

Ray-Optical Prediction of Radio-Wave Propagation Characteristics in Tunnel Environments— Part 1: Theory

Y. Hwang, Y. P. Zhang, and Robert G. Kouyoumjian

Abstract—A tunnel is modeled as congregates of walls, with the wall being approximated by uniform impedance surface. The aim is to get a solution for a canonical problem of a wedge with uniform impedance surface. The diffraction by a right-angle wedge with different impedance boundary conditions at its two surfaces is first considered. A functional transformation is used to simplify the boundary conditions. The eigenfunction solutions for the transformed functions are replaced by integral representations, which are then evaluated asymptotically by the modified Pauli–Clemmow method of steepest descent. The asymptotic solution is interpreted ray optically to obtain the diffraction coefficient for the uniform geometrical theory of diffraction (UTD). The obtained diffraction coefficients are related directly to Keller diffraction coefficients in uniform version. The total field is continuous across the shadow of the geometrical optics fields.

Index Terms—Geometrical theory of diffraction, radio propagation.

I. INTRODUCTION

TUNNELS are common in metropolitan cities, mountain areas, or under the sea. This paper is to study the propagation characteristics of tunnels by the uniform geometrical theory of diffraction (UTD). The walls of tunnels are approximated by uniform impedance surface. Although the diffraction of electromagnetic waves by a perfectly-conducting wedge has been studied extensively [1], few practical results exist for the case where the surface of the wedge is subject to the impedance boundary condition.

Malyughinets [2], [3] considered the scalar diffraction by an arbitrary-angle wedge, with different impedance boundary conditions on its two surfaces and the solution being restricted to plane wave illumination. The solution was obtained by a method analogous to the one used by Peters [4] and Senior [5]. With this method, the differential equation and boundary conditions were expressed as a difference equation for the determination of a regular function of which the real part represented the velocity potential. The exact solution, thus obtained, can be applied to the problem for the plane wave illumination. Difficulty, however, was experienced in the computation of

Manuscript received May 23, 1995; revised February 15, 1997.

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Publisher Item Identifier S 0018-926X(98)07054-9.

the integrals in the solution since they were improper, with meromorphic functions as their integrands.

Felson [6] solved the scalar diffraction problem for an arbitrary-angle wedge when the surface impedance was proportional to the radial variable ρ . Such special boundary condition had made the problem separable, with the Green's function obtainable by standard methods.

Mohsen and Hamid [7] adopted a different approximation in their approach to the diffraction of electromagnetic waves by a perfectly conducting arbitrary-angle wedge covered by a thin dielectric slab on one surface. They treated the diffraction of an E -polarized plane wave normally incident on the edge. It was assumed that the dielectric slab was thin with the relative permittivity not much larger than unity and that the slab was in the illuminated region. By imposing an approximate boundary condition $E^s \sim \Gamma E^i$ at the dielectric-covered surface, where Γ represented the plane wave reflection coefficient for a dielectric slab on an infinite ground plane the solution was obtained in an integral form, which was then evaluated asymptotically in terms of Fresnel integrals.

Burnside *et al.* [8] used a heuristic approach to modify the diffraction coefficients of a perfectly conducting wedge to a finite conductivity wedge by incorporating the reflection coefficient of the impedance surface in the diffraction coefficients. As a result, the fields were continuous across the reflection and shadow boundaries. Such field, however, does not satisfy the boundary condition.

Rojas [9] presented a uniform asymptotic solution for the electromagnetic diffraction by a wedge with impedance surfaces and included angles equal to 0, 90, 180, 270°. The incident field was a plane wave of arbitrary polarization, obliquely incident to the axis of the wedge. The formal solution was obtained by the generalized reflection method. The integral was then evaluated asymptotically to yield the geometrical optics fields, surface wave field, and the diffracted field. The numerical solution was obtained by computing the Maliuzhinets functions.

Pelosi *et al.* [10] used an incremental length diffraction coefficient formulation for the canonical problem of a locally tangent wedge with surface impedance boundary condition on its faces. The solution was used to determine the incremental contribution to the field which arose at any point of the edge. The results were deduced in a rigorous fashion from a Sommerfeld spectral integral representation of the exact solution for the canonical wedge problem. The scattered field

was decomposed into physical optics, surface wave, and fringe components due to the combined incident and surface wave fields.

In our approach, the difficulty of a mixed boundary condition at a right-angle wedge is overcome by a simple functional transformation [11]. The transformation is made in such a way that the impedance boundary condition is replaced by the simpler Neumann or Dirichlet conditions. Thus, the Green's functions resulting from the transformation can be found in the usual way. The final solution is obtained by an inverse transformation. This method was first proposed by Lewy [12] and Stoker [13], who studied problems in water wave theory. The same idea has been applied by Karp and Karal [14], [15], Chu [16], Karal *et al.* [17], and Chu *et al.* [18] for solving electromagnetic diffraction problems for a right-angle wedge, where one of the wedge surfaces sustained a surface wave, while the other was perfectly conducting. They considered a wedge illuminated by a normally incident plane wave, a magnetic line source on the vertex or by an incident surface wave. In all cases, two important constants in their solutions were found by using continuity conditions for the total field and its derivative across one of the positive axis. They completed their solution by determining the far-zone field. In the case of a normally incident plane wave, their diffracted field became singular at the reflection and shadow boundaries, because they did not obtain a uniform asymptotic approximation. Karp [19] also obtained a two-dimensional (2-D) Green's function for a right-angle wedge with an impedance boundary condition, which did not support surface waves. The calculation of the far-zone field depended on one of the constants in his solution. The constant was given by an expression which was difficult to compute. We have determined that this constant is one.

In this paper, the transformation technique is used to solve the diffraction of scalar waves. The scalar solution is related to the electromagnetic problem. Integral representations are obtained from the eigenfunction expansions of the Green's functions. They are evaluated asymptotically via the modified Pauli-Clemmow method of steepest descent to obtain a far-zone approximation valid at both the shadow and reflection boundaries. The solutions are cast into the form of the geometrical optics fields and diffracted field. The geometrical optics field, which comes from pole contributions to the integral, consists of the incident field and the reflected field. Saddle point contributions from the integral yield the diffracted field. The geometrical optics field is discontinuous across the shadow and reflection boundaries; however, the total field, which is the sum of the geometrical optics field and the diffracted field, is continuous.

The diffraction of scalar waves is treated in Section II. The cylindrical wave illumination is considered first. The plane wave illumination can be treated as a special case of the cylindrical wave illumination. The spherical wave illumination can also be reduced to a two-dimensional (2-D) problem via a Fourier integral transformation. For the sake of clarity, we consider first the wedge with one side of the surface having a uniform impedance boundary condition and the other having either Dirichlet or Neumann boundary

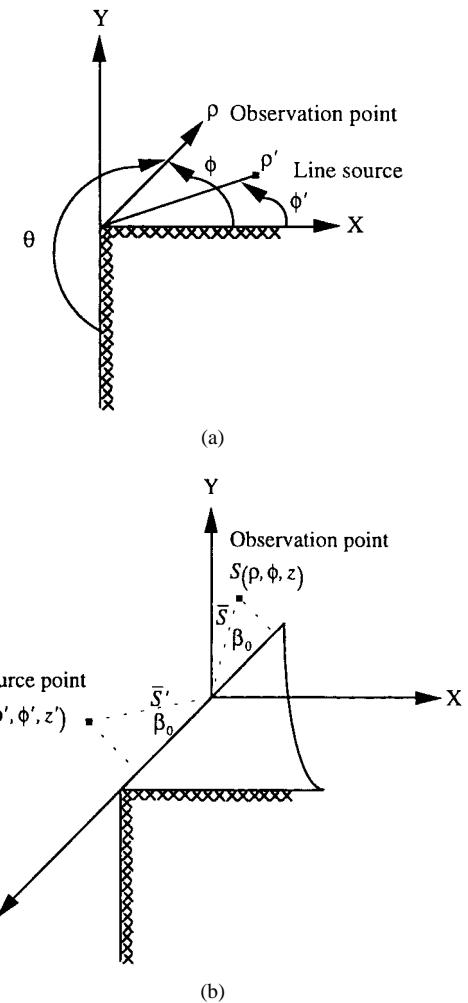


Fig. 1. A two-impedance wall right-angle wedge. (a) A line source illumination. (b) A point source illumination.

condition. The analysis can easily be extended to a wedge with two different impedance surfaces. The relation of scalar diffraction to electromagnetic diffraction is also presented. Singularity of the transformation operator can be overcome and is discussed in Section III. It is also shown that the functional transformation technique can be extended in an approximate way to treat an arbitrary-angle wedge, provided that one of its surfaces satisfies a Dirichlet boundary condition. Some numerical results and comparison with other methods are shown in Section III. The surface wave is not analyzed in the paper for it can be derived from the canonical problem of a locally tangent infinite impedance surface. The excited surface wave has an important effect only when the field is close to the surface. The diffracted field due to a surface wave has been given by Chu [16]–[18].

II. DIFFRACTION BY A RIGHT-ANGLE IMPEDANCE WEDGE

Consider a z directed, uniform line source of unit strength radiating in the presence of a right-angle impedance wedge ($n = 1.5$) with its edge also oriented in the z direction, as shown in Fig. 1. One side of the wedge surface has a uniform impedance boundary condition; the other has a Dirichlet or Neumann boundary condition. The total field $U(\bar{\rho}, \bar{\rho}', \xi_1, \xi_2)$,

which consists of the incident field and the scattered field, satisfies the scalar wave equation

$$(\nabla_t^2 + k^2)U(\rho, \rho', \xi_1, \xi_2) = -\delta(x - x')\delta(y - y'). \quad (1)$$

The surface impedance boundary condition is

$$\left(\frac{\partial}{\partial y} + \xi_1\right)U = 0; \quad \phi = 0 \quad (2)$$

$$U = 0; \quad \phi = n\pi \quad (3a)$$

$$\frac{\partial U}{\partial x} = 0; \quad \phi = n\pi. \quad (3b)$$

In addition, it has to satisfy the Sommerfeld radiation condition and the Meixner edge condition.

∇_t^2 is the 2-D Laplacian operator, $\delta(x - x')$ is the Dirac Delta function, and k is the wave number of the linear, homogeneous, isotropic medium surrounding the wedge. A time dependence $\exp(j\omega t)$ is assumed and suppressed.

By making the functional transformation

$$V = \left(\frac{\partial}{\partial y} + \xi_1\right)U \quad (4)$$

and using the properties that the operators $(\nabla_t^2 + k^2)$ and $((\partial/\partial y) + \xi_1)$ commute and $(\partial/\partial y)\delta(y - y') = -(\partial/\partial y')\delta(y - y')$, V satisfies

$$(\nabla_t^2 + k^2)V = \left(\frac{\partial}{\partial y'} - \xi_1\right)\delta(x - x')\delta(y - y') \quad (5)$$

together with the boundary conditions

$$V = 0; \quad \phi = 0 \quad (6a)$$

$$\begin{cases} V = 0; \\ \frac{\partial V}{\partial x} = 0; \quad \phi = n\pi \end{cases} \quad (6b)$$

and the Sommerfeld radiation condition. By introducing the Green's function which satisfies

$$(\nabla_t^2 + k^2)G = -\delta(x - x')\delta(y - y'); \quad 0 < \phi \leq n\pi \quad (7)$$

$$G = 0; \quad \phi = 0 \quad (8)$$

and conditions (3), the Sommerfeld radiation condition and the Meixner edge condition show that

$$G_a = -\frac{j}{4n} \sum_{m=0}^{\infty} \varepsilon_m J_{2m/2n}(k\rho_{<}) H_{2m/2n}^{(2)}(k\rho_{>}) \cdot \left[\cos \frac{2m}{2n}(\phi - \phi') - \cos \frac{2m}{2n}(\phi + \phi') \right]; \quad \text{for (3a)} \quad (9)$$

$$G_b = -\frac{j}{2n} \sum_{m=0}^{\infty} J_{(2m+1)/2n}(k\rho_{<}) H_{(2m+1)/2n}^{(2)}(k\rho_{>}) \cdot \left[\cos \frac{2m+1}{2n}(\phi - \phi') - \cos \frac{2m+1}{2n}(\phi + \phi') \right]; \quad \text{for (3b)} \quad (10)$$

$-((\partial/\partial y) - \xi_1)G$ is then the particular solution of V . By constructing another Green's function W , which satisfies the

conditions (1), (3), the Sommerfeld radiation condition, the Meixner edge condition, and the boundary condition

$$\frac{\partial W}{\partial y} = 0; \quad \phi = 0 \quad (11)$$

it can be shown that

$$W_a = -\frac{j}{4n} \sum_{m=0}^{\infty} J_{(2m+1)/2n}(k\rho_{<}) H_{(2m+1)/2n}^{(2)}(k\rho_{>}) \cdot \left[\cos \frac{2m+1}{2n}(\phi - \phi') + \cos \frac{2m+1}{2n}(\phi + \phi') \right]; \quad \text{for (3a)} \quad (12)$$

$$W_b = -\frac{j}{2n} \sum_{m=0}^{\infty} \varepsilon_m J_{2m/2n}(k\rho_{<}) H_{2m/2n}^{(2)}(k\rho_{>}) \cdot \left[\cos \frac{2m}{2n}(\phi - \phi') + \cos \frac{2m}{2n}(\phi + \phi') \right]; \quad \text{for (3b)} \quad (13)$$

here $0 < \phi, \phi' < n\pi$; $0 \leq \rho, \rho' \leq \infty$, and $\varepsilon_m = 1$ for $m = 0$, $\varepsilon_m = 2$ for $m \neq 0$. $J_{v_m}(k\rho)$ and $H_{v_m}^{(2)}(k\rho)$ represent the Bessel function of the first kind and Hankel function of the second kind, respectively, and $\rho_{<} = \rho$ for $\rho < \rho'$, $\rho_{<} = \rho'$ for $\rho > \rho'$, $\rho_{>} = \rho'$ for $\rho > \rho'$, $\rho_{>} = \rho$ for $\rho < \rho'$. The function $H = ((\partial G/\partial y') + (\partial W/\partial y))$ is a solution of the homogeneous wave equation. The complete solution for V can then be expressed as

$$V = -\left(\frac{\partial}{\partial y'} - \xi_1\right)G + C\left(\frac{\partial G}{\partial y'} + \frac{\partial W}{\partial y}\right). \quad (14)$$

Since G and $\partial W/\partial y$ are zero on $\phi = 0$, this implies $(C - 1)\partial G/\partial y' = 0$. While $\partial G/\partial y'$ is not zero, C must equal to one. Equation (14) becomes

$$V = \frac{\partial W}{\partial y} + \xi_1 \frac{\partial G}{\partial y'} \quad (15)$$

U , V , G , and W can be expressed in the asymptotic forms as

$$U \approx \frac{\exp(-jk\rho)}{\sqrt{\rho}} u(\phi; x', y') \quad (16)$$

$$V \approx \frac{\exp(-jk\rho)}{\sqrt{\rho}} v(\phi; x', y') \quad (17)$$

$$G \approx \frac{\exp(-jk\rho)}{\sqrt{\rho}} g(\phi; x', y') \quad (18)$$

$$W \approx \frac{\exp(-jk\rho)}{\sqrt{\rho}} w(\phi; x', y'). \quad (19)$$

Here, G , W , and V are known functions and U is to be found. Since the far-zone field is of interest, only those terms which are of $O(1/\sqrt{\rho})$ are retained and the operator $\partial/\partial y$ can then be approximated by $-jk \sin \phi$; thus,

$$(-jk \sin \phi + \xi_1)U \approx V \quad (20)$$

which leads to

$$U \approx \frac{\xi_1 G - jk \sin \phi W}{\xi_1 - jk \sin \phi}. \quad (21)$$

In the Appendix, it is shown that G and W are transformed into the integral representations and computed asymptotically

for the large parameter $k\rho\rho' / (\rho + \rho')$. From the asymptotic solution, we can identify the contribution from the physical poles, which yield the geometrical optics components. The saddle-point contribution yields the diffracted field, which is related to the incident field upon the edge and is written as

$$U^d \approx U^i D_{a,b} \frac{\exp(-jk\rho)}{\sqrt{\rho}} \quad (22)$$

in which

$$U^i \approx -\frac{j}{4} \sqrt{\frac{2j}{\pi k\rho'}} \exp(-jk\rho') \quad (23)$$

and $D_{a,b}$ are the diffraction coefficients given as

$$D_a = \frac{\xi_1 D_s - jk \sin \phi D_{g1}}{\xi_1 - jk \sin \phi}; \quad \text{for (3a)} \quad (24a)$$

$$D_b = \frac{\xi_1 D_{g2} - jk \sin \phi D_h}{\xi_1 - jk \sin \phi}; \quad \text{for (3b)} \quad (24b)$$

where

$$D_{g1} = -\frac{\text{Exp}\left(-j\frac{\pi}{4}\right)}{2n\sqrt{2k\pi}} \cdot \left\{ 2 \sin \frac{\pi}{2n} \left[\cos \frac{\beta^-}{2n} + \cos \frac{\beta^+}{2n} \right] + [\alpha_g^+(\beta^-)F(Ka^+(\beta^-)) + \alpha_g^-(\beta^-)F(Ka^-(\beta^-))] + [\alpha_g^+(\beta^+)F(Ka^+(\beta^+)) + \alpha_g^-(\beta^+)F(Ka^-(\beta^+))] \right\} \quad (25)$$

$$D_{g2} = -\frac{\text{Exp}\left(-j\frac{\pi}{4}\right)}{2n\sqrt{2k\pi}} \cdot \left\{ 2 \sin \frac{\pi}{2n} \left[\cos \frac{\beta^-}{2n} - \cos \frac{\beta^+}{2n} \right] + [\alpha_g^+(\beta^-)F(Ka^+(\beta^-)) + \alpha_g^-(\beta^-)F(Ka^-(\beta^-))] - [\alpha_g^+(\beta^+)F(Ka^+(\beta^+)) + \alpha_g^-(\beta^+)F(Ka^-(\beta^+))] \right\} \quad (26)$$

where

$$\alpha_g^\pm = \frac{\cos^2\left(\frac{\pi \pm \beta}{2n}\right)}{\sin\left(\frac{\pi \pm \beta}{2n}\right)} \quad (27)$$

$$D_s = [\alpha^+(\beta^-)F(Ka^+(\beta^-)) + \alpha^-(\beta^-)F(Ka^-(\beta^-))] - [\alpha^+(\beta^+)F(Ka^+(\beta^+)) + \alpha^-(\beta^+)F(Ka^-(\beta^+))] \quad (28)$$

$$D_h = [\alpha^+(\beta^-)F(Ka^+(\beta^-)) + \alpha^-(\beta^-)F(Ka^-(\beta^-))] + [\alpha^+(\beta^+)F(Ka^+(\beta^+)) + \alpha^-(\beta^+)F(Ka^-(\beta^+))] \quad (29)$$

where

$$\alpha^\pm = \text{Cot}\left(\frac{\pi \pm \beta}{2n}\right). \quad (30)$$

Note that D_s and D_h are the Keller diffraction coefficients in uniform version for the horizontal and vertical polarization for a perfectly conducting wedge. In the preceding equations

$$\beta = \beta^\pm = \phi \pm \phi' \quad (31)$$

$$K = \frac{k\rho\rho'}{\rho + \rho'} \quad (32)$$

$$F(Ka^\pm(\beta)) = 2j\sqrt{Ka^\pm(\beta)} \text{Exp}(jkKa^\pm(\beta)) \cdot \int_{\sqrt{Ka^\pm(\beta)}}^{\infty} \text{Exp}(-t^2) dt \quad (33)$$

where the positive branch of the square root is taken and

$$a^\pm(\beta) = 1 + \text{Cos}(-\beta + 2nN^\pm\pi). \quad (34)$$

The value of N^\pm is determined by the integer which most closely satisfies

$$2nN^\pm - \beta = \pm\pi. \quad (35)$$

For the case of a right-angle wedge with two impedance surfaces, we make another functional transformation

$$Q = \left(\frac{\partial}{\partial x} - \xi_2 \right) V. \quad (36)$$

By repeating the same procedure, we obtain the diffraction coefficients, as shown in (37) at the bottom of the page, where $\Theta = n\pi - \theta$. The plane wave illumination can be treated as a special case of the cylindrical wave illumination. By letting $\rho' \rightarrow \infty$ in the line source case and factoring out $-(j/4)\sqrt{2j/\pi k\rho'}\text{Exp}(-jk\rho')$, the result for the plane wave case can be obtained. The diffracted field is given by

$$U^d \approx D_{a,b} \frac{\text{Exp}(-jk\rho)}{\sqrt{\rho}} \quad (38)$$

for a plane wave of unit amplitude and with its phase reference located at the edge, where D is given in (24) with the large parameter $K = k\rho\rho' / (\rho + \rho') \rightarrow k\rho$, since $\rho' \rightarrow \infty$.

The spherical wave illumination is related to the cylindrical case by applying the Fourier transform with respect to the z variable. For a source of unit strength, the total field U_a due to spherical waves incident on the wedge satisfies

$$(\nabla_t^2 + k_t^2)U_a(\bar{s}, \bar{s}') = -\delta(x - x')\delta(y - y')\delta(z - z'). \quad (39)$$

$$D = \frac{jk \sin \theta (jk D_h \sin \phi - \xi_1 D_{g2}) - \xi_2 (jk D_{g1} \sin \phi - \xi_1 D_s)}{(jk \sin \theta - \xi_2)(jk \sin \phi - \xi_1)} \quad (37)$$

The Fourier transform of the above equation yields

$$(\nabla^2 + k_t^2) \tilde{U}_a(\rho, \phi, h; \rho', \phi', z') = -\delta(x-x')\delta(y-y')e^{-jhz'} \quad (40)$$

where $\nabla^2 = \nabla_t^2 + (\partial^2/dz^2)$ is used because of the special geometry of the wedge, $k_t^2 = k^2 - h^2$ and (U, \tilde{U}) are Fourier transform pair. The z variation of the function U is removed. Rewriting the above equation as

$$(\nabla^2 + k_t^2) \tilde{U}_a(\rho, \phi, h; \rho', \phi', z')e^{-jhz'} = -\delta(x-x')\delta(y-y') \quad (41)$$

allows one to identify $\tilde{U}_a(\rho, \phi, h; \rho', \phi', z')e^{-jhz'}$ as $U_a(\rho, \phi')$, which is a 2-D scalar Green's function of (1). The diffracted field can be given by

$$U^d \approx \left(\frac{D}{\sin \beta_0} \right) \frac{\text{Exp}(-jkS')}{4\pi S'} \sqrt{\frac{S'}{S(S+S')}} \text{Exp}(-jks) \quad (42)$$

for a spherical wave of unit amplitude, where D is given in (24) with the large parameter $K = kSS'/(S+S')\sin^2 \beta_0$. The $1/\sin \beta_0$ factor in (42) is to be expected because of the conical spreading of the diffracted rays. This conical spreading is a consequence of the generalized Fermat's principle.

The solutions of the above scalar cylindrical wave problems can be directly related to some electromagnetic cylindrical wave problems [20], [21]. The diffracted fields can be expressed as

$$\begin{bmatrix} E_{\text{II}}^d \\ E_{\perp}^d \end{bmatrix} \approx \begin{bmatrix} D_a & 0 \\ 0 & D_b \end{bmatrix} \begin{bmatrix} E_{\text{II}}^i(Q_E) \\ E_{\perp}^i(Q_E) \end{bmatrix} \frac{\exp(-jk\rho)}{\sqrt{\rho}} \quad (43)$$

$$\begin{bmatrix} H_{\perp}^d \\ H_{\text{II}}^d \end{bmatrix} \approx \begin{bmatrix} D_b & 0 \\ 0 & D_a \end{bmatrix} \begin{bmatrix} H_{\perp}^i(Q_E) \\ H_{\text{II}}^i(Q_E) \end{bmatrix} \frac{\exp(-jk\rho)}{\sqrt{\rho}} \quad (44)$$

where $E_{\text{II}}^d = \bar{E}^d \bullet \hat{z}$, $E_{\perp}^d = \bar{E}^d \bullet \hat{\phi}$, $E_{\text{II}}^i(Q_E) = \bar{E}^i(Q_E) \bullet \hat{z}$, and $E_{\perp}^i(Q_E) = \bar{E}^i(Q_E) \bullet (-\hat{\phi})$; $H_{\perp}^d = \bar{H}^d \bullet \hat{\phi}$, $H_{\text{II}}^d = \bar{H}^d \bullet \hat{z}$, $H_{\perp}^i(Q_E) = \bar{H}^i(Q_E) \bullet (-\hat{\phi})$, and $H_{\text{II}}^i(Q_E) = \bar{H}^i(Q_E) \bullet \hat{z}$.

III. DISCUSSIONS AND NUMERICAL RESULTS

Fig. 2 compares the computed radiation patterns between our formulation (shown by solid curve) and that of Tiberio *et al.* [22], shown by discrete points, for a line source at $k\rho' = 10$, $\phi' = 30^\circ$ illuminating a right-angle wedge with two equal face impedances with $\xi_1 = \xi_2 = -0.25jk$. Our computed radiation pattern is given by solid curve while the singularity is presented by dashed line. The prediction is again compared with that of Tiberio *et al.* [22].

The diffraction coefficient becomes singular when $-jk \sin(\theta, \phi) + \xi = 0$ due to the approximation of the transformation $(\partial/\partial(x, y)) + \xi$. The V in (35) cannot be divided by zero to obtain U . When $((\partial/\partial y) + \xi)U = 0$, C in (14) should be zero. This singularity can then be removed by factoring out the same factor $-jk \sin \phi + \xi$ in (14), which leads to $U = G$. The diffraction coefficient becomes

$$D = \begin{cases} \frac{(jkD_h \sin \phi - \xi_1 D_{g2})}{(jk \sin \phi - \xi_1)} & \text{for } jk \sin \theta - \xi_2 = 0 \\ \frac{(jkD_h \sin \theta - \xi_2 D_{g1})}{(jk \sin \theta - \xi_2)} & \text{for } jk \sin \phi - \xi_1 = 0 \end{cases} \quad (45)$$

and is finite. Fig. 3 presents the correction of singularity that takes place for a TM case for a line source at $k\rho' = 10$,

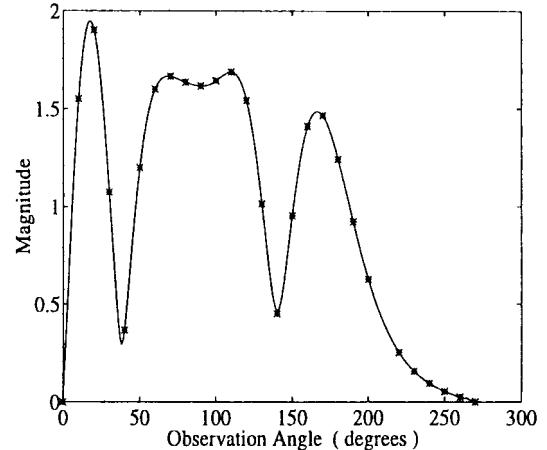


Fig. 2. Pattern of the total field TE case.

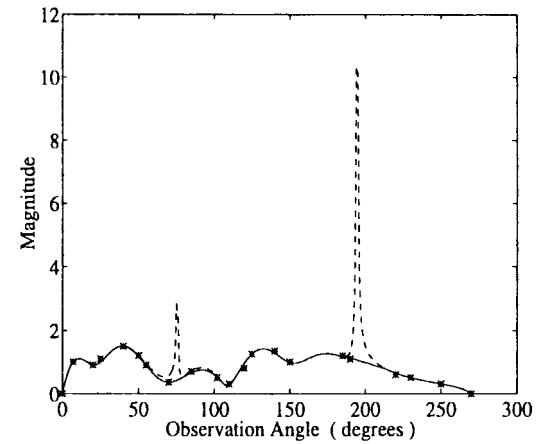


Fig. 3. Pattern of the total field TM case.

$\phi' = 30^\circ$ illuminating a right-angle wedge with two equal face impedances with $\xi_1 = \xi_2 = -0.25jk$. Our computed radiation pattern is given by solid curve while the singularity is presented by dashed line. The prediction is again compared with that of Tiberio *et al.* [22].

The formulation can be extended in an approximate way to an arbitrary-angle wedge where one of its surfaces satisfies the Dirichlet boundary condition. When the approximation $\partial/\partial y \approx \sin \phi (\partial/\partial \rho)$ is made, $V = 0$ at $\phi = n\pi$ and leads to $U = 0$ at $\phi = n\pi$, an approximation solution for U can be obtained by following the same procedure as that used in a right-angle wedge.

IV. CONCLUSIONS

The diffraction by a right-angle wedge with different surface boundary conditions has been derived. A functional transformation was used to simplify the boundary conditions. The eigenfunction solutions for the transformed functions were replaced by integral representations, which were evaluated asymptotically by the modified Pauli-Clemmow method of steepest descent. The solution was interpreted in terms of the geometrical optics fields and the diffracted field. The diffracted field is equal to the incident field on the edge multiplied by the associated uniform impedance diffraction coefficients. The

diffraction coefficients are related directly to Keller diffraction coefficients in uniform version for a perfectly conducting wedge. In the case where the impedance surface supports a surface wave, the surface wave can be derived from the canonical problem of a locally tangent, infinite impedance surface. The diffracted field due to the surface wave is then added to give a total radiation field. We have shown that the functional transformation technique can be extended in an approximate way to an arbitrary-angle wedge where one of its surfaces satisfies the Dirichlet boundary condition. Furthermore, when the transformation operator becomes singular, constant C should be zero. The diffraction coefficients will be used in analyzing the propagation characteristics in tunnels in Part 2.

APPENDIX

In (21) the far zone form of U was shown to be

$$U \approx \frac{\xi_i G - jk \sin \phi W}{\xi_1 - jk \sin \phi} \quad (\text{A.1})$$

where G_a , G_b , W_a , and W_b are known special Green's functions given in (9), (10), (12), and (13). An integral representation for the product $J_{v_m}(k\rho')H_{v_m}^{(2)}(k\rho)$ is given by [21]

$$\begin{aligned} J_{v_m}(k\rho')H_{v_m}^{(2)}(k\rho) \\ = -\frac{1}{\pi j} \int_0^{c-j\infty} \text{Exp}\left[\frac{1}{2}(t - k^2(\rho^2 + \rho'^2)t^{-1})\right] \\ \cdot I_{v_m}\left(\frac{k^2\rho\rho'}{t}\right) \frac{dt}{t} \end{aligned} \quad (\text{A.2})$$

where $c > 0$, $v_m > -1$ and $|\rho'| < |\rho|$. $I_{v_m}(k^2\rho\rho'/t)$ represents the modified cylindrical Bessel function of the first kind. The integral representation for $I_{v_m}(k^2\rho\rho'/t)$ is given by [20]

$$I_{v_m}\left(\frac{k^2\rho\rho'}{t}\right) = -\frac{1}{2\pi} \int_{\gamma-j\infty}^{\gamma+j\infty} \text{Exp}\left[\frac{k^2\rho\rho'}{t} \text{Cos}\zeta + jv_m\zeta\right] d\zeta \quad (\text{A.3})$$

where $-\pi < \gamma < 0$ and $\pi < \gamma < 2\pi$. Thus

$$I_{v_m}\left(\frac{k^2\rho\rho'}{t}\right) = -\frac{1}{2\pi} \int_L \text{Exp}\left[\frac{k^2\rho\rho'}{t} \text{Cos}\zeta + jv_m\zeta\right] d\zeta \quad (\text{A.4})$$

or

$$I_{v_m}\left(\frac{k^2\rho\rho'}{t}\right) = -\frac{1}{2\pi} \int_L \text{Exp}\left[\frac{k^2\rho\rho'}{t} \text{Cos}\zeta - jv_m\zeta\right] d\zeta. \quad (\text{A.5})$$

Let us rewrite (10) and (12) as

$$G_b = g(\bar{\rho}, \bar{\rho}'; \beta^-) - g(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.6})$$

$$W_a = g(\bar{\rho}, \bar{\rho}'; \beta^-) + g(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.7})$$

where

$$\begin{aligned} g(\bar{\rho}, \bar{\rho}'; \beta) = -\frac{j}{6} \sum_{m=0}^{\infty} J_{(2m+1)/2n}(k\rho')H_{(2m+1)/2n}^{(2)}(k\rho) \\ \cdot \left[\text{Exp}\left(j \frac{2m+1}{2n} \beta\right) + \text{Exp}\left(-j \frac{2m+1}{2n} \beta\right) \right]. \end{aligned} \quad (\text{A.8})$$

Substituting (A.3)–(A.5) into (A.8), we obtain

$$\begin{aligned} g(\bar{\rho}, \bar{\rho}'; \beta) = -\frac{1}{12\pi^2} \int_0^{c-j\infty} & \left[\text{Exp}\left[\frac{1}{2}(t - k^2(\rho^2 + \rho'^2)t^{-1})\right] \right. \\ & \cdot \left\{ \int_L \left[\text{Exp}\left[\frac{k^2\rho\rho'}{t} \text{Cos}\zeta\right] \right. \right. \\ & \left. \left. \cdot \sum_{m=0}^{\infty} \text{Exp}\left[j \frac{2m+1}{2n} (\zeta + \beta)\right] \right] d\zeta \right. \\ & + \int_{L'} \left[\text{Exp}\left[\frac{k^2\rho\rho'}{t} \text{Cos}\zeta\right] \right. \\ & \left. \left. \cdot \sum_{m=0}^{\infty} \text{Exp}\left[-j \frac{2m+1}{2n} (\zeta + \beta)\right] \right] d\zeta \right\} \cdot \frac{dt}{t}. \end{aligned} \quad (\text{A.9})$$

By noting that

$$\begin{aligned} \sum_{m=0}^{\infty} \text{Exp}\left[j \frac{2m+1}{2n} (\zeta + \beta)\right] \\ = -\frac{1}{2j} \frac{\text{Cos}^2\left(\frac{\zeta + \beta}{2n}\right)}{\text{Sin}\left(\frac{\zeta + \beta}{2n}\right)} + \text{Sin}\left(\frac{\zeta + \beta}{2n}\right) \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} \sum_{m=0}^{\infty} \text{Exp}\left[-j \frac{2m+1}{2n} (\zeta + \beta)\right] \\ = \frac{1}{2j} \frac{\text{Cos}^2\left(\frac{\zeta + \beta}{2n}\right)}{\text{Sin}\left(\frac{\zeta + \beta}{2n}\right)} + \text{Sin}\left(\frac{\zeta + \beta}{2n}\right) \end{aligned} \quad (\text{A.11})$$

and from [20] that

$$-\frac{1}{2} \int_{c-j\infty}^0 \text{Exp}\left[\frac{t}{2} - \frac{Z^2}{2t}\right] \frac{dt}{t} = K_0(jZ) \quad (\text{A.12})$$

(A.9) can be simplified to

$$\begin{aligned} g(\bar{\rho}, \bar{\rho}'; \beta) = \frac{1}{j12\pi^2} \int & \left[\frac{\text{Cos}^2\left(\frac{\zeta + \beta}{2n}\right)}{\text{Sin}\left(\frac{\zeta + \beta}{2n}\right)} + \text{Sin}\left(\frac{\zeta + \beta}{2n}\right) \right] \\ \cdot K_0[jZ(\zeta)] d\zeta \end{aligned} \quad (\text{A.13})$$

where $K_0(jZ)$ is the modified cylindrical Bessel function of the second kind of order zero with argument jZ , and

$Z(\zeta) = k\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \zeta}$. For large Z , $K_0[jZ(\zeta)]$ can be replaced by its large argument approximation

$$K_0[jZ(\zeta)] \approx \sqrt{\frac{\pi}{2jZ(\zeta)}} \text{Exp}(-jZ(\zeta)). \quad (\text{A.14})$$

The saddle points of interest occur at $\zeta_s = \pm\pi$. In the neighborhood of these saddle points

$$\begin{aligned} \text{Exp}[-jkZ(\zeta)] &= \text{Exp}\left[-jk\sqrt{(\rho + \rho')^2 - 2\rho\rho'(1 + \cos \zeta)}\right] \\ &\approx \text{Exp}\left[-jk(\rho + \rho')\left(1 - \frac{\rho\rho'}{(\rho + \rho')^2}(1 + \cos \zeta)\right)\right]. \end{aligned} \quad (\text{A.15})$$

Thus

$$\begin{aligned} g(\bar{\rho}, \bar{\rho}'; \beta) &\approx \text{Exp}[-jk(\rho + \rho')] \\ &\bullet \int_{L-L'} \sqrt{\frac{2}{jk\pi\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \zeta}}} \\ &\bullet F_1(\zeta, \beta) \text{Exp}[Kf(\zeta)] d\zeta \end{aligned} \quad (\text{A.16})$$

where

$$F_1(\zeta, \beta) = \frac{1}{j6\pi} \left[\frac{\cos^2\left(\frac{\zeta + \beta}{2n}\right)}{\sin\left(\frac{\zeta + \beta}{2n}\right)} + \sin\left(\frac{\zeta + \beta}{2n}\right) \right] \quad (\text{A.17})$$

$$f(\zeta) = j(1 + \cos \zeta) \quad (\text{A.18})$$

and K is the large parameter

$$K = \frac{k\rho\rho'}{\rho + \rho'}. \quad (\text{A.19})$$

The integral in (A.16) is in the proper form to be evaluated by the method of steepest descent for a large parameter. The saddle points of $f(\zeta)$ occur at

$$\left. \frac{d}{d\zeta} f(\zeta) \right|_{\zeta=\zeta_s} = 0 \quad (\text{A.20})$$

but only $\zeta_s = \pm\pi$ is considered because the steepest descent paths through $\zeta_s = \pm\pi$ allow us to close the $(L - L')$ contour. Note that when K is large, the approximation of (A.14) is justified in the neighborhood of these saddle points. Fig. 4 shows the locations of the steepest descent paths through the saddle points at $\zeta_s = \pm\pi$. Therefore,

$$\begin{aligned} \text{Exp}[-jk(\rho + \rho')] &\bullet \int_{L-L'} \sqrt{\frac{2}{jk\pi\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \zeta}}} \\ &\bullet F_1(\zeta, \beta) \text{Exp}[Kf(\zeta)] d\zeta \\ &= -\text{Exp}[-jk(\rho + \rho')] \\ &\bullet \left[\int_{SDP(\pi)} + \int_{DP(-\pi)} \sqrt{\frac{2}{jk\pi\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \zeta}}} \right. \\ &\quad \left. \bullet F_1(\zeta, \beta) \text{Exp}[Kf(\zeta)] d\zeta \right] \\ &+ [2\pi j \sum \text{The residues of the integrand}] \end{aligned}$$

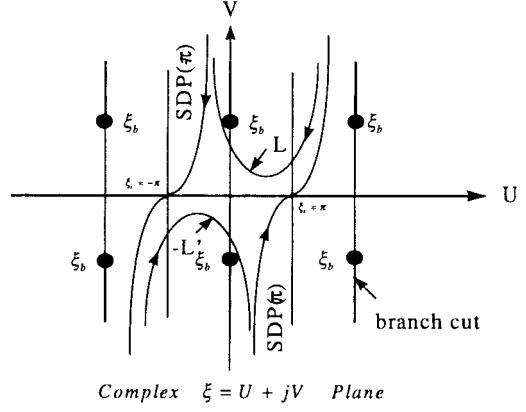


Fig. 4. Steepest descent paths and the complex ξ -plane topology.

$$\begin{aligned} &\text{enclosed by } L - L' \text{ and } SDP(\pm\pi) \\ &+ \text{Branch cut contributions, if any.} \end{aligned} \quad (\text{A.21})$$

The pole singularities occur at

$$\zeta_p = -\beta + 2nN\pi, \quad N = 0, \pm 1, \pm 2, \dots \quad (\text{A.22})$$

The residues corresponding to ζ_p are evaluated only for those $|\zeta_p| \leq \pi$. Let the contribution of the poles to g be denoted by g^p

$$\begin{aligned} g^p(\bar{\rho}, \bar{\rho}'; \beta) &= \sqrt{\frac{2}{jk\pi\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \zeta_p}}} \\ &\bullet \text{Exp}\left[-jk\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \zeta_p}\right] \\ &\bullet \cos N\pi \bullet H(\pi - |\beta + 2nN\pi|) \end{aligned} \quad (\text{A.23})$$

where

$$H(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{2}, & t = 0 \\ 1, & t > 0. \end{cases}$$

The contribution of the saddle points to g , which is denoted by g^d , is derived via the modified steepest descent method [21] where the pole singularity close to the saddle point is taken into consideration. It can be shown that

$$\begin{aligned} g^d(\bar{\rho}, \bar{\rho}'; \beta) &\approx \left[-\frac{j}{4} \sqrt{\frac{2j}{\pi k\rho}} \text{Exp}(-jk\rho) \right] \\ &\bullet d_g(\beta) \frac{\text{Exp}(-jk\rho)}{\sqrt{\rho}} \end{aligned} \quad (\text{A.24})$$

where

$$\begin{aligned} d_g(\beta) &= -\frac{\text{Exp}\left(-j\frac{\pi}{4}\right)}{2n\sqrt{2k\pi}} \\ &\bullet \left\{ \left[\sin \frac{(\pi + \beta)}{2n} + \sin \frac{(\pi - \beta)}{2n} \right] \right\} \end{aligned}$$

$$+ \left[\frac{\cos^2\left(\frac{\pi+\beta}{2n}\right)}{\sin\left(\frac{\pi+\beta}{2n}\right)} F(Ka^+(\beta)) + \frac{\cos^2\left(\frac{\pi-\beta}{2n}\right)}{\sin\left(\frac{\pi-\beta}{2n}\right)} F(Ka^-(\beta)) \right] \right\} \quad (\text{A.25})$$

with

$$F(Ka^\pm(\beta)) = 2j\sqrt{Ka^\pm(\beta)} \exp(jKa^\pm(\beta)) \int_{\sqrt{Ka^\pm(\beta)}}^{\infty} \cdot \exp(-t^2) dt$$

where the positive branch of the square root is taken, and

$$a^\pm(\beta) = 1 + \cos(-\beta + 2nN^\pm\pi).$$

The value of N^\pm is determined by the integer, which most closely satisfies

$$2nN^\pm - \beta = \pm\pi.$$

Substituting (A.23) and (A.24) into (A.6) and (A.7), we obtain

$$G_b^p(\bar{\rho}, \bar{\rho}') = g^p(\bar{\rho}, \bar{\rho}'; \beta^-) - g^p(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.26})$$

$$G_a^d(\bar{\rho}, \bar{\rho}') = g^d(\bar{\rho}, \bar{\rho}'; \beta^-) - g^d(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.27})$$

$$W_a^p(\bar{\rho}, \bar{\rho}') = g^p(\bar{\rho}, \bar{\rho}'; \beta^-) + g^p(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.28})$$

$$W_a^d(\bar{\rho}, \bar{\rho}') = g^d(\bar{\rho}, \bar{\rho}'; \beta^-) + g^d(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.29})$$

where the superscript p denotes the contribution from the pole singularities and the superscript d denotes the contribution from the saddle points.

Equations (9) and (14) have been evaluated asymptotically by Pathak and Kouyoumjian [21] in which G_a and W_b are given as

$$G_a^p(\bar{\rho}, \bar{\rho}') = I^p(\bar{\rho}, \bar{\rho}'; \beta^-) - I^p(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.30})$$

$$G_a^d(\bar{\rho}, \bar{\rho}') = I^d(\bar{\rho}, \bar{\rho}'; \beta^-) - I^d(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.31})$$

$$W_b^p(\bar{\rho}, \bar{\rho}') = I^p(\bar{\rho}, \bar{\rho}'; \beta^-) + I^p(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.32})$$

and

$$W_b^d(\bar{\rho}, \bar{\rho}') = I^d(\bar{\rho}, \bar{\rho}'; \beta^-) + I^d(\bar{\rho}, \bar{\rho}'; \beta^+) \quad (\text{A.33})$$

where

$$I^p(\bar{\rho}, \bar{\rho}'; \beta) = \sqrt{\frac{2}{jk\pi\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos\zeta_p}}} \cdot \exp\left[-jk\sqrt{\rho^2\rho'^2 - 2\rho\rho' \cos\zeta_p}\right] \cdot H(\pi - |\beta + 2nN\pi|) \quad (\text{A.34})$$

$$I^d(\bar{\rho}, \bar{\rho}'; \beta) \approx \left[-\frac{j}{4} \sqrt{\frac{2j}{\pi k\rho'}} \exp(-jk\rho) \right] \cdot d(\beta) \frac{\exp(-jk\rho)}{\sqrt{\rho}} \quad (\text{A.35})$$

with

$$d(\beta) = -\frac{\exp\left(-j\frac{\pi}{4}\right)}{2n\sqrt{2k\pi}} \cdot \left\{ \left[\left[\cot\left(\frac{\pi+\beta}{2n}\right) F(Ka^+(\beta)) + \cot\left(\frac{\pi-\beta}{2n}\right) F(Ka^-(\beta)) \right] \right] \right\}. \quad (\text{A.36})$$

Now, let us consider the pole singularity contribution for the case $N = 0$ and $\beta^- = \phi - \phi'$. The equation $|\phi - \phi'| < \pi$, i.e., $\phi < \pi + \phi'$, describes an illuminated region for a cylindrical wave illumination on the wedge. The spatial factor

$$\frac{\exp\left[-jk\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi')}\right]}{\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi')}}$$

indicates that this pole singularity contribution yields the incident field. Substituting (A.26), (A.28), (A.30), and (A.32) into (A.1) yields

$$U^i = -\frac{j}{4} \sqrt{\frac{2j}{\pi k|\bar{\rho} - \bar{\rho}'|}} \exp[-jk|\bar{\rho} - \bar{\rho}'|].$$

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