Direct Singular Integral Equation Methods in Scattering and Propagation in Strip- or Slot-Loaded Structures

John L. Tsalamengas, *Member, IEEE*

***Abstract*— Problems of three-dimensional (3-D) scattering/hybrid-wave propagation for strip- or slot-loaded structures are often formulated in terms of systems of singular integral-integrodifferential equations (SIE-SIDE) of the first kind. Proper handling of the singular part of the kernels constitutes a major difficulty in carrying out method of moments (MoM). Three powerful techniques explored in the present paper provide efficient solutions by direct recourse to the theory of singular integral equations. In contrast to low-frequency methods wherein similar concepts are utilized for electrically narrow strips/slots, the proposed procedures are applicable uniformly to the whole range of widths from very narrow to very wide scatterers with remarkable accuracy. Numerical results are presented to validate and compare to one another the various numerical codes.**

***Index Terms*— Electromagnetic propagation, electromagnetic scattering.**

I. Introduction

METHOD of moments (MoM)-oriented direct singular integral equation techniques (DSIET) are useful in problems of three-dimensional (3-D) diffraction/hybrid-wave propagation for two-dimensional structures loaded by infinite strips or slots. With their help, the major problem of computation of singular or slowly convergent integrals inherent in the implementation of conventional MoM is faced head on. In addition, the edge conditions are automatically incorporated. As a result, filling the matrix elements—via efficient analytical expressions—becomes most expeditious.

DSIET [1], [2] are only one group of methods out of many for solving singular integral-integrodifferential equations (SIE-SIDE). In this context, exact and approximate Wiener–Hopf techniques are limited to a few canonical geometries [3]. Reduction or regularization (indirect) methods [4], [5] (in general, rather complex and multistage closely connected with the classical "Riemann–Hilbert–Carleman" problem) may be used to obtain equivalent second-kind regular Fredholm equations [6], [7]. Dual-series (or integral)-equation methods have also been very successfully used [8].

In this paper, we investigate two systems, each consisting of two coupled SIE commonly encountered in problems of hybrid-wave propagation/3-D diffraction for layered media loaded by strips or slots. The solution to these systems is obtained by three versions of the DSIET. The first version (Section III) is based on [9]. The second version (Section IV) is an extension of the method used in [7] and [10], whereas the third one (Section V) uses interpolation polynomials in the way described in [11] and [12]. (Variants of the last two methods have been also used in [13]–[16]). The validity, accuracy, and efficiency of these DSIET are amply demonstrated in Section VI by detailed numerical results and comparisons. Their use is exemplified in Section VII in connection with the practical problem of 3-D scattering by a strip right on the interface between two dissimilar media. Further applications to more complex configurations will be presented in forthcoming papers.

II. Statement of the Problem

Consider the following two systems of 2×2 SIE-SIDE of the first kind

$$A_1 \int_{-w}^{w} f_1(x) G_q(x-s)\,dx + A_2 \frac{d}{ds} \int_{w}^{w} f_2(x) G_q(x-s)\,dx = g_1(s) \tag{1a}$$

$$A_3 \frac{d}{ds} \int_{-w}^{w} f_1(x) G_q(x-s)\,dx + A_4 \left(k_c^2 + \frac{d^2}{ds^2}\right) \int_{w}^{w} f_2(x) G_q(x-s)\,dx = g_2(s) \tag{1b}$$

($|s| \le w$), designated by the subscript (or superscript) q ($q = 1, 2$). Here, A_j ($j = 1, 2, 3, 4$) are multiplicative constants, g_1 and g_2 are known continuous functions, f_1 and f_2 are unknowns, and

$$G_1(x-s) = G_1(x-s;k) \equiv H_0^{(2)}(k|x-s|) \tag{2a}$$

$$G_2(x-s) = G_2(x-s;k_0,k_1) \equiv \frac{1}{k_0^2-k_1^2} \sum_{n=0}^{1} (-1)^n \cdot k_n^2 [H_0^{(2)}(k_n|x-s|) + H_2^{(2)}(k_n|x-s|)] \tag{2b}$$

where the Hankel functions $H_m^{(2)}(\cdot)$ are of the second kind. In actual problems, the scalar constants k_0, k_1, k, k_c would denote suitably selected wavenumbers.

Manuscript received June 14, 1994; revised March 15, 1998.

The author is with the Department of Electrical and Computer Engineering, National Technical University of Athens, Zografou, GR-157 73 Athens, Greece.

Publisher Item Identifier S 0018-926X(98)05786-X.

The kernels $G_q(x-s)$ $(q=1,2)$ possess logarithmic singularities. Change in the order of integration and single differentiation (d/ds) is permissible, leading to Cauchy-type singular integrals. In (1b), however, the operator d^2/ds^2 cannot be moved behind the integral sign since the resulting integral would then diverge; that is, the SIDE (1b) cannot be reduced to a SIE.

Suppose that the unknowns $f_1(x)$ and $f_2(x)$ possess endpoint singularities of the form $[1-(x/w)^2]^{\mp 1/2}$, respectively, dictated by physical constraints (edge conditions). It is then natural to expand the unknown functions in terms of Chebyshev polynomials:

$$f_1(x) = w_1(t)\sum_{N=0}^{\infty} a_N T_N(t)$$
$$f_2(x) = w_2(t)\sum_{N=0}^{\infty} b_N U_N(t); \qquad t = x/w \tag{3}$$

where a_N and b_N are expansion constants and $w_1(t) = 1/w_2(t) = (1-t^2)^{-1/2}$.

III. The First DSIET

We change variables $x = wt, s = w\tau$ $(-1 \le t, \tau < 1)$, insert (3) in (1), multiply both sides of (1a) [of (1b)] by $w_1(\tau)T_M(\tau)$ [by $w_2(\tau)U_M(\tau)$], and integrate from $\tau=-1$ to $\tau=1$. We thus obtain an infinite linear algebraic system of the form

$$wA_1\sum_{N=0}^{\infty} a_N A_{MN}^{(q)} + A_2\sum_{N=0}^{\infty} b_N B_{MN}^{(q)} = c_M^{(1)}$$
$$A_3\sum_{N=0}^{\infty} a_N C_{MN}^{(q)} + \frac{1}{w}A_4\sum_{N=0}^{\infty} b_N D_{MN}^{(q)} = c_M^{(2)} \tag{4}$$

$M = 0,1,2,\cdots; q = 1$ or $q=2$.

The present method for evaluating the matrix elements, based on Neumann-type expansions of the kernels and on the analytical evaluation of all singular integrals encountered, is given in detail in [9]. The final results take the following concise form:

$$A_{MN}^{(1)}(kw) = \sum_{q=0}^{\infty}\varepsilon_q[I(M,q)\Xi_I(N,q) - j2\pi^{-2} \cdot \sum_{p=0}^{\infty}\varepsilon_p I(p,q)\Lambda_a(p,q,M,N)] \tag{5}$$

$$A_{MN}^{(2)}(k_0w,k_1w) = [k_0^2\Phi_{MN}(k_0w) - k_1^2\Phi_{MN}(k_1w)]/(k_0^2-k_1^2) \tag{6}$$

$$\Phi_{MN}(\kappa w) = \sum_{q=0}^{\infty}\varepsilon_q\left[-j\pi^{-2}\sum_{p=0}^{\infty}\varepsilon_p I(p,q)\sum_{n=-2,0,2}(1+\delta_{n0}) \cdot \Lambda_a(p,q+n,M,N) + I(N,q)S_M\{\lambda_q\}\right] \quad (\kappa = k_0,k_1) \tag{7}$$

$$C_{MN}^{(1)}(kw) = \sum_{q=0}^{\infty}\varepsilon_q\left[kw\tilde{J}(M,q)\Xi_I(N,q) + (j/\pi) \cdot \sum_{p=0}^{\infty}\varepsilon_p I(p,q)\Theta_a(p,q,0,M,N)\right] \tag{8}$$

$$C_{MN}^{(2)}(k_0w,k_1w) = [k_0^2\Gamma_{MN}(k_0w) - k_1^2\Gamma_{MN}(k_1w)]/(k_0^2-k_1^2) \tag{9}$$

$$\Gamma_{MN}(\kappa w) = \sum_{q=0}^{\infty}\varepsilon_q\left[\frac{j}{2\pi}\sum_{p=0}^{\infty}\varepsilon_p I(p,q)\sum_{n=-2,0,2}(1+\delta_{n0}) \cdot \Theta_a(p,q,n,M,N) + \kappa w I(N,q)S_M\{\tilde{h}_q\}\right] \quad (\kappa = k_0,k_1) \tag{10}$$

$$D_{MN}^{(1)}(k_c^2,kw) = \sum_{q=0}^{\infty}\varepsilon_q\left[\Xi_J(N,q)H(M,q) - 4j\pi^{-2} \cdot \sum_{p=0}^{\infty}\Theta_b(p,q,0,M,N)\right] \tag{11}$$

$$D_{MN}^{(2)}(k_c^2,k_0w,k_1w) = [k_0^2P_{MN}(k_c^2,k_0w) - k_1^2P_{MN}(k_c^2,k_1w)]/(k_0^2-k_1^2) \tag{12}$$

$$P_{MN}(k_c^2,\kappa w) = \sum_{q=0}^{\infty}\varepsilon_q\left[-4j\pi^{-2}\sum_{p=0}^{\infty}\sum_{n=-2,0,2}\frac{1}{\varepsilon_n}\Theta_b(p,q,n,M,N) + J(N,q)S_M\{V_q\}\right] \quad (\kappa = k_0,k_1) \tag{13}$$

$$B_{MN}^{(q)} = -C_{NM}^{(q)} \qquad (q=1,2). \tag{14}$$

Here, $\delta_{nm} = 0$ if $n \ne m, \delta_{nn} = 1$ (Kronecker delta) and $\varepsilon_n = 2 - \delta_{n0}$. All other symbols involved are defined in Appendix A.

Remarks:

1) As a consequence of the Neumann-type expansions used for the kernels, results (5)–(14) are applicable uniformly to the whole range of kw, k_0w and k_1w. In contrast, use of power series expansions for the kernels is limited due to well known severe roundoff errors to the lower regime of these parameters [9].
2) In addition to (14), we note the symmetry relations $A_{MN}^{(q)} = A_{NM}^{(q)}, D_{MN}^{(q)} = D_{NM}^{(q)}$. Furthermore, $A_{MN}^{(q)} = 0 = D_{MN}^{(q)}$ when $M+N$ is odd and $B_{MN}^{(q)} = 0 = C_{MN}^{(q)}$ when $M+N$ is even. These relations reduce considerably the number of matrix elements that need separate computation.
3) As seen, all matrix elements involve series of the form $S = \Sigma_{m=0}^{\infty}\ \Sigma_{n=0}^{\infty}\ d_{mn}J_{(m+n)/2}\ (\kappa w/2)\ J_{(m-n)/2}$ $(\kappa w/2)$ $(\kappa \equiv k, k_0, k_1)$ solely, where d_{mn} is of order 1

or smaller. Thus, in view of the asymptotic expression $J_\nu(x) \underset{\nu\to\pm\infty}{\longrightarrow} [ex/(2\nu)]^\nu/\sqrt{2\pi\nu}$, the terms of S diminish very rapidly as the summation indexes increase, the large ones being those with small values of m, n. Furthermore, in generating the matrix elements the only functions to be evaluated are the Bessel functions $J_m(\kappa w/2)$ $(m = 0, 1, 2, \cdots)$ of prescribed argument. Finally, problems due to roundoff errors are completely avoided as explained in detail in [9]. The efficiency of the present DSIET is further discussed in Sections VI–VII.

IV. The Second DSIET

In this section, we describe an alternative method to evaluate the matrix elements in (4).

Let $t = x/w, \tau = s/w$ $(-1 \le t, \tau \le 1)$. We define the analytic functions

$$P_q(x-s) = j\frac{\pi}{2}G_q(x-s) - \ln|\tau - t| \qquad (q = 1, 2) \tag{15a}$$

$$\begin{aligned} 2P_1(0) &= j\pi\Gamma_0(kw) \\ 2P_2(0) &= j\pi[k_0^2\Gamma_1(k_0w) - k_1^2\Gamma_1(k_1w)]/(k_0^2 - k_1^2) + 1 \end{aligned} \tag{15b}$$

with Γ_0 and Γ_1 defined in the Appendix (91) and introduce the simplifying notation

$$\mathcal{L}_j(P_q) = \int_{-1}^{1} f_j[x(t)]P_q[x(t) - s(\tau)]\,dt \qquad (j = 1, 2). \tag{16}$$

Then, with the help of the results [9]

$$\begin{aligned} \xi_N(\tau) &\equiv \int_{-1}^{1} w_1(t)T_N(t)\ln|\tau - t|\,dt \\ &= -\pi\delta_{N0}\ln 2 - \frac{\pi}{N}T_N(\tau) \end{aligned} \tag{17}$$

$$\begin{aligned} \zeta_N(\tau) &\equiv \int_{-1}^{1} w_2(t)U_N(t)\ln|\tau - t|\,dt \\ &= \frac{1}{2}[\xi_N(\tau) + \pi T_{N+2}(\tau)/(N+2)] \end{aligned} \tag{18}$$

$$\begin{aligned} \zeta'_N(\tau) &= \int_{-1}^{1} w_2(t)U_N(t)\frac{dt}{\tau - t} = \pi T_{N+1}(\tau) \\ \xi'_N(\tau) &= -\pi U_{N-1}(\tau) \qquad (N > 0), 0 \quad (N = 0) \end{aligned} \tag{19}$$

(1a) becomes

$$\begin{aligned} A_1w&\left[\mathcal{L}_1(P_q) + \sum_{N=0}^{\infty} a_N\xi_N(\tau)\right] \\ &+ A_2\left[\mathcal{L}_2\left(\frac{d}{d\tau}P_q\right) + \pi\sum_{N=0}^{\infty} b_NT_{N+1}(\tau)\right] \\ &= j\frac{\pi}{2}g_1(w\tau). \end{aligned} \tag{20}$$

The primes in (19) denote differentiation with respect to τ. In (17), π/N is set by convention equal to zero for $N = 0$.

Equation (1b) can be treated in the same way. In connection with the second integral in (1b), care is needed, however, in carrying out the second derivative d^2/ds^2. In order to avoid high-order singularities, the proper decomposition of $G_p(x - s)$—suitable for handling the term $(d^2/d\tau^2)\mathcal{L}_2(G_q)$—has the form

$$\begin{aligned} j\frac{\pi}{2}&G_q[x(t) - s(\tau)] \\ &= (1 - (\tau - t)^2C_q)\ln|\tau - t| + (P_q[x(t) - s(\tau)] \\ &\quad + (\tau - t)^2C_q\ln|\tau - t|) \end{aligned} \tag{21}$$

where

$$C_q = \frac{(kw)^2}{4}(q = 1), \frac{1}{8}[(k_0w)^2 + (k_1w)^2](q = 2). \tag{22}$$

Using (21), we get

$$\frac{d^2}{d\tau^2}\int_{-1}^{1} f_2[x(t)]G_q[x(t) - s(\tau)]\,dt = -j\frac{2}{\pi}[I(\tau) + \mathcal{L}_2(L_q)] \tag{23}$$

where

$$I(\tau) = \frac{d^2}{d\tau^2}\int_{-1}^{1} f_2[x(t)](1 - (\tau - t)^2C_q)\ln|\tau - t|\,dt \tag{24}$$

whereas $L_q(x - s)$ $(q = 1, 2)$ are new analytic functions defined by

$$\begin{aligned} L_q(x - s) &= L_q[x(t) - s(\tau)] \\ &= j\frac{\pi}{2}\frac{d^2}{d\tau^2}G_q[x(t) - s(\tau)] + \frac{1}{(\tau - t)^2} \\ &\quad + (3 + 2\ln|\tau - t|)C_q \end{aligned} \tag{25}$$

$$\begin{aligned} L_1(0) &= -j\pi C_1\Gamma_1(kw) \\ 2L_2(0) &= -C_2 + j\frac{\pi}{2}\frac{w^2}{k_0^2 - k_1^2}\sum_{n=0}^{1}(-1)^n \\ &\quad \cdot k_n^4\left[-\Gamma_1(k_nw) + \frac{1}{2}\Gamma_2(k_nw)\right]. \end{aligned} \tag{26}$$

$I(\tau)$ may be evaluated as follows:

$$\begin{aligned} I(\tau) &= \frac{d}{d\tau}\int_{-1}^{1} f_2[x(t)](-2(\tau - t)C_q\ln|\tau - t| \\ &\quad + [1 - (\tau - t)^2C_q]/(\tau - t))\,dt \\ &= -2C_q\int_{-1}^{1} f_2[x(t)]\ln|\tau - t|\,dt \\ &\quad - 3C_q\int_{-1}^{1} f_2[x(t)]\,dt + \frac{d}{d\tau}\int_{-1}^{1} f_2[x(t)]\frac{dt}{\tau - t} \\ &= \sum_{N=0}^{\infty} b_N\left\{-C_q\left[\frac{3}{2}\pi\delta_{N0} + 2\zeta_N(\tau)\right]\right. \\ &\qquad \left. + \pi(N+1)U_N(\tau)\right\}. \end{aligned} \tag{27}$$

Equation (1b) then becomes

$$A_3\left[\mathcal{L}_1\left(\frac{d}{d\tau}P_q\right)+\sum_{N=0}^{\infty}a_N\xi'(\tau)\right]+\frac{1}{w}\cdot A_4\Bigg\{[(k_cw)^2\mathcal{L}_1(P_q)+\mathcal{L}_1(L_q)]+\sum_{N=0}^{\infty}\cdot b_N\Bigg[(k_cw)^2\zeta_N(\tau)+\Bigg\{-C_q\left[\frac{3}{2}\pi\delta_{N0}+2\zeta_N(\tau)\right]+\ \pi(N+1)U_N(\tau)\Bigg\}\Bigg]\Bigg\}=j\frac{\pi}{2}g_2(w\tau). \tag{28}$$

Multiplying both sides of (20) and (28) by $w_1(\tau)T_M(\tau)$ and by $w_2(\tau)U_M(\tau)$, respectively, and integrating from $\tau=-1$ to $\tau=1$ one obtains again the linear system (4). In carrying out the integrations in the last step, the contributions to the matrix elements due to $\mathcal{L}_j(P_q),\mathcal{L}_j(L_q)$ and similar terms in (20) and (28) are here obtained by numerical integration using the quasi-analytical Lobatto's integration formulas [18]. For instance

$$\int_{-1}^{1}w_1(t)T_N(t)P_q[x(t)-s(\tau)]\,dt=\frac{\pi}{L}\sum_{n=1}^{L}T_N(t_n)P_q[x(t_n)-s(\tau)] \tag{29}$$

$$\int_{-1}^{1}w_2(t)U_N(t)P_q[x(t)-s(\tau)]=\frac{\pi}{L+1}\sum_{n=1}^{L}(1-\hat{t}_n^2)U_N(\hat{t}_n)P_q[x(\hat{t}_n)-s(\tau)]dt \tag{30}$$

$$t_n=\cos\vartheta_n,\quad \vartheta_n=\frac{2n-1}{2L}\pi;\quad \hat{t}_n=\cos\hat{\vartheta}_n$$
$$\hat{\vartheta}_n=\frac{n\pi}{L+1}. \tag{31}$$

L in series (29)–(30) is any fixed integer selected as high as needed to ensure convergence.

The final expressions of the matrix elements take the form

$$A_{MN}^{(q)}=2j\pi\Bigg\{\delta_{N0}\delta_{M0}\ln 2+\frac{\delta_{MN}}{2N}-\frac{1}{L^2}\sum_{n=1}^{L}T_N(t_n)\cdot\sum_{m=1}^{L}T_M(t_m)P_q[x(t_n)-s(t_m)]\Bigg\} \tag{32}$$

($\delta_{MN}/(2N)$ is set by convention equal to zero for $N=0$)

$$C_{MN}^{(q)}=-2j\pi\Bigg\{-\frac{1}{2}\delta_{M(N-1)}+\frac{1}{L(L+1)}\sum_{n=1}^{L}T_N(t_n)\cdot\sum_{m=1}^{L}(1-\hat{t}_m^2)U_M(\hat{t}_m)\frac{d}{d\tau}P_q[x(t_n)-s(\tau)]_{\tau=\hat{t}_m}\Bigg\}=-B_{NM}^{(q)} \tag{33}$$

$$D_{MN}^{(q)}=-j\frac{\pi}{2}\Bigg\{-[(k_cw)^2\ln 2+C_q(3-2\ln 2)]\delta_{M0}\delta_{N0}+2(N+1)\delta_{MN}+\frac{1}{2}[(k_cw)^2-2C_q]\cdot\left[-\frac{1}{N}s_{M-N+1}\delta_{(|M-N+1|-1)0}+\frac{1}{N+2}s_{M-N-1}\cdot\delta_{(|M-N-1|-1)0}\right]+\left(\frac{2}{L+1}\right)^2\sum_{n=1}^{L}(1-\hat{t}_n^2)U_N(\hat{t}_n)\cdot\sum_{m=1}^{L}(1-\hat{t}_m^2)U_M(\hat{t}_m)[(k_cw)^2P_q[x(\hat{t}_n)-s(\hat{t}_m)]+L_q[(x\hat{t}_n)-s(\hat{t}_m)]]\Bigg\},\qquad s_m=sgn(m). \tag{34}$$

In (33),

$$\frac{d}{d\tau}P_q[x(t)-s(\tau)]=\begin{cases} j\frac{\pi}{2}\frac{d}{d\tau}G_q[x(t)-s(\tau)]-\frac{1}{\tau-t} & (t\neq\tau)\\ 0 & (t=\tau).\end{cases} \tag{35}$$

Note: The second DSIET, although very similar in spirit with the first DSIET discussed in the preceding section, has quite different performance characteristics. Let $\nu=\{(L+1)/2\}[\{L/2\}+1]$, with $\{m/n\}$ denoting the integer part of m/n, then L_q in (34) as well as P_q in each of (32) and (34) have to be evaluated ν times, taking into account the symmetry relations $F_{MN}^{(q)}=0$ when $M+N$ is odd; $F_{MN}^{(q)}=F_{NM}^{(q)}$ $(F\equiv A,D)$. Finally, $dP_q/d\tau$ in (33) needs $L(L+1)-2\nu$ separate computations. This, in conjunction with the comparatively large values of L needed to achieve a prescribed high accuracy, renders the present approach very time consuming as will be made clear in Section VI by specific examples. So, in spite of its simplicity, the second DSIET has inferior performance characteristics in comparison with the first one.

V. The Third DSIET

The method to be explored in this section, based on rather different principles, uses the following expansions:

$$f_1(x)\equiv w_1(t)F_1(t)=w_1(t)\sum_{N=0}^{L-1}a_NT_N(t)$$
$$f_2(x)\equiv w_2(t)F_2(t)=w_2(t)\sum_{N=0}^{L-1}b_NU_N(t) \tag{36}$$

$t=x/w$. Here, unlike (3), L is finite, i.e., only finite series are used from the beginning.

Following step by step the procedure outlined in the preceding section, we get equations identical with (20) and (28) in which all infinite series $\Sigma_{N=0}^{\infty}$ are replaced by corresponding finite sums $\Sigma_{N=0}^{L-1}$. From now on we depart and attempt a discretization of (20) and (28) in the form of a finite, linear algebraic system of equations for unknowns the values $F_1(t_n)$

and $F_2(\hat{t}_n)$ $(n = 1, 2, \cdots, L)$; the discrete points t_n and $\hat{t}_n$ (zeros of $T_L(t)$ and $U_L(t)$, respectively) are given by (31). To this end we first re-express a_N and b_N using the summation formulas [12]

$$a_N = \frac{\varepsilon_N}{L} \sum_{n=1}^{L} T_N(t_n) F_1(t_n)$$
$$b_N = \frac{2}{L+1} \sum_{n=1}^{L} (1 - \hat{t}_n^2) U_N(\hat{t}_n) F_2(\hat{t}_n) \tag{37}$$

(derived from (36) using the orthogonality of Chebyshev polynomials in conjunction with Lobatto's integration formulas). Next, using Lobatto's integration formulas, we obtain

$$\int_{-1}^{1} f_1[x(t)] R_q[x(t) - s(\tau)]\, dt = \frac{\pi}{L} \sum_{n=1}^{L} F_1(t_n) R_q[x(t_n) - s(\tau)] \tag{38}$$

$$\int_{-1}^{1} f_2[x(t)] R_q[x(t) - s(\tau)]\, dt = \frac{\pi}{L+1} \sum_{n=1}^{L} (1 - \hat{t}_n^2) F_2(\hat{t}_n) R_q[x(\hat{t}_n) - s(\tau)] \tag{39}$$

where $R_q(x - s)$ may represent either of $P_q(x - s)$, $(d/d\tau)P_q[x(t) - s(\tau)]$, or $L_q(x - s)$. Insert (37)–(39) in both (20) and (28). Set successively, $\tau = t_1, t_2, \cdots, t_L$ in (20) and $\tau = \hat{t}_1, \hat{t}_2, \cdots \hat{t}_L$ in (28). Then, after some term rearrangement, we obtain

$$wA_1 \sum_{n=1}^{L} F_1(t_n) \tilde{A}_{mn}^{(q)} + A_2 \sum_{n=1}^{L} F_2(\hat{t}_n) \tilde{B}_{mn}^{(q)} = j\frac{\pi}{2} g_1(wt_m) \tag{40}$$

$$A_3 \sum_{n=1}^{L} F_1(t_n) \tilde{C}_{mn}^{(q)} + \frac{1}{w} A_4 \sum_{n=1}^{L} F_2(\hat{t}_n) \tilde{D}_{mn}^{(q)} = j\frac{\pi}{2} g_2(w\hat{t}_m) \tag{41}$$

$(m = 1, 2, \cdots, L; q = 1$ or $q = 2)$. The matrix elements assume several forms as described below.

Expressions of $\tilde{A}_{mn}^{(1)}(kw), \tilde{A}_{mn}^{(2)}(k_0 w, k_1 w)$

$$\begin{aligned}\tilde{A}_{mn}^{(q)} = \tilde{A}_{nm}^{(q)} &= \frac{\pi}{L}\Bigg[-\ln 2 - 2\sum_{N=1}^{L-1} \frac{1}{N} T_N(t_m) T_N(t_n) + P_q(|wt_m - wt_n|)\Bigg] \\ &= \frac{\pi}{L}\Bigg[-\ln 2 - \mathcal{A}(|m-n|) - \mathcal{A}(|m+n-1|) + P_q(|wt_m - wt_n|)\Bigg]; \qquad q = 1, 2\end{aligned} \tag{42}$$

where in the last step we introduced the quantity

$$\mathcal{A}(\ell) = \sum_{N=1}^{L-1} \frac{1}{N} \cos\left(\frac{N}{L}\pi\ell\right). \tag{43}$$

Since P_q is given in closed form, filling the matrix elements via (42) requires simply the evaluation of $\mathcal{A}(\ell)$ for $\ell = 0, 1, \cdots, 2L - 1$.

Expressions of $\tilde{B}_{mn}^{(q)}(kw), \tilde{C}_{mn}^{(q)}(kw)$

$$\tilde{B}_{mn}^{(q)} = \frac{\pi}{L+1}(1 - \hat{t}_n^2)\Bigg[2\sum_{N=0}^{L-1} U_N(\hat{t}_n) T_{N+1}(t_m) + \frac{d}{d\tau} P_q[x(\hat{t}_n) - s(\tau)]_{\tau = t_m}\Bigg] \tag{44}$$

$$\tilde{C}_{mn}^{(q)} = \frac{\pi}{L}\Bigg[-2\sum_{N=1}^{L-1} T_N(t_n) U_{N-1}(\hat{t}_m) + \frac{d}{d\tau} P_q[x(t_n) - s(\tau)]|_{\tau = \hat{t}_m}\Bigg]; \qquad q = 1, 2. \tag{45}$$

Using the relation [19]

$$D_L(x) \equiv \sum_{N=0}^{L-1} \sin[(N+1)x] = \sin\left(\frac{L+1}{2}x\right) \sin\left(\frac{Lx}{2}\right) \operatorname{cosec}\left(\frac{x}{2}\right) \tag{46}$$

we obtain the closed-form expressions

$$\tilde{B}_{mn}^{(q)} = \frac{\pi}{L+1} \sin\hat{\vartheta}_n \Bigg[\sin\hat{\vartheta}_n \frac{d}{d\tau} P_q[x(\hat{t}_n) - s(\tau)]|_{\tau = t_m} + D_L(\hat{\vartheta}_n - \vartheta_m) + D_L(\hat{\vartheta}_n + \vartheta_m)\Bigg] \tag{47}$$

$$\tilde{C}_{mn}^{(q)} = \frac{\pi}{L}\Bigg[\frac{d}{d\tau} P_q[x(t_n) - s(\tau)]|_{\tau = \hat{t}_m} - \frac{1}{\sin\hat{\vartheta}_m} \cdot (D_{L-1}(\hat{\vartheta}_m - \vartheta_n) + D_{L-1}(\hat{\vartheta}_m + \vartheta_n))\Bigg]; \quad q = 1, 2. \tag{48}$$

Expressions of $\tilde{D}_{mn}^{(1)}(k_c^2, kw), \tilde{D}_{mn}^{(2)}(k_c^2, k_0 w, k_1 w)$

$$\begin{aligned}\tilde{D}_{mn}^{(q)} = \frac{\pi}{L+1}(1 - \hat{t}_n^2)\Bigg\{&(k_c w)^2 P_q[x(\hat{t}_n) - s(\hat{t}_m)] + L_q[x(\hat{t}_n) - s(\hat{t}_m)] + [(k_c w)^2 - 2C_q] \\ &\cdot \left[-\ln 2 + \frac{1}{2} T_2(\hat{t}_m)\right] - 3C_q + [(k_c w)^2 - 2C_q] \\ &\cdot \sum_{N=1}^{L-1} U_N(\hat{t}_n)\left[-\frac{T_N(\hat{t}_m)}{N} + \frac{T_{N+2}(\hat{t}_m)}{N+2}\right] \\ &+ 2\sum_{N=0}^{L-1} (N+1) U_N(\hat{t}_m) U_N(\hat{t}_n)\Bigg\}; \quad q = 1, 2.\end{aligned} \tag{49}$$

Let us define

$$\mathcal{A}^{(m)}(\ell) = \frac{1}{2} \sum_{N=1}^{L-1} \frac{1}{N+m} e^{jN\pi|\ell|/(L+1)} = C^{(m)}(\ell) + jS^{(m)}(\ell) \qquad (m = 0, 2). \tag{50}$$

TABLE I
VALUES OF THE LEADING MATRIX ELEMENT $A_{00}^{(1)}$ AS OBTAINED BY THE FIRST DSIET [(5)] AND BY THE SECOND DSIET [(32)] FOR $kw = 0.5\pi$

First DSIET

ε	n_{max}	q_{max}	p_{max}	$A_{00}^{(1)}$	CPU
10^{-5}	5	8	6	5.5356851780217+j1.5768834234042	0.0
10^{-10}	8	14	9	5.5356851780386+j1.5768818926162	0.0
10^{-15}	10	18	11	5.5356851780386+j1.5768818926202	0.0
10^{-20}	10	20	11	5.5356851780386+j1.5768818926202	0.0

Second DSIET

L	$A_{00}^{(1)}$	CPU
20	5.5356851780386+j1.5770276716025	0.1
30	5.5356851780386+j1.5769250554212	0.3
50	5.5356851780386+j1.5768912123676	0.9
100	5.5356851780386+j1.5768830574086	3.6
200	5.5356851780386+j1.5768820382131	14
500	5.5356851780386+j1.5768819019380	90
1000	5.5356851780386+j1.5768818937849	343
2000	5.5356851780386+j1.5768818927657	1378

Then (49) recasts into the following computationally most convenient form

$$\begin{aligned}\tilde{D}_{mn}^{(q)} = \frac{\pi}{L+1}\sin\hat{\vartheta}_n\Bigg\{&\sin\hat{\vartheta}_n\Big[(k_c w)^2 P_q[x(\hat{t}_n) - s(\hat{t}_m)]\\ &+ L_q[x(\hat{t}_n) - s(\hat{t}_m)] + [(k_c w)^2 - 2C_q]\\ &\cdot\left[-\ln 2 + \frac{1}{2}\cos(2\hat{\vartheta}_m)\right] - 3C_q\Big]\\ &+ [(k_c w)^2 - 2C_q]\cdot[\mathrm{sgn}\,(n-m)(-\cos\hat{\vartheta}_n\\ &\cdot S^{(0)}(|m-n|) + \cos(\hat{\vartheta}_n - 2\hat{\vartheta}_m)S^{(2)}(|m-n|))\\ &- \sin\hat{\vartheta}_n C^{(0)}(|m-n|) + \sin(\hat{\vartheta}_n - 2\hat{\vartheta}_m)\\ &\cdot C^{(2)}(|m-n|) - \cos\hat{\vartheta}_n S^{(0)}(m+n)\\ &+ \cos(\hat{\vartheta}_n + 2\hat{\vartheta}_m)S^{(2)}(m+n) - \sin\hat{\vartheta}_n\\ &\cdot C^{(0)}(m+n) + \sin(\hat{\vartheta}_n + 2\hat{\vartheta}_m)C^{(2)}(m+n)]\\ &+ \left[\Gamma\left(\frac{\pi}{L+1}|m-n|\right) - \Gamma\left(\frac{\pi}{L+1}|m+n|\right)\right]\Big/\\ &\sin\hat{\vartheta}_m\Bigg\}. \qquad (51)\end{aligned}$$

Here [19]

$$\begin{aligned}\Gamma(x) &\equiv \sum_{N=0}^{L-1}(N+1)\cos[(N+1)x]\\ &= L\cos(Lx) + \frac{L\sin\left(\frac{2L-1}{2}x\right)}{2\sin(x/2)} - \frac{1-\cos(Lx)}{4\sin^2(x/2)} \qquad (52)\end{aligned}$$

with $\Gamma(0) = L(L+1)/2$. Filling the matrix elements $\tilde{D}_{mn}^{(q)}$ via (51) requires simply the evaluation of $\mathcal{A}^{(0)}(\ell)$ and $\mathcal{A}^{(2)}(\ell)$ for $\ell = 0, 1, \cdots, 2L$.

The simplicity of the third DSIET is striking. The matrix elements assume either closed-form expressions $(\tilde{B}_{mn}^{(q)}, \tilde{C}_{mn}^{(q)})$ or expressions solely involving single finite series. The convergence of this algorithm is discussed just below.

VI. Validation of the Algorithms and Comparisons

The validity of the algorithms developed will be demonstrated by comparing to one another their corresponding results.

In order to show the relative advantages of the first and of the second DSIET, in Table I we present the values of the leading matrix element $A_{00}^{(1)}$ for $kw = \pi/2$ as obtained by these two methods along with the elapsed CPU times in seconds on a Silicon Graphics workstation. In the case of the first DSIET and for a given absolute accuracy ε we also show the required truncation sizes $p_{\max}$, $q_{\max}$, and $n_{\max}$ for the series over p, q, and n in each of (5) and the Appendix (90).

Inspection of this table reveals that the first DSIET reaches the final values of the matrix elements very quickly and in a remarkably stable manner. In contrast, the second DSIET approaches these same values asymptotically. The needed large values of the parameter L in each of (32)–(34) lead to excessive central processing unit (CPU) times, which render the second DSIET impractical, especially when a high degree of accuracy is required.

The behavior of all other matrix elements was found to be quite similar.

From now on we are restricted to the comparison between the first and the third DSIET. This is carried out in Table II, where $|a_n|$ $(n = 0{-}8)$ are shown as obtained; (a) from (4) for the case $q = 1, A_1 = 1, A_2 = 0 = A_3 = A_4$ and for several truncation sizes N_r [N_r is the number of basis functions used in each of (3)] and (b) from $\{(37), (40)\}$ for several values of L. (In this example, $a_n = 0$ for $n =$ odd since $g_1(x)$ was chosen to be even).

One clearly observes from Table II that results obtained on the basis of the first DSIET settle down to their final values very quickly and in an extremely stable manner. Thus, e.g., a value of $N_r = 14$ suffices to obtain an accuracy to within ten significant digits. In contrast, the corresponding values

TABLE II
$|a_n|$ $(n = 0, 2, 4, 6, 8)$ AS OBTAINED FROM THE SOLUTION OF (1a) FOR $(A_1 = 1, A_2 = 0, q = 1, g_1(x) = 1/k, kw = \pi)$ BY THE FIRST AND BY THE THIRD DSIET

First DSIET						
N_r	$\lvert a_0\rvert$	$\lvert a_2\rvert$	$\lvert a_4\rvert\times10^1$	$\lvert a_6\rvert\times10^2$	$\lvert a_8\rvert\times10^4$	CPU (s)
8	0.3234133427	0.2060992449	0.1999407833	0.1333104558		
10	0.3234133437	0.2060992487	0.1999404955	0.1331863936	0.5168987221	0.2
12	0.3234133437	0.2060992487	0.1999404954	0.1331863714	0.5167125076	0.2
14	0.3234133437	0.2060992487	0.1999404954	0.1331863714	0.5167125076	0.2
Third DSIET						
L	$\lvert a_0\rvert$	$\lvert a_2\rvert$	$\lvert a_4\rvert\times10^1$	$\lvert a_6\rvert\times10^2$	$\lvert a_8\rvert\times10^4$	CPU (s)
10	0.3235898283	0.2056472308	0.1999005586	0.1288142088	0.4577906306	0.2
20	0.3234345980	0.2060442450	0.1999588788	0.1328151436	0.5130877222	0.3
40	0.3234159737	0.2060924170	0.1999434478	0.1331443389	0.5163365552	1.1
60	0.3234141215	0.2060972268	0.1999414053	0.1331741451	0.5166049795	1.5
100	0.3234135116	0.2060988122	0.1999406958	0.1331837554	0.5166897008	4
200	0.3234133647	0.2060991941	0.1999405206	0.1331860457	0.5167096787	15
400	0.3234133463	0.2060992418	0.1999404986	0.1331863307	0.5167121547	62

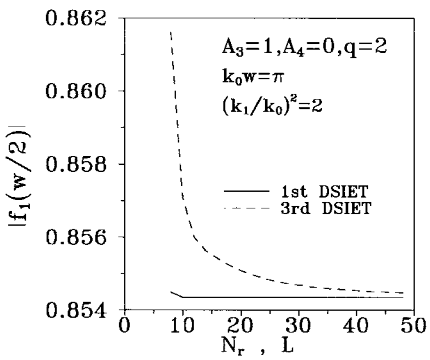

Fig. 1. $|f_1(x = w/2)|$ versus N_r as obtained from the solution of (1b) for $A_3 = 1, A_4 = 0, q = 2, g_2(x) = 1, k_1/k_0 = \sqrt{2}$. (_____ first DSIET; ----- third DSIET).

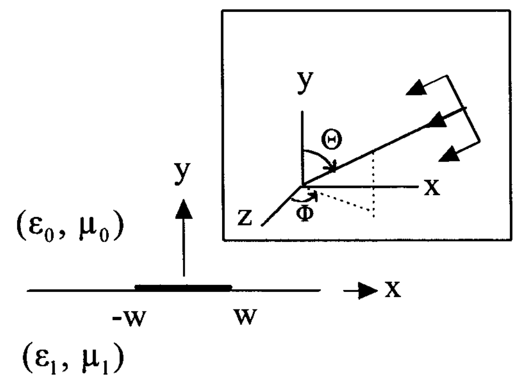

Fig. 2. A strip at the interface between two dielectric half-spaces, illuminated by an obliquely incident plane wave.

obtained on the basis of the third DSIET take on their final values for considerably greater matrix sizes. Very illustrative in this respect is also Fig. 1, which shows $|f_1(x)|$ at $x = w/2$ obtained from (1) for the case $A_3 = 1, A_4 = 0 = A_1 = A_2, q = 2, g_2(x) = 1, k_0 w = \pi, k_1/k_0 = \sqrt{2}$ and for several matrix sizes. Clearly, the third DSIET exhibits an almost asymptotic convergence ending up with the same results as those obtained earlier by the first DSIET.

VII. Illustration of the Application of the Algorithms

Consider (Fig. 2) a strip of width $2w$ right on the interface between the dielectric half-spaces $(\varepsilon_i, \mu_i, k_i = \omega\sqrt{\varepsilon_i \mu_i})$ $(i \equiv 0, 1)$. Let $E_z^{\text{inc}}(\overline{r}) = E_0 \exp(j\overline{k}^{\text{inc}} \cdot \overline{r})$, $H_z^{\text{inc}}(\overline{r}) = H_0 \exp(j\overline{k}^{\text{inc}} \cdot \overline{r})$ be the z components of an arbitrarily polarized plane wave that is obliquely incident in the direction of $\overline{k}^{\text{inc}}(k_0, \Theta, \Phi) = k_0(\sin\Theta \sin\Phi \hat{x} + \cos\Theta \hat{y} + \sin\Theta\cos\Phi \hat{z}) = k_x\hat{x} + k_y\hat{y} + \beta\hat{z}$. The surface current density induced on the strip, denoted by $(J_x(x)\hat{x} + J_z(x)\hat{z})e^{j\beta z}$, satisfies the system

$$\int_{-w}^{w} dx' \int_{-\infty}^{\infty} e^{-ju(x-x')} \begin{pmatrix} Z_1(u) & Z_2(u) \\ Z_2(u) & Z_3(u) \end{pmatrix} \begin{pmatrix} J_x(x') \\ J_z(x') \end{pmatrix} du = 2\pi \begin{pmatrix} E_x^{\text{exc}}(0) \\ E_z^{\text{exc}}(0) \end{pmatrix} e^{jk_x x} \tag{53}$$

$(|x| \leq w)$. Here, $E^{\text{exc}}(y)\exp(jk_x x + j\beta z)$ denotes the (known) field excited by the incident wave in the absence of the strip, while

$$Z_1(u) = \beta^2/Y^h - u^2/Y^e, \quad Z_2(u) = u\beta(1/Y^h + 1/Y^e)$$
$$Z_3(u) = u^2/Y^h - \beta^2/Y^e \tag{54}$$

$$Y^p = (\mathcal{Y}_0^p + \mathcal{Y}_1^p)/(u^2 + \beta^2) \qquad (p \equiv e, h)$$
$$\mathcal{Y}_i^e = -j\omega\varepsilon_i/\gamma_i, \quad \mathcal{Y}_i^h = -j\gamma_i/(\omega\mu_i) \qquad (i \equiv, 0, 1) \tag{55}$$

$$\gamma_i = \gamma_i(u) = (u^2 - \kappa_i^2)^{1/2}$$
$$\kappa_i^2 = k_i^2 - \beta^2; \quad -\frac{\pi}{2} < \arg(\gamma_i) \leq \frac{\pi}{2}. \tag{56}$$

Singulariy Extraction

One easily verifies that $Z_j(u)$ vary as $|u|^{2-j}$ $(j = 1, 2, 3)$ when $u \to \pm\infty$. This implies the divergence or (at best) the conditional convergence of the real-axis spectral integrals in (53). Thus, (53) becomes rather impractical in the course of conventional MoM. To recast it into a convenient form, taking advantage of the preceding DSIET without necessarily departing from real-axis integration, one may proceed as follows.

Let us introduce the shorthand notation

$$\varepsilon = \varepsilon_1/\varepsilon_0, \quad \mu = \mu_1/\mu_0, \xi_1 = (1-\mu^2)/(\varepsilon-\mu)$$
$$\xi_2 = (\varepsilon\mu - 1)/(\varepsilon-\mu), \quad \xi_3 = (\mu^2\kappa_0^2 - \kappa_1^2)/(\varepsilon-\mu) \tag{57}$$

$$f_p(u) = 1/(\gamma_1 + p\gamma_0), \quad A_p = 2(p-1)/(p+1)^2$$
$$B_p = 4/(p+1)^2 \qquad (p \equiv \varepsilon, \mu) \tag{58}$$

then, after some lengthy straightforward algebra, we obtain

$$Z_1(u) = -j\{[\xi_1 f_\mu(u) + \xi_2 f_\varepsilon(u)]u^2 - k_0^2 \mu f_\mu(u)\}/(\omega\varepsilon_0)$$
$$Z_2(u) = ju\beta[\xi_1 f_\mu(u) + \xi_2 f_\varepsilon(u)]/(\omega\varepsilon_0)$$
$$Z_3(u) = -j[\xi_3 f_\mu(u) + \xi_2 \beta^2 f_\varepsilon(u)]/(\omega\varepsilon_0). \tag{59}$$

The decomposition

$$f_p(u) = \frac{A_p}{2\gamma_0} + \frac{B_p}{\gamma_0+\gamma_1} - \frac{A_p}{2}F_p(u)$$
$$F_p(u) = \frac{(k_0^2 - k_1^2)^2}{\gamma_0(\gamma_0+\gamma_1)^3(\gamma_1 + p\gamma_0)} \qquad (p \equiv \varepsilon, \mu) \tag{60}$$

in conjunction with [17, eqs. (14) and (15)] help recast (53) in the form

$$4\omega\varepsilon_0 E_x^{\rm exc}(0)e^{jk_x x} = \mathcal{K}^{12}(x) + \left(k_A^2\frac{d^2}{dx^2} + k_c^2\right)\mathcal{J}_x^1(x) + \left(k_B^2\frac{d^2}{dx^2} + k_D^2\right)\mathcal{J}_x^2(x) + j\beta \cdot \frac{d}{dx}[k_A^2\mathcal{J}_z^1(x) + k_B^2\mathcal{J}_z^2(x)] \tag{61}$$

$$4\omega\varepsilon_0 E_z^{\rm exc}(0)e^{jk_x x} = \mathcal{K}^{23}(x) + j\beta\frac{d}{dx}[k_A^2\mathcal{J}_x^1(x) + k_B^2\mathcal{J}_x^2(x)] - T_A\mathcal{J}_z^1(x) - T_B\mathcal{J}_z^2(x) \tag{62}$$

($|x| \leq w$), where

$$k_A^2 = \xi_1 A_\mu + \xi_2 A_\varepsilon, \quad k_B^2 = \xi_1 B_\mu + \xi_2 B_\varepsilon$$
$$k_C^2 = \mu k_0^2 A_\mu, \quad k_D^2 = \mu k_0^2 B_\mu$$
$$T_Q = \xi_3 Q_\mu + \beta^2 \xi_2 Q_\varepsilon \qquad (Q \equiv A, B) \tag{63}$$

$$\mathcal{K}^{ij}(x) = \frac{1}{2\pi}\int_{-w}^{w} dx' \int_{-\infty}^{\infty} e^{-ju(x-x')}[R_i(u)J_x(x') + R_j(u)J_z(x')]\,du \tag{64}$$

$$R_1 = 2j\{[\xi_1 A_\mu F_\mu(u) + \xi_2 A_\varepsilon F_\varepsilon(u)]u^2 - k_0^2\mu A_\mu F_\mu(u)\} \tag{65}$$

$$R_2 = -2j\beta u[\xi_1 A_\mu F_\mu(u) + \xi_2 A_\varepsilon F_\varepsilon(u)]$$
$$R_3 = 2j[\xi_3 A_\mu F_\mu(u) + \beta^2 \xi_2 A_\varepsilon F_\varepsilon(u)] \tag{66}$$

$$\mathcal{J}_p^1(x) = \int_{-w}^{w} J_p(x')G_1(x-x';\kappa_0)\,dx'$$
$$\mathcal{J}_p^2(x) = \int_{-w}^{w} J_p(x')G_2(x-x';\kappa_0,\kappa_1)\,dx' \qquad (p \equiv x, z) \tag{67}$$

with G_1 and G_2 defined in (2).

Discretization of (61) and (62) by the First DSIET

We set $x = wt, x' = w\tau$ and identify $J_z(x)$ and $J_x(x)$ with $f_1(x)$ and $f_2(x)$ of (3), respectively. Then, from (61) and (62) on the basis of the first DSIET we end up with the following linear algebraic system:

$$\sum_{N=0}^{\infty}(b_N R_{MN}^{xx} + a_N R_{MN}^{xz}) = 4\omega\varepsilon_0 c_M^{(1)})$$
$$\sum_{N=0}^{\infty}(b_N R_{MN}^{zx} + a_N R_{MN}^{zz}) = 4\omega\varepsilon_0 C_M^{(2)} \tag{68}$$

$(M = 0, 1, 2, \cdots, \infty)$ where

$$R_{MN}^{xx} = \left\{-I^{11}[M,N;R_1] + \frac{1}{w}[k_A^2 D_{MN}^{(1)}(k_C^2/k_A^2, \kappa_0 w) + k_B^2 D_{MN}^{(2)}(k_D^2/k_B^2, \kappa_0 w, \kappa_1 w)]\right\} d_{MN}^{+} \tag{69a}$$

$$R_{MN}^{xz} = \{-I^{10}[M,N;R_2] + j\beta[k_A^2 C_{MN}^{(1)}(\kappa_0 w) + k_B^2 C_{MN}^{(2)}(\kappa_0 w, \kappa_1 w)]\}\, d_{MN}^{-} = -R_{NM}^{zx} \tag{69b}$$

$$R_{MN}^{zz} = \{-I^{00}[M,N;R_3] - w[T_A A_{MN}^{(1)}(\kappa_0 w) + T_B A_{MN}^{(2)}(\kappa_0 w, \kappa_1 w)]\}\, d_{MN}^{+} \tag{69c}$$

$$c_M^{(1)} = \frac{\pi}{2}j^M[J_M(k_x w) + J_{M+2}(k_x w)]E_x^{\rm exc}(0)$$
$$c_M^{(2)} = \pi j^M J_M(k_x w)E_z^{\rm exc}(0) \tag{70}$$

$$d_{MN}^{\pm} = [1 \pm (-1)^{M+N}]/2. \tag{71}$$

The integrals I^{mn} $(m, n = 0, 1)$ given in Appendix B have to be evaluated numerically. They converge as $1/u^6$, i.e, very strongly. For a given truncation size N_r (the number of basis functions used for J_x, J_z) the needed numerical integrations reduce to $(N_r + 1) \times (N_r + 2)$. This is a consequence of Appendix (93), (94) and the symmetry relation $I^{00}(M,N;R) = I^{00}(N,M;R)$ in conjunction with the vanishing of most of the matrix elements (depending on weather $M + N$ is odd or even). When $N_r = 9$, e.g., 110 numerical integrations suffice to fill all the 400 matrix elements.

Discretization of (61) and (62) by the Third DSIET

We identify now $J_z(x)$ and $J_x(x)$ with $f_1(x)$ and $f_2(x)$ of (36). Then, on the basis of the third DSIET, one obtains the linear algebraic system

$$\sum_{n=1}^{L}(\tilde{R}_{mn}^{xx}F_2(\hat{t}_n) + \tilde{R}_{mn}^{xz}F_1(t_n)) = 4\omega\varepsilon_0 E_x^{\rm exc}(0)e^{jk_x w\hat{t}_m} \tag{72a}$$

$$\sum_{n=1}^{L}(\tilde{R}_{mn}^{zx}F_2(\hat{t}_n) + \tilde{R}_{mn}^{zz}F_1(t_n)) = 4\omega\varepsilon_0 E_z^{\rm exc}(0)e^{jk_x w t_m} \tag{72b}$$

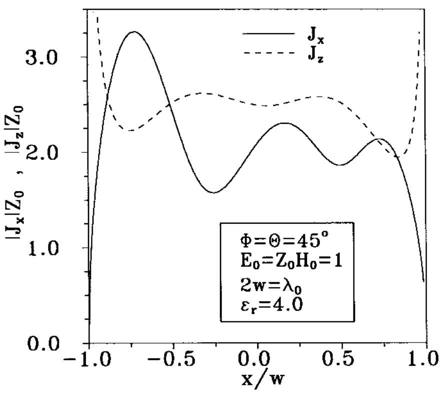

Fig. 3. Induced current densities versus x/w for the structure of Fig. 2.

$(m = 1, 2, \cdots, L)$ where

$$\tilde{R}^{xx}_{mn} = J^a[\hat{t}_m, \hat{t}_n; R_1] + \frac{2}{j\pi w}[k_A^2 \tilde{D}^{(1)}_{mn}(k_c^2/k_A^2, \kappa_a w) + k_B^2 \tilde{D}^{(2)}_{mn}(k_D^2/k_B^2, \kappa_0 w, \kappa_1 w)] \tag{73}$$

$$\tilde{R}^{xz}_{mn} = J^b[\hat{t}_m, t_n; R_2] + \frac{2}{\pi}\beta[k_A^2 \tilde{C}^{(1)}_{mn}(\kappa_0 w) + k_B^2 \tilde{C}^{(2)}_{mn}(\kappa_0 w, \kappa_1 w)] \tag{74}$$

$$\tilde{R}^{zx}_{mn} = J^a[t_m, \hat{t}_n; R_2] + \frac{2}{\pi}\beta[k_A^2 \tilde{B}^{(1)}_{mn}(\kappa_0 w) + k_B^2 \tilde{B}^{(2)}_{mn}(\kappa_0 w, \kappa_1 w)] \tag{75}$$

$$\tilde{R}^{zz}_{mn} = J^b[t_m, t_n; R_3] - \frac{2w}{j\pi}[T_A \tilde{A}^{(1)}_{mn}(\kappa_0 w) + T_B \tilde{A}^{(2)}_{mn}(\kappa_0 w, \kappa_1 w)]. \tag{76}$$

The integrals $J^q(t, \tau; R_j \ \ (q \equiv a, b; j = 1, 2, 3)$ given in Appendix B converge uniformly as $1/|u|^{3+j}$.

To further appreciate the convergence characteristics of the algorithms, Fig. 3 shows $|J_x(x)|Z_0$ and $|J_z(x)|Z_0$ $(Z_0 = \sqrt{\mu_0/\varepsilon_0})$ versus x as obtained by the first DSIET for $N_r = 8$ (the parameter values are $2w = \lambda_0, \varepsilon_r \equiv \varepsilon_1/\varepsilon_0 = 4, \mu_1 = \mu_0, \Phi = \Theta = 45°, E_0 = Z_0 H_0 = 1$). These results were also independently rederived by the third DSIET for $L = 12$. As seen, both methods yield indistinguishable final results for comparatively small matrix sizes.

Evaluation of the spectral integrals I^{mn} $(m, n = 0, 1)$ and J^q $(q \equiv a, b, c)$ is a crucial factor affecting the efficiency of the proposed algorithms. The observed rapid convergence of these integrals is a very important feature minimizing CPU time. To see this, Fig. 4(a) shows the relative error in evaluating $I^{11}(0, 0, R_1)$, $I^{00}(0, 0, R_3)$, and $I^{10}(1, 0, R_2)$. The relative error is defined by $[I^{mn}\{\infty\} - I^{mn}\{u_{\max}\}]/I^{mn}\{\infty\} \times 100$. Here, $I^{mn}\{u_{\max}\}$ is the value of the integral when the integration is carried out from $u = 0$ to $u = u_{\max}; I^{mn}\{\infty\}$ is approximated by $I^{mn}\{100k_1\}$. For the sake of comparison the relative error of $I^{00}(0, 0, Z_3)$ is also indicated. As seen, this last integral, which results from a brutal MoM solution of (53) (without singularity extraction), requires large values of $u_{\max}/k_1$. In contrast, the relative error of $I^{mn}(M, N, R_j)$ $(m, n = 0, 1; j = 1, 2, 3)$ diminishes very rapidly as $u_{\max}$ increases. This is more clearly shown in Fig. 4(b), which results after enlarging the encircled area near the origin in Fig. 4(a). The aforementioned very strong convergence of the procedure leads to very accurate evaluation of the integrals for extremely small values of $u_{\max}/k_1$.

VIII. Conclusions

Three powerful DSIET have been applied to the solution of systems of singular integral equations frequently encountered in scattering and propagation problems related to strip/slot-loaded structures. The first and the second DSIET are very similar in spirit, yet the latter is considerably more time consuming when a high accuracy is desirable. The great computational advantage of the third DSIET is the simplicity of the analytical—either closed form or in terms of simple finite series expressions of the matrix elements, which makes it easily programmable. From the standpoint of accuracy, the first DSIET yields results, which settle down to their final values in a very stable manner requiring very small matrix sizes. These same values are approached asymptotically by the third (and by the second) DSIET at the expense of much greater matrix sizes. In conclusion, the first and the third DSIET are promising alternatives, useful in treating a very wide class of 3-D problems for strip/slot-loaded structures.

Appendix A

Definition of the Symbols Introduced in (5)–(13)

Throughout this Appendix, the entry κw $(\kappa w \equiv kw, k_0 w, k_1 w)$ is specified by the left side of (5), (7), (8), (10), (11), and (13). Let (77), as shown at the bottom of the page. The quantities defined in (78)–(88) below comprise simple linear combinations of $I(p, q)$:

$$J(p, q) = \frac{1}{2}[I(p, q) - I(p + 2, q)]$$
$$\tilde{J}(M, q) = \frac{1}{2}[J(M, q - 1) - J(M, q + 1)] \tag{78}$$

$$h_q(M, m) = J(M, q + m) + J(M, q - m)$$
$$\tilde{h}_q(M, m) = \tilde{J}(M, q + m) + \tilde{J}(M, q - m) \tag{79}$$

$$\lambda_q(M, m) = I(M, q + m) + I(M, q - m)$$
$$V_q(M, m) = H(M, m + q) + (-1)^q H(M, m - q) \tag{80}$$

$$H(M, q) = (k_c w)^2 J(M, q) + \frac{1}{4}(\kappa w)^2 [h_q(M, 2) - h_q(M, 0)] \tag{81}$$

$$I(p, q) = I(p, q; \kappa w) = \pi J_{\frac{p+q}{2}}\left(\frac{\kappa w}{2} J_{\frac{\frac{q-p}{2}(\kappa w}{2)}}\right) \quad (p + q \text{ even}), \quad 0 \quad (p + q \text{ odd}). \tag{77}$$

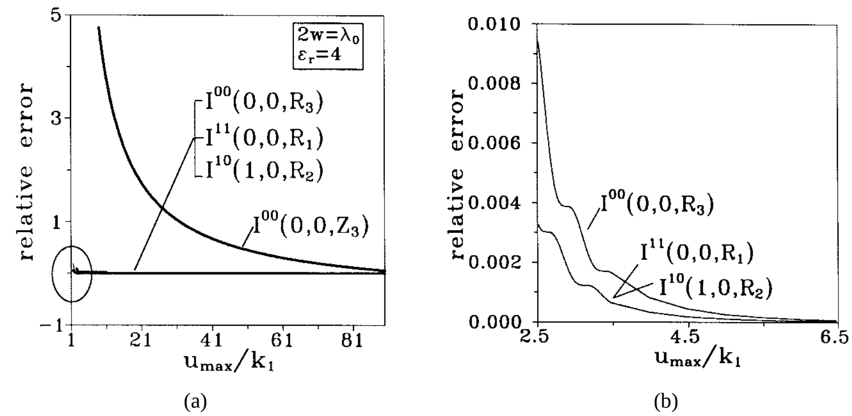

Fig. 4. (a) Convergence of the spectral integrals $I^{11}(0,0,R_1)$, $I^{10}(1,0,R_2)$, $I^{00}(0,0,R_3)$, and $I^{00}(0,0,Z_3)$. (b) More detailed examination of the convergence of the first three of the above integrals.

$$\begin{aligned} I_a(m,n,q) &= \frac{1}{2}[I(|m-n|,q)+I(m+n,q)] \\ J_a(q,m,n) &= \frac{1}{2}[I(|m-n|,q)-I(m+n+2,q)] \end{aligned} \tag{82}$$

$$J_b(m,n,q) = \frac{1}{2}[J(m+n,q)+s_{m-n+1}J(|n-m-1| - 1,q)]; \qquad s_m = \operatorname{sgn}(m) \tag{83}$$

$$\Lambda_J(p,q,M,N) = \frac{\pi}{2}\left[-\delta_{(N-p)0}J(M,q)\ell n2 - \frac{J_b(M,|p-N|,q)}{|N-p|} + \frac{J_b(M,p+N+2,q)}{p+N+2}\right] \tag{84}$$

$$\Lambda_I(p,q,M,N) = -\frac{\pi}{2}\left[(1+\delta_{p0})\delta_{Np}J(M,q)\ell n2 + \frac{J_b(M,|p-N|,q)}{|N-p|} + \frac{J_b(M,p+N,q)}{p+N}\right] \tag{85}$$

$$\Lambda_a(p,q,M,N) = -\frac{\pi}{2}\left[(1+\delta_{p0})\delta_{Np}I(M,q)\ell n2 + \frac{I_a(M,|p-N|,q)}{|N-p|} + \frac{I_a(M,p+N,q)}{p+N}\right]. \tag{86}$$

$$\begin{aligned} \Theta_a(p,q,n,M,N) = \frac{-\kappa w}{\pi}[&\Lambda_I(p,q+n-1,M,N) \\ &- \Lambda_I(p,q+n+1,M,N) \\ &+ J_a(q+n,M,N+p-1) \\ &+ J_a(q+n,M,|N-p|-1) \end{aligned} \tag{87}$$

$$\begin{aligned} \Theta_b(p,q,n,M,N) = {} & (k_c w)^2 J(p,q)\Lambda_J(p,q+n,M,N) \\ & + \frac{1}{2}\varepsilon_p I(p,q)\Bigg(-\frac{(\kappa w)^2}{4}\sum_{\ell=-2,0,2} \\ & \cdot \frac{1}{\varepsilon_\ell}(-1)^{\ell/2}[\Lambda_J(0,q+\ell+n,M, \\ & N+p) + s_{N-p+1}\Lambda_j(0,q+\ell+n, \\ & M,|p-N-1|-1)] - \frac{\pi}{2}\kappa w \\ & \cdot \sum_{\ell=-1,1}\ell[J_b(M,p+N+1, \\ & q+n+\ell) + s_{N-p+1} \\ & \cdot J_b(M,|p-N-1|,q+n+\ell)] + \frac{\pi}{2} \\ & \cdot [(p+N+1)J_a(q+n,M,p+N) \\ & - (p-N-1) \\ & \cdot J_a(q+n,M,|p-N-1|-1)]\Bigg) \end{aligned} \tag{88}$$

In (5)–(13), $S_M\{f\}$ $(f \equiv \lambda_q, h_q, V_q), \Xi_I$, and Ξ_J are short-hand symbols for

$$\begin{aligned} S_M\{f\} = {} & \frac{1}{2}[\Gamma_1(\kappa w)f(M,0)+\Gamma_2(\kappa w)f(M,2)] \\ & + j\pi^{-1}\Bigg\{2\sum_{m=1}^{\infty}\frac{(-1)^m f(M,2m+2)}{m(m+1)(m+2)} \\ & - \sum_{m=0}^{\infty}\Bigg[\frac{f(M,2m)}{(2m+1)(2m+2)} \\ & + 2\frac{f(M,2m+2)}{(2m+1)(2m+3)} + \frac{f(M,2m+4)}{(2m+2)(2m+3)}\Bigg]\Bigg\} \end{aligned} \tag{89}$$

$$\begin{aligned} \Xi_L(N,q) = {} & \Gamma_0(\kappa w)L(N,q) + j\frac{2}{\pi} \\ & \cdot \sum_{n=1}^{\infty}\frac{(-1)^n}{n}[L(N,q+2n)+L(N,q-2n)] \\ & (L \equiv I,J) \end{aligned} \tag{90}$$

$(\Xi_L(N,q)=0$ for $N+q$ odd). All these series are very rapidly (exponentially) convergent. In (89) and (90)

$$\begin{aligned} \Gamma_0(\kappa w) &= 1 - j\frac{2}{\pi}\left(\ln\frac{\kappa w}{2}+\gamma\right) \\ \Gamma_m(\kappa w) &= \Gamma_0(\kappa w) + j\frac{2}{m\pi} \qquad (m=1,2) \\ \gamma &= 0.7721\cdots. \end{aligned} \tag{91}$$

APPENDIX B

Definition of I^{mn} $(m, n = 0, 1)$

$$I^{mn}[M,N;G] = -\pi w(-1)^{M} j^{M+N}(M+1)^{m}(N+1)^{n} \cdot \int_0^\infty G(u)\frac{J_{M+m}(wu)}{(wu)^m}\frac{J_{N+n}(wu)}{(wu)^n}\,du \tag{92}$$

We note that

$$I^{11}[M,N;G] = \tfrac{1}{4}\{I^{00}[M,N;G] + I^{00}[M+2,N;G] + I^{00}[M,N+2;G] + I^{00}(M+2,N+2;G]\} \tag{93}$$

$$I^{10}(M,N;G] = \frac{1}{2}\{I^{00}[M,N;G] + I^{00}[M+2,N;G]\}. \tag{94}$$

Definition of J^q $(q \equiv a, b, c)$

$$J^a[t,\hat{t}_n;G] = \frac{w}{2(L+1)}(1-\hat{t}_n^2)J^c[t,\hat{t}_n;G]$$
$$J^b[t,t_n;G] = \frac{w}{2L}J^c[t,t_n;G]$$
$$J^c[t,\tau;G] = \int_{-\infty}^{\infty} G(u)\exp\left[-jw(t-\tau)u\right]du. \tag{95}$$

REFERENCES

[1] A. I. Kalandiya, *Mathematical Methods of Two-Dimensional Elasticity*. Moscow, Russia: Mir, 1982.

[2] V. Z. Parton and P. I. Perlin, *Integral Equations in Elasticity*. Moscow, Russia: Mir, 1982.

[3] R. Mitra and S. W. Lee, *Analytical Techniques in the Theory of Guided Waves*. New York: Macmillan, 1971.

[4] N. I Muskhelishvili, *Singular Integral Equations*. Groningen, Holland: Noordhoff, 1953.

[5] F. D. Gakhov, *Boundary Value Problems*. Oxford, U.K.: Pergamon, 1966.

[6] J. G. Fikioris, J. L. Tsalamengas, and G. J. Fikioris, "Exact solutions for shielded lines by the Carleman–Vekua method," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 21–33, Jan. 1989.

[7] A. Matsushima and T. Itakura, "Singular integral equation approach to electromagnetic scattering from a finite periodic array of conducting strips," *JEWA*, vol. 5, no. 6, pp. 545–562, 1991.

[8] L. Lewin, "The use of singular integral equations in the solution of waveguide problems," in *Advances of Microwaves*, Leo Young, Ed. New York: Academic, 1966, vol. 1.

[9] J. L. Tsalamengas and J. G. Fikioris, "Efficient solutions for scattering from strips and slots in the presence of a dielectric half-space: Extension to wide scatterers—Part I: Theory," *J. Appl. Phys.*, vol. 70, no. 3, pp. 1121–1131, Aug. 1991.

[10] A. Frenkel, "External modes of two-dimensional thin scatterers," *Proc. Inst. Elect. Eng.*, vol. 130, pt. H, no. 3, pp. 209–214, 1983.

[11] S. Krenk, "On the use of the interpolation polynomial for solutions of singular integral equations," *Quart. Appl. Math.*, vol. 33, pp. 479–483, Jan. 1975.

[12] ——, "On quadrature formulas for singular integral equations of the first and the second kind," *Quart. Appl. Math.*, vol. 33, pp. 225–232, Oct. 1975.

[13] E. I. Veliev, "Numerical-analytical methods of solution of integral equations of two-dimensional diffraction problems," in *Math. Methods Electromagn. Theory, Proc. 3rd Int. School Seminal*, Crimea, Ukraine, Apr. 1990.

[14] Z. T. Nazarchuk, "Singular integral equations in two-dimensional diffraction problems," in *Math. Methods Electromagn. Theory Proc. 3rd Int. School Seminal*, Crimea, Ukraine, Apr. 1990.

[15] Z. Nazarchuk and O. Ovsyannikov, "Electromagnetic scattering by screens in an open planar waveguide," *JEWA*, vol. 8, no. 11, pp. 1481–1498, 1994.

[16] R. Kress, "Numerical solution of boundary integral equations in time-harmonic electromagnetic scattering," *Electromagn.*, vol. 10, pp. 1–20, 1990.

[17] J. L. Tsalamengas, "TE scattering by conducting strips right on the planar interface of a three-layered medium," *IEEE Trans. Antennas Propagat.*, vol. 45, Dec. 1993.

[18] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1972.

[19] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, 4th ed. New York: Academic, 1965 (English transl., A. Jeffrey).

John L. Tsalamengas (M'87), for a photograph and biography, see p. 555 of the May 1993 issue of this TRANSACTIONS.