Space-Domain Method of Moments Solution for a Strip on a Dielectric Slab

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***Abstract*— This paper presents a space-domain method of moments (MoM) solution to the problem of a strip dipole on a dielectric slab. The solution involves the use of a special junction basis function which models the nearly singular polarization currents in the vicinity of the strip/dielectric junction.**

***Index Terms*—Moment methods, printed circuit antennas.**

I. Introduction

THIS paper presents a space-domain method of moments (MoM) [1] solution to the problem of a strip dipole on a dielectric slab or substrate. There is an enormous literature on the MoM solution for printed circuit antennas [2, pt. 6], [3, pt. 4], however, virtually all is based upon a spectral-domain hybrid MoM/Green's function formulation [4]. The advantages of the spectral-domain approach are that the unknowns in the MoM solution are limited to the currents on the printed strips and the presence of the dielectric substrate is accounted for in an exact manner by including the substrate Green's function in the kernel of the integral equation. By contrast, this paper deals with a standard MoM space-domain solution which has a free-space Green's function in the kernel of the integral equation and requires unknowns on both the printed strips and in the volume of the dielectric slab.

For printed antennas on an infinite substrate, the spectral-domain solution is more efficient than the space-domain solution since it does not require unknowns in the substrate and is also more accurate since it employs the exact Green's function for the substrate. A disadvantage of the spectral-domain approach is that it is cumbersome to include the effects of a more complicated radiating environment such as the effects of the support structure on which the printed antennas are mounted. By contrast, a traditional strength of space-domain MoM solutions has been their ability to deal with complex geometries. For example, [5] is a general-purpose space-domain MoM code capable of treating almost arbitrary combinations of thin wires, polygonal plates and polygonal dielectric volumes. If such a code could treat the strip to dielectric junction problem, then it could model such problems as a printed antenna on a hand-held transmitter box in the vicinity of a human operator.

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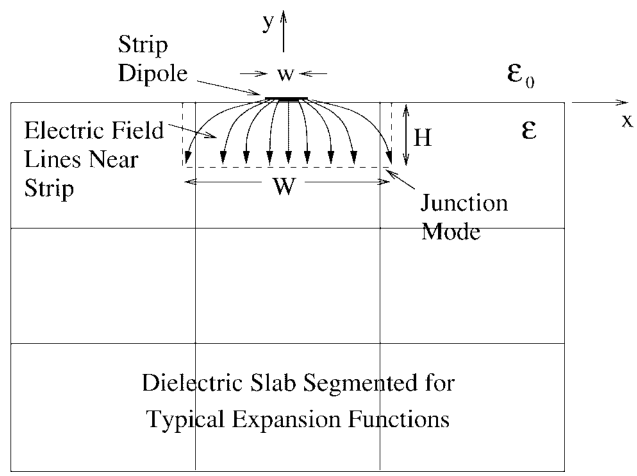

Fig. 1. The end view of a strip antenna on a dielectric slab.

II. Strip to Dielectric-Junction Mode

Fig. 1 shows the end view of a thin strip dipole on the surface of an ungrounded dielectric slab. This paper assumes that the MoM solution for the dielectric employs the volume current approach in which the dielectric slab is replaced by the volume polarization currents [6], [7]

$$\mathbf{J} = j\omega(\epsilon - \epsilon_0)\mathbf{E}^T \tag{1}$$

where $\mathbf{E}^T$ is the total electric field, ω is the radian frequency, and ϵ and ϵ_0 are the permittivities of the slab and of free-space, respectively. The dielectric slab is shown segmented into rectangular volumes for the purpose of defining MoM expansion functions for the polarization current in the dielectric volume. As illustrated in the figure, the electric field $\mathbf{E}^T$ or, equivalently, the volume polarization current $\mathbf{J}$ is nearly singular in the vicinity of the thin strip to dielectric junction. Since the usual dielectric expansion functions are typically of size $0.1 \rightarrow 0.25$ dielectric wavelengths, they are unable to model these rapidly varying currents. In principle, one could attempt to model the rapidly varying currents by greatly increasing the density of dielectric expansion functions in the vicinity of the strip. However, this would increase the number of unknowns, and also possibly lead to numerical difficulties associated with the use of electrically very small modes [8], [9]. In analogy with the attachment modes used for wire to plate junctions [10]–[12], the solution presented here is to employ special strip to dielectric junction expansion functions placed just beneath the strip in the dielectric volume.

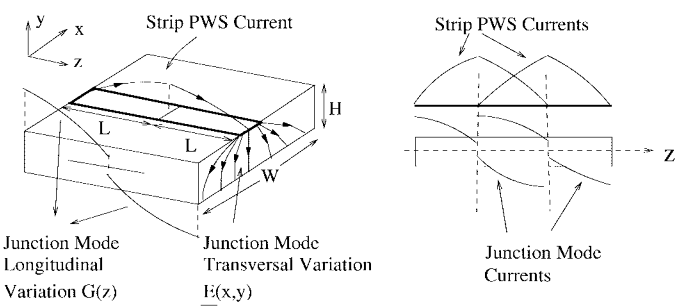

Fig. 2. A strip expansion function and its associated junction mode.

Fig. 2 shows a strip-expansion function of width ω and length $2L$, which, for simplicity, is assumed to be $\hat{\mathbf{z}}$ directed and centered about the origin. For piecewise sinusoidal expansion functions, the strip current is given by

$$\mathbf{J}^S(z) = \hat{\mathbf{z}}J^S(z) = \hat{\mathbf{z}}\frac{\sin k(L-|z|)}{\omega \sin kL}|x| \leq \omega/2$$
$$y = 0, \ |z| \leq L \tag{2}$$

where k is the free-space wavenumber. Each strip-surface patch-expansion function will have its associated junction mode in the dielectric volume just below it. The choice of the shape function for the junction mode is a compromise between accuracy and simplicity of implementation. For simplicity, we employ a junction-mode shape function, which is a separable function of the transverse coordinates (x, y) and the longitudinal coordinate z, i.e.,

$$\mathbf{J}^J(x, y, z) = \mathbf{E}(x, y)G(z) \ |x| \leq W/2, \ 0 \geq y \geq -H$$
$$|z| \leq L. \tag{3}$$

A. Transverse Variation of the Junction Mode

Since the extreme near-zone fields are dominated by the strip charges, we will take the transverse variation of the junction mode to be that of a two-dimensional (2-D) strip of charge of width w. Since we are only interested in the shape function and do not care how the fields divide between the air and slab regions, the strip charge will be assumed to be in free-space. Referring to the insert in Fig. 3, the charge fields are given by

$$E_x(x_w, y_w) = \frac{\rho_s}{4\pi\epsilon_0} \ln\left[\frac{(x_w+0.5)^2+y_w^2}{(x_w-0.5)+y_w^2}\right] \tag{4}$$

$$E_y(x_w, y_w) = \frac{\rho_s}{2\pi\epsilon_0}\left[\arctan\left(\frac{x_w+0.5}{y_w}\right) - \arctan\left(\frac{x_w-0.5}{y_w}\right)\right] \tag{5}$$

where the normalized coordinates are $x_w = x/w$ and $y_w = y/w$. Fig. 3 illustrates that the rapid transverse variation of the field is confined to a distance of a few strip widths from the strip. Further, for x_w or $y_w \gg 1$, the magnitude of the fields becomes negligible. It is thus concluded that the size of the junction mode (in the transverse directions) must be at least a few strip widths in order to model the large and rapidly varying portion of the currents. Also, there is no need to extend the size beyond several strip widths since the fields are becoming small and slowly varying and, thus, can be modeled with the usual dielectric expansion functions.

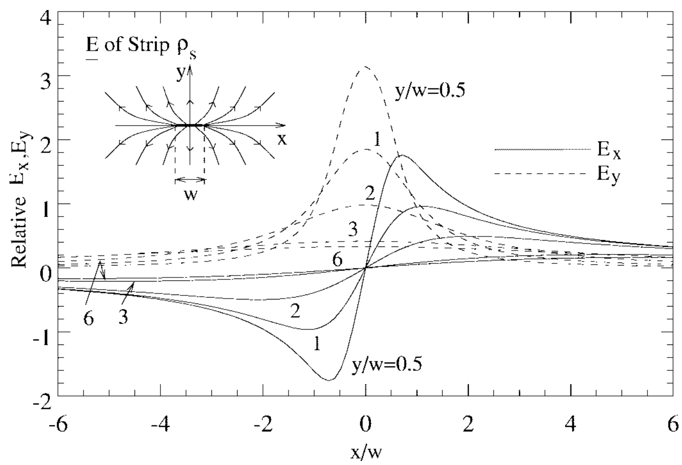

Fig. 3. The transverse variation of the junction mode is that of a 2-D strip of charge in free-space.

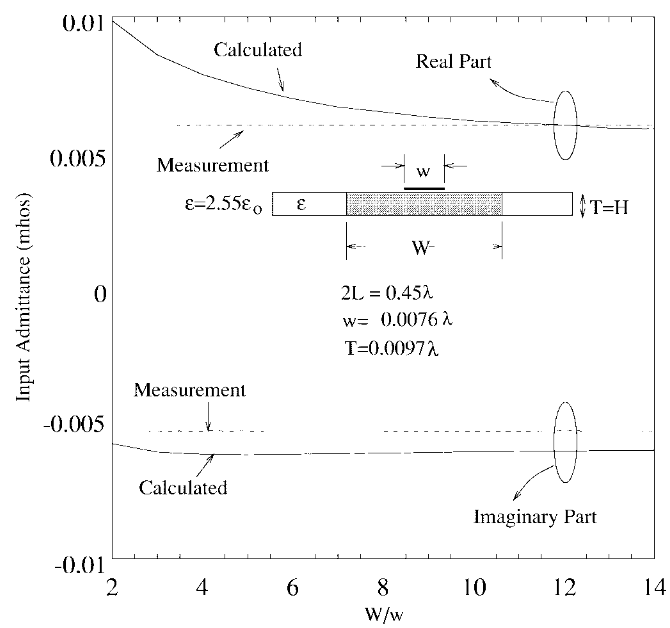

Fig. 4. The input admittance for a strip antenna on a dielectric slab versus the width of the junction mode.

B. Longitudinal Variation of the Junction Mode

To obtain the longitudinal variation of the junction mode, we observe that very close to the strip the strength of the fields is proportional to the strip surface charge density, i.e., $G(z) \propto \rho_s(z)$. Since surface current and surface charge density are related by the equation of continuity

$$\nabla \cdot \mathbf{J}^S = -j\omega\rho_s \tag{6}$$

the longitudinal variation of the junction mode is $J^{S'}(z)$ ($'$ implies d/dz).

Combining this with the transverse variation from (4) and (5), the junction mode of (3) is

$$\mathbf{J}^J(x, y, z) = [\hat{\mathbf{x}}E_x(x_w, y_w) + \hat{\mathbf{y}}E_y(x_w, y_w)]J^{S'}(z). \tag{7}$$

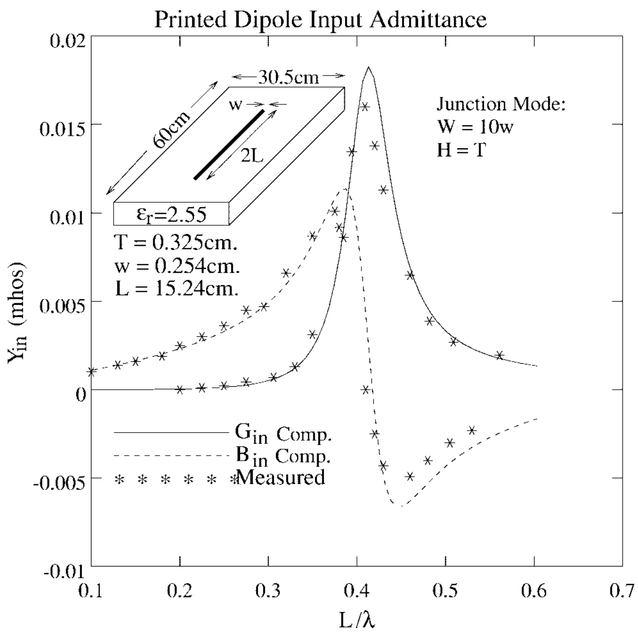

Fig. 5. The input admittance for a strip antenna on a dielectric slab.

For the piecewise sinusoidal expansion function of (2), the longitudinal variation of the junction mode is

$$G(z) = J^{S\prime}(z) = -k\,\mathrm{sgn}(z)\frac{\cos k(L-|z|)}{w \sin kL} \tag{8}$$

and, thus, as illustrated in Fig. 2, the junction-mode current is discontinuous about $z = 0$. However, since the junction mode only contains transverse directed current, this discontinuity does not produce any unphysical surface charge. Discontinuities at $|x| = W/2$ and $y = -H$ will produce unphysical surface charges, however, if $W/w \gg 1$ and $H/w \gg 1$, these should be small.

III. Numerical Results

Referring to the insert in Fig. 5, all of the numerical results in this section will be for a strip dipole of length $2L = 15.24$ cm and width $w = 0.254$ cm on a dielectric slab of thickness $T = 0.325$ cm and relative permittivity $\epsilon_r = \epsilon/\epsilon_0 = 2.55$. Fig. 4 illustrates the proper choice of the size of the junction mode. At a frequency of $f = 895$ MHz, where the strip length $2L = 0.45\ \lambda$, the solid line in Fig. 4 shows the input admittance of the strip dipole versus the width of the junction mode, W. Since the slab is electrically thin, the height of the junction mode is fixed at $H = T$. Note that as $W/w \rightarrow 0$ (i.e., the junction mode is eliminated), the input conductance diverges from the measured value [13], thus illustrating the need for a junction mode. Note also that for W greater than several strip widths, the computed and measured results are in close agreement. It is thus concluded that the transverse dimensions of the junction mode are not critical as long as they are chosen as at least several strip widths. Finally, with a junction mode of size $W/w = 10$ and $H = T$, Fig. 5 shows good agreement between the computed and measured input admittance as a function of frequency.

IV. Conclusions

This paper has demonstrated the use of a space-domain MoM junction mode to model the nearly singular fields in the vicinity of a strip/dielectric slab junction. In order to be able to treat arbitrary printed circuit antennas in a space-domain MoM code, it will also be necessary to treat the problems of a strip on a grounded slab and coupled-strip antennas.

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I. Tekin, for a photograph and biography, see p. 562 of the April 1998 issue of this Transactions.

E. H. Newman (S'67–M'74–SM'86–F'89), for a photograph and biography, see p. 1258 of the August 1997 issue of this Transactions.