current $I(0.3L, \omega)$ on a wire with dimensions $L = 1$ m and $a = 2$ mm, excited by a delta gap at $z_g = L/2$ with a Gaussian pulse with $T_p = 0.3$ ns.

order modes of the wire are dominant, while at late times the current is dominated by the first mode of the wire.

When a longer pulse duration is chosen, $T_p = 2.0$ ns, only the first natural mode of the wire is excited. This single mode is approximated most successfully by the time-domain model. The numerical result and the result obtained by the time-domain model are almost indistinguishable. This is hardly surprising, since the time-domain model contains three real-valued parameters, which suffices to describe an exponent $A\exp(s_n t)$ with a real-valued amplitude $A$ and a complex frequency $s_n$. Next, we will derive an analytical expression for the radiated electric field.

## V. Radiated Field for Voltage Excitation

The current $\mathcal{I}(z, t)$ on the wire caused by the delta-gap excitation produces a radiated field which in the far field is given by

$$\mathcal{E}^r(\mathbf{r},t) = \frac{\mu}{4\pi}\int_0^L \partial_t \mathcal{I}(z', t - R/c)\frac{\mathbf{R}\times(\mathbf{R}\times\mathbf{u}_z)}{R^3}\,dz'. \quad (24)$$

This far-field representation is obtained by neglecting those terms in the integrand of the field representation (5) which are proportional to $R^{-2}$ and $R^{-3}$ [3, pp. 378–383], [22], [23]. The analytical expression for the radiated field, indicated with $\mathcal{E}^r_\delta(\mathbf{r},t)$, is found by substituting the analytical expression for the current (16) in (24). To achieve that we rewrite the current as

$$\begin{aligned}\mathcal{I}_\delta(z,t) = h\left(t - \frac{|z-z_g|}{c}\right) + f^+\left(t - \frac{z+z_g}{c}\right) \\ + f^-\left(t + \frac{z+z_g}{c}\right) + g^+\left(t - \frac{z-z_g}{c}\right) \\ + g^-\left(t + \frac{z-z_g}{c}\right)\end{aligned} \quad (25)$$

where $h$, $f$, and $g$ are readily identified as the different terms in (16). The function $h$ is equal to the first term, the functions $f^\pm$ indicate the summation terms with an odd number of reflections, while the functions $g^\pm$ indicate the summation terms with an even number of reflections. The "$+$" and "$-$" superscripts indicate whether the waves are propagating in the positive or in the negative $z$ direction. The time derivative of the current can now be written as a sum of the time derivatives of $h$, $f$, and $g$. Because the current consists of a sum of traveling waves, the time derivatives of $h$, $f$, and $g$ can be rewritten in terms of space derivatives

$$\begin{aligned}&\partial_t h\left(t - \frac{R}{c} - \frac{|z'-z_g|}{c}\right) \\ &\quad = \frac{-c}{(z'-z)/R + \operatorname{sgn}(z'-z_g)}\partial_{z'} h\left(t - \frac{R}{c} - \frac{|z'-z_g|}{c}\right)\end{aligned} \quad (26)$$

$$\begin{aligned}&\partial_t f^\pm\left(t - \frac{R}{c} \mp \frac{z'+z_g}{c}\right) \\ &\quad = \frac{-c}{(z'-z)/R \pm 1}\partial_{z'} f^\pm\left(t - \frac{R}{c} \mp \frac{z'+z_g}{c}\right)\end{aligned} \quad (27)$$

$$\begin{aligned}&\partial_t g^\pm\left(t - \frac{R}{c} \mp \frac{z'-z_g}{c}\right) \\ &\quad = \frac{-c}{(z'-z)/R \pm 1}\partial_{z'} g^\pm\left(t - \frac{R}{c} \mp \frac{z'-z_g}{c}\right)\end{aligned} \quad (28)$$

where the chain rule of differentiation has been used and where $\operatorname{sgn}(z)$ is the sign function. The integral in (24) is approximated by taking only the boundary term which is found after integrating by parts. The integral term is neglected because it is proportional to $R^{-2}$. Neglecting this term is consistent with the approximations used to obtain the far-field representation (24). The analytical expression for the radiated field now assumes the form

$$\begin{aligned}4\pi Y \mathcal{E}^r_\delta(\mathbf{r},t) &= \left(p^-_{z_g} - p^+_{z_g}\right)h(\tau_{z_g})\mathbf{F}_{z_g} \\ &+ \left[-p^+_0 q^+_0(\tau_0) + p^-_0\left(h\left(\tau_0 - \frac{z_g}{c}\right) + q^-_0(\tau_0)\right)\right]\mathbf{F}_0 \\ &+ \left[p^+_L\left(h\left(\tau_L - \frac{L-z_g}{c}\right) + q^+_L(\tau_L)\right) + p^-_L\ q^-_L(\tau_L)\right]\mathbf{F}_L\end{aligned} \quad (29)$$

where

$$\mathbf{R}_\zeta = \mathbf{R}|_{z'=\zeta}, \quad R_\zeta = |\mathbf{R}_\zeta|, \quad (30)$$

$$\mathbf{F}_\zeta = \frac{\mathbf{R}_\zeta \times (\mathbf{R}_\zeta \times \mathbf{u}_z)}{R^3_\zeta} \quad (31)$$

$$\tau_\zeta = t - \frac{R_\zeta}{c} \quad (32)$$

$$p^\pm_\zeta = \left(\frac{z-\zeta}{R_\zeta} \mp 1\right)^{-1} \quad (33)$$

$$q^\pm_\zeta(t) = f^\pm\left(t \mp \frac{\zeta + z_g}{c}\right) + g^\pm\left(t \mp \frac{\zeta - z_g}{c}\right) \quad (34)$$

with $\zeta = (0, z_g, L)$. From (29), it can be observed that the radiated field is represented in terms of spherical waves that originate either from the delta gap at $z = z_g$ or from the ends of the wire at $z = 0, L$. This is consistent with numerical results that have appeared in the literature (see, e.g., Miller and Landt [13]).

The approximate expression (29) for the electric-field strength were compared with results from a complete numerical computation as wel as from the frequency-domain model. Several combinations of $T_p$, $z_g$, and $\mathbf{r}$, with $|\mathbf{r}|$ in orders of a single wire length, were considered. Both reference results have been obtained by substituting the accompanying currents in the exact representation for the radiated electric field (5) and evaluating the integral in this equation numerically. The numerical current is obtained by solving Hallén's thin wire integral equation with the so called *marching-on-in-time method* [4], while the frequency-domain approximated current is given by (15), (19), and (20). The time-domain result of this latter current is obtained with the aid of the so-called inverse Fast Fourier Transformation (iFFT). In all cases, (29) turned out to provide a relatively accurate approximation of the actual field. Fig. 5 shows a representive result for a wire with length $L = 1$ m and radius $a = 2$ mm, which is excited by a delta gap at $z_g = L/4$ with a Gaussian pulse of the form (22) with $T_p = 0.3$ ns. The components $\mathcal{E}_x$ and $\mathcal{E}_y$ of the radiated electric field are not shown, but the results are comparable to those of the $\mathcal{E}_z$ component shown in Fig. 5. The pulse duration was chosen as $T_p = 0.3$ ns, which implies that several modes of the wire are excited. Because $z_g = L/4$ some "even" modes are excited as well. The identity represented in (23) becomes valid for $n = 1, 2, 3, \ldots, 15\backslash\{4, 8, 12\}$. We already have concluded that, when several modes of the wire are excited, the higher order modes are approximated better than the first one (Figs. 3 and 4). This explains the accurate representation of the early-time radiated electric field and the less accurate representation of the late-time radiated electric field obtained with the time-domain model.

When only the first mode of the wire is excited ($T_p = 2.0$ ns) the current is approximated very well. It appears that the result for the radiated electric field, obtained with the time-domain model, is better than the result obtained with the frequency-domain model and is almost indistinguishable from the numerical result. Comparable results have also been obtained for other points of observation. Therefore it can be concluded that, although the expression for the radiated field is essentially a far-field approximation, it turns out to be valid

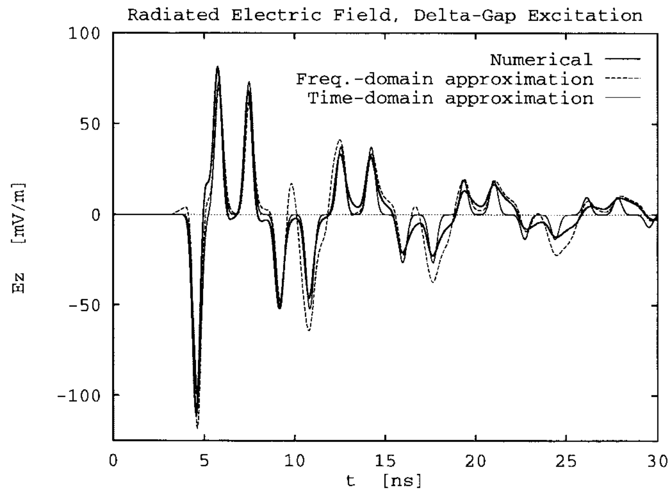


Fig. 5. Radiated electric field at $\mathbf{r} = (1.0, 0.0, 0.5)L$. The wire has dimensions $L = 1$ m and $a = 2$ mm and is excited by a delta gap at $z_g = L/4$ with a Gaussian pulse with $T_p = 0.3$ ns.

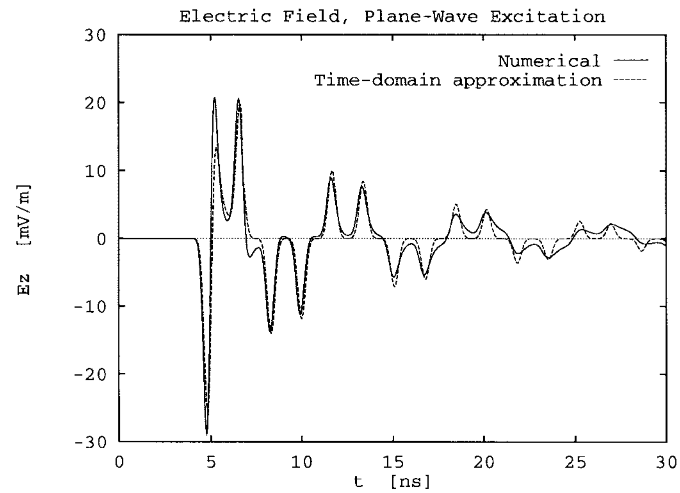


Fig. 7. Scattered electric field at $\mathbf{r} = (1.0, 0.0, 0.5)L$. The wire has dimensions $L = 1$ m and $a = 2$ mm and is excited by a plane wave. The time derivative of the Gaussian pulse is used and $T_p = 0.3$ ns. The angle of incidence is $\theta = 60°$, while the angle of polarization $\eta = 0°$.

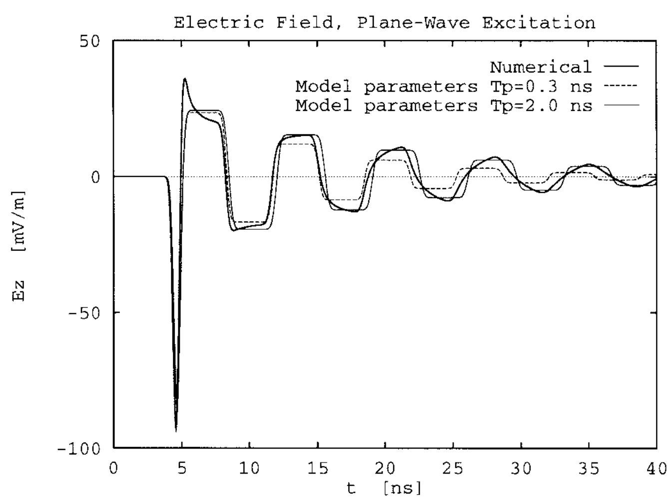


Fig. 6. Scattered electric field at $\mathbf{r} = (1.0, 0.0, 0.5)L$. The wire has dimensions $L = 1$ m and $a = 2$ mm and is excited by a plane Gaussian pulse with $T_p = 0.3$ ns. The angle of incidence is $\theta = 90°$, while the angle of polarization $\eta = 0°$.

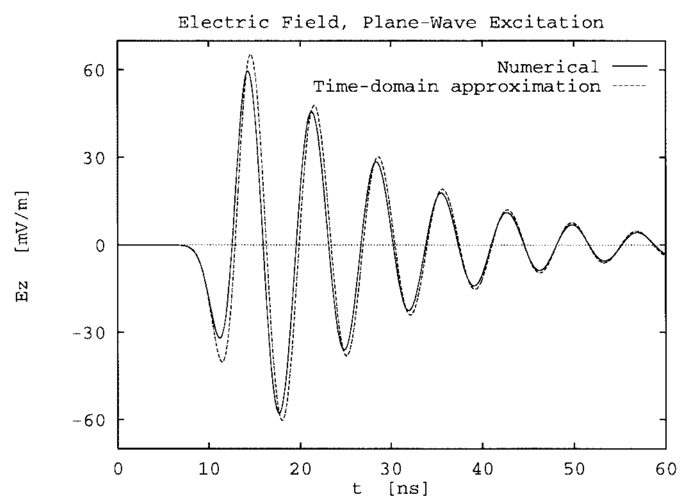


Fig. 8. Scattered electric field at $\mathbf{r} = (1.0, 0.0, 0.5)L$. The wire has dimensions $L = 1$ m and $a = 2$ mm and is excited by a plane Gaussian pulse with $T_p = 2.0$ ns. The angle of incidence is $\theta = 60°$, while the angle of polarization $\eta = 0°$.

from distances of the order of a single wire length. Finally, it can be concluded that with the time-domain model, which has an approximated expression for the current as well as for the radiated electric field, results are obtained which have the same or even better accuracy than the results obtained with the frequency-domain model and an exact representation for the radiated field. Therefore, only the time-domain model (16) and (29) will be used to derive an approximate expression for the scattered electric field caused by an incident plane wave.

## VI. Scattered Field for Plane-Wave Excitation

The approximated scattered electric field, indicated by $\mathcal{E}_p^s(\mathbf{r}, t)$, which results from a plane-wave excitation is derived directly from the radiated field due to a delta-gap excitation. A plane-wave excitation can be envisaged as the cumulative effect of a continuous distribution of delta-gap sources on a finite wire, provided that the sources are excited with the correct delay time. Therefore the scattered electric field can be written as

$$\mathcal{E}_p^s(\mathbf{r}, t) = \cos\eta \sin\theta \int_0^L \mathcal{E}_\delta^r\left(\mathbf{r}, z_g, t - \frac{z_g \cos\theta}{c}\right) dz_g \quad (35)$$

where $\theta$ is the angle of incidence and $\eta$ is the angle of polarization. For the terms of the radiated field $\mathcal{E}_\delta^r(\mathbf{r}, t)$, which depend on $z_g$ only via the time delay, the integral for the scattered electric field can be evaluated analytically. For the first term of the radiated field $\mathcal{E}_\delta^r(\mathbf{r}, t)$ this is not possible since the dependence on $z_g$ is more complicated. Approximating this integral by taking only the boundary term after integrating by parts, as done with the approximated solution for the radiated field, gives only an accurate result when the incoming field is perpendicular to the wire ($\theta = 90°$). Because in all other situations the approximation is very poor, this integral is evaluated numerically with the aid of a repeated trapezoidal rule. In Fig. 6 some results of the approximated scattered electric field together with the numerical result are shown. The wire with length $L = 1$ m and radius $a = 2$ mm is excited by a normally incident plane Gaussian pulse with $T_p = 0.3$ ns. Initially, the high-frequency model parameters were used, which were determined for $T_p = 0.3$ ns. Observing the results of the scattered electric field in the frequency domain reveals that the higher modes of the wire become less dominant compared with the first mode of the wire, in spite of the fact that the same number of modes are excited as in the case of the delta-gap excitation. Therefore, we expected that a more accurate result would be obtained when model parameters are used that represent this first mode of the wire. This is achieved by taking the model parameters found for $T_p = 2.0$ ns. From Fig. 6 it can be concluded that with these parameters an acceptable accurate approximation is obtained. When we want to use the model parameters belonging to the higher order modes, we have to suppress the first natural mode of the wire. This can for example be achieved by taking the following pulse shape for the delta-gap source

$$\mathcal{V}_g(t) = 2\left(4 - \frac{t}{T_p}\right)\exp\left[-\left(\frac{t}{T_p} - 4\right)^2\right]. \tag{36}$$

Apart from a factor of $T_p$ this pulse is the time derivative of the Gaussian pulse specified in (22). The results for the scattered electric field can be observed in Fig. 7. From this figure it can be concluded that, although a far-field approximation has been used and the distance from the wire is in the order of a wire length, a very accurate approximation is reached. When the pulse duration is chosen as $T_p = 2.0$ ns, only the first natural mode is excited which always leads to results which are almost indistinguishable from the numerical ones. An example of single-mode excitation is depicted in Fig. 8.

## VII. Conclusion

In this paper approximate representations have been presented for the current along a perfectly conducting straight thin wire and the corresponding electric field around the wire. The current is approximated in terms of pulsed waves that travel along the wire with the velocity of the exterior medium. At the ends of the wire, these pulses are partially reflected, with a constant reflection coefficient and delay time. These parameters and the strength of the current at the excitation point were determined for frequency-domain as well as time-domain descriptions of the model. For the parameters in the frequency-domain model expressions have been found in literature. The time-domain model parameters have been determined by comparing the approximate current at a single point with the results of a numerical computation with the marching-on-in-time method, for the case of delta-gap excitation.

A delta-gap excitation was investigated first. For the radiated electric field the most accurate approximation is found from the time-domain description of the model parameters. Although the expression for the approximated electric field is essentially a far-field approximation, it turns out to be valid from distances of the order of a single wire length. The most accurate approximation is reached when only the first natural mode of the wire is excited. When more modes are excited, the approximation becomes less accurate but still very acceptable. When only the first natural mode of the wire is excited, other values of the model parameters are required. The model parameters also depend on the dimensions of the wire. This dependence is almost linear, when they are considered as a function of $\ln(L/a)$. This means that we only have to compute the model parameters for certain values of $\ln(L/a)$, store these values in a table, and use interpolation for the intermediate points.

Approximate expressions for the scattered electric field outside the wire were obtained by treating the excitation of a continuous superposition of delta-gap sources along the wire. If the delta-gap sources are excited with the correct delay times, the total response is the superposition of all the individual responses. These responses, in turn, were described by the time-domain far field approximation mentioned above. The approximation for the scattered electric field contains an integral over the initial pulse, which must be computed numerically, and closed-form contributions from all reflected pulses. The scattered electric field around the wire is approximated best when the pulse of the plane wave excites only the first natural mode of the wire. In this case the analytical and exact results are almost indistinguishable. When the pulse shape is Gaussian and more natural modes of the wire are excited, the contribution from the higher order modes is less important than for a delta-gap excitation. So, the best approximation is still obtained with the model parameters corresponding to the first natural mode of the wire. When the excitation of the first mode is suppressed, e.g. by choosing a different pulse shape, a very accurate approximation is reached by using the parameters corresponding to the higher order modes. So, it seems likely that, if even better results for the current as well as the scattered electric field are desired, the time-domain model should be divided in a single-mode part and a higher mode part with accompanying model parameters. The total current and/or scattered field will then be acquired as the sum of the single-mode result and the higher mode result.

The scattered field consists of spherical waves caused by a direct current wave along the wire and by repeated reflections of such waves at the end points. At a sufficient distance from the wire, each of these constituents again reduces to a plane wave. Therefore, the expression for the scattered electric field derived in this paper is capable of describing multiple-scattering effects. This makes the model suitable for several “statistical” applications, such as analyzing the scattering of an electromagnetic wave by a cloud of metal wires, also referred to as chaff, or determining the effective electromagnetic properties of composite media. To investigate multiple scattering with the model proposed in this paper, it is recommended to investigate first the scattering of a small number of wires. In that case, it would still be possible to compare numerically and analytically obtained results.

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