On the Locally Continuous Formulation of Surface Doublets

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***Abstract*— Exact (locally continuous) formulation of doublets and particularly rooftop basis functions based on unitary vector concept are presented. Basic properties of such a formulation are examined showing many advantages when compared with classical (approximate) formulation. In particular, in the case of rooftop basis functions based on exact formulation, the shape quality factor is defined and optimal shapes of quadrilateral patches are determined. If such quadrilaterals are used for modeling of general structures, the number of unknowns needed in the analysis is almost halved when compared with modeling by triangular doublets.**

***Index Terms*— Basis functions, boundary integral methods, integral equations.**

I. Introduction

STARTING from the equivalence theorem, any composite metallic and dielectric structure can be analyzed by using the surface integral equation (SIE). Such integral equations are usually solved by the method of moments (MoM) [1]. The MoM is applied in two principal steps: geometrical modeling and modeling of currents.

Geometrical modeling of surfaces is usually performed by flat quadrilaterals and triangles [2]–[10]. Having in mind that flat quadrilateral cannot be defined by four arbitrary points in the space, it is obvious that such elements are not suitable for modeling of curved surfaces. It is shown that flat quadrilaterals are very suitable for modeling of polygonal plates [6], [7], while triangles are recommended for modeling of curved surfaces [5], [8], [9].

Currents are usually approximated by subdomain basis functions in the form of doublets [2]–[10]. Doublets are basis functions defined over two neighboring patches in the following way: 1) continuity equation across the junction of these patches is automatically satisfied and 2) current component normal to other edges is equal to zero. In the case of triangular doublets, the continuity equation is exactly satisfied in all points along the junction [5]. However, in the case of general quadrilaterals, the continuity equation is not satisfied exactly (i.e., in a local way), but only approximately (i.e., in a global way) [3], [6], [7]. Namely, total current flowing out from one doublet arm is equal to the total current flowing into another doublet arm. (In what follows such formulation of doublets will be termed as "approximate.") As a result, line charge exists along the junction of these patches. These fictious charges obviously decrease approximating potentials of doublets. Only in the case of rectangular doublets is the continuity equation satisfied in a local way [2], [4]. That is the reason, among others, why it is desirable that as many as possible doublets are rectangular [6]. Hence, some methods combine rectangular and triangular doublets [10].

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Flexibility of basis functions can be significantly improved if the so-called bilinear surfaces are used instead of flat quadrilaterals and entire-domain approximation is used instead of subdomain approximation [11], [12]. For the lowest order of approximation, these entire-domain basis functions degenerate into doublets that exactly satisfy the continuity equation at junctions. (In what follows, such formulation of doublets will be termed as "exact.")

The purpose of this paper is twofold. First, it is intended to present alternative expressions of doublets and rooftop basis functions that exactly satisfy continuity equation at the junctions, particularly convenient for subdomain modeling. Second, it is intended to investigate basic properties of such doublets and to compare them with other types of doublets.

II. Desired Properties of Doublets

In the general case, doublet basis functions can be written as

$$\boldsymbol{J}_s(\boldsymbol{r}) = \begin{Bmatrix} J_{s1}(u_1, v)\boldsymbol{i}_{u1}, & \boldsymbol{r} \in S_1 \\ -J_{s2}(u_2, v)\boldsymbol{i}_{u2}, & \boldsymbol{r} \in S_2 \end{Bmatrix} \tag{1}$$

where J_{s1} and J_{s2} are current densities and $\boldsymbol{i}_{u1}$ and $\boldsymbol{i}_{u2}$ are unit vectors defined over two neighboring isoparametric surfaces S_1 and S_2. Parametric equations of these surfaces can be written in the general form as

$$\boldsymbol{r} = \boldsymbol{r}(u, v) \qquad 0 \le u \le 1 \qquad 0 \le v \le 1 \tag{2}$$

where u and v are local parametric coordinates, as shown in Fig. 1(a). Note that two patches are interconnected along common v-coordinate line $u_1 = u_2 = 1$.

It is desirable that doublets have the same good properties as rectangular doublets. Current density vector over a rectangular doublet arm can be written as

$$\boldsymbol{J}_s(u, v) = \frac{f(u, v)}{l_v}\boldsymbol{i}_u \qquad f(0, v) = 0 \qquad f(1, v) = 1 \tag{3}$$

where l_v is the width of a doublet along the v-coordinate line and $f(u, v)$ is an arbitrary smooth function. The above current density vector is normalized in such a manner that the total current flowing through the interconnection is equal to unity. Such basis functions satisfy the continuity equation at the junction not only in the global, but also in the local way.

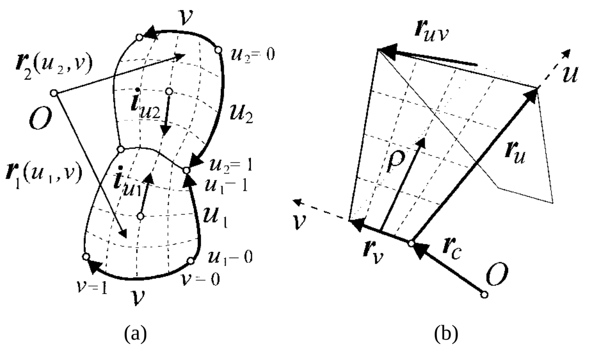

Fig. 1. Doublet composed of (a) isoparametric surfaces and (b) flat quadrilaterals.

TABLE I
LIST OF DESIRED PROPERTIES OF DOUBLETS

no.	short name	description
P1	global continuity equation	total current flowing out the first arm is equal to the total current flowing into the second arm
P2	local continuity equation	at each point of the junction of two arms, the normal current component flowing out the first arm is equal to the normal current component flowing into the second arm
P3	constant junction component	current component normal to the junction of two doublet arms is constant at the junction
P4	constant current distribution	possibility of approximating constant current density vector
P5	constant charge distribution	possibility of approximating constant charge distribution

In addition, the current component normal to the junction is constant.

Current and charge distributions over each rectangle are modeled by one or more overlapping doublets. For sufficiently small doublets these distributions should be approximately constant. Such distributions can be successfully modeled by doublets known as rooftop basis functions or sinusoidal doublets. These doublets are obtained if function $f(u, v)$ is adopted in the following forms, respectively:

$$f(u, v) = u \qquad f(u, v) = \frac{\sin(\beta l_u u)}{\sin(\beta l_u)} \tag{4}$$

where l_u is the length of the u-coordinate line. Good properties of a rectangular doublet, which are also desirable in the case of any other doublet type, are summarized in Table I.

III. APPROXIMATE AND EXACT FORMULATION

The expression for current distribution of a flat quadrilateral doublet can be written in the same form as in the case of rectangular doublet, except that l_v is function of the u-coordinate and $\boldsymbol{i}_u$ is function of the v-coordinate [6], [7]. The current component normal to the interconnection is

$$J_s(1, v)_{norm} = J_s(1, v) \sin\alpha_{uv}(1, v) = \frac{\sin\alpha_{uv}(1, v)}{l_v(1)} \tag{5}$$

where $\sin\alpha_{uv}$ is angle between u- and v-coordinate lines. Since such a formulation of doublet does not satisfy property P2 (local continuity equation), it is referred to as approximate formulation.

In order to satisfy property P1, normalization constant C is introduced in (3) (e.g., by posing that total current of unit magnitude flows through the interconnection). Finally, by expressing l_v and $\boldsymbol{i}_u$ in terms of unitary vectors $\boldsymbol{a}_u = d\boldsymbol{r}/du$ and $\boldsymbol{a}_v = d\boldsymbol{r}/dv$, current distribution over one doublet arm, and corresponding charge distribution can be written in the form

$$\begin{aligned}\boldsymbol{J}_s(u, v) &= C\frac{f(u, v)}{|\boldsymbol{a}_u||\boldsymbol{a}_v|}\boldsymbol{a}_u \\ \rho_s(u, v) &= \frac{j}{\omega|\boldsymbol{a}_u \times \boldsymbol{a}_v|}\frac{\partial}{\partial u}\left\{f(u, v)\frac{|\boldsymbol{a}_u \times \boldsymbol{a}_v|}{|\boldsymbol{a}_u||\boldsymbol{a}_v|}\right\}.\end{aligned} \tag{6}$$

It is obvious that such basis functions satisfy only the first of five desired properties.

In order to satisfy property P2 (local continuity equation), above approximate formulation should be slightly modified as

$$\begin{aligned}\boldsymbol{J}_s(u, v) &= \frac{f(u, v)}{l_v(u)\sin\alpha_{uv}(u, v)}\boldsymbol{i}_u(v) \\ & f(0, v) = 0 \qquad f(1, v) = 1.\end{aligned} \tag{7}$$

By expressing l_v, $\boldsymbol{i}_u$, and $\sin\alpha_{uv}$ in terms of unitary vectors, current distribution over one doublet arm, and corresponding charge distribution can be written as

$$\boldsymbol{J}_s(u, v) = \frac{f(u, v)}{|\boldsymbol{a}_u \times \boldsymbol{a}_v|}\boldsymbol{a}_u \quad \rho_s = \frac{j}{\omega|\boldsymbol{a}_u \times \boldsymbol{a}_v|}\frac{\partial f(u, v)}{\partial u}. \tag{8}$$

It is obvious that such doublet satisfies not only property P2 (local continuity equation), but also properties P1 (global continuity equation) and P3 (constant junction current). By proper adoption of function $f(u, v)$, it is possible to satisfy properties P4 and P5. Hence, such a formulation of the doublet is termed as exact formulation.

Note that (6) and (8) are valid not only for flat quadrilateral doublets but also for doublets defined over isoparametric surfaces given by (2). Starting from these general expressions, various types of doublets can be obtained. For example, well-known triangular rooftop basis function is a special case of the general doublet described by (8). In what follows, exact formulation of flat quadrilateral rooftop basis functions will be considered.

IV. ROOFTOP BASIS FUNCTIONS

Let us consider a doublet made of flat quadrilaterals, as shown in Fig. 1(b). Parametric equation of each doublet arm (2) can be written in the form

$$\begin{aligned}\boldsymbol{r}(u, v) &= \boldsymbol{r}_c + \boldsymbol{r}_u u + \boldsymbol{r}_v v + \boldsymbol{r}_{uv} uv \\ & 0 \le u \le 1 \qquad 0 \le v \le 1\end{aligned} \tag{9}$$

where vectors $\boldsymbol{r}_c$, $\boldsymbol{r}_u$, $\boldsymbol{r}_v$, and $\boldsymbol{r}_{uv}$ can be easily determined starting from position vectors of quadrilateral vertices. (Note that above equation can also define the so-called bilinear surface if the position vectors do not belong to a plane [11], [12].) Let us define the position vector according to the free edge $\boldsymbol{\rho}$ as

$$\boldsymbol{\rho} = u\boldsymbol{a}_u = u(\boldsymbol{r}_u + \boldsymbol{r}_{uv} v). \tag{10}$$

Combining (4), (6), (8), and (10), approximate and exact formulation of rooftop basis functions are obtained as

$$\boldsymbol{J}_s(u, v) = \frac{C}{|\boldsymbol{a}_u||\boldsymbol{a}_v|}\boldsymbol{\rho} \qquad \boldsymbol{J}_s(u, v) = \frac{1}{|\boldsymbol{a}_u \times \boldsymbol{a}_v|}\boldsymbol{\rho}. \tag{11}$$

In order to examine if exact formulation satisfies properties P4 (constant current distribution) and P5 (constant charge distribution), let us transform u- and v-parametric coordinates to U- and V-parametric coordinates as $U = u - 0.5$ and $V = v - 0.5$. In this case, the parametric equation of bilinear surface (9) retains the same form except that all small letters u and v should be replaced by large letters U and V. Further, in the case of flat bilinear surfaces, vector $\boldsymbol{r}_{UV}$ can be expressed as a linear combination of vectors $\boldsymbol{r}_U$ and $\boldsymbol{r}_V$ in the form

$$\boldsymbol{r}_{UV} = \alpha \boldsymbol{r}_U + \beta \boldsymbol{r}_V \qquad -2 < \alpha, \beta < 2. \tag{12}$$

Having in mind that

$$|\boldsymbol{a}_U \times \boldsymbol{a}_V| = (1 + \beta U + \alpha V)|\boldsymbol{r}_U \times \boldsymbol{r}_V| \quad S = |\boldsymbol{r}_U \times \boldsymbol{r}_V| \tag{13}$$

where S is the surface area of a flat quadrilateral, current, and charge distribution over one doublet arm can be written as

$$\boldsymbol{J}_s(U, V) = \frac{1}{1 + \beta U + \alpha V} \frac{1}{S} \boldsymbol{\rho}$$
$$\rho_s(U, V) = \frac{1}{1 + \beta U + \alpha V} \frac{1}{S}. \tag{14}$$

Let us consider the current distribution over a flat quadrilateral shared by four overlapping doublets. [Two of them (u doublets) enable the approximation of the u-current component, and another two (v doublets) enable the approximation of the v-current component.] Current distributions equivalent to the pair of the u doublet $\boldsymbol{J}_{sU}$ and to the pair of the v doublets $\boldsymbol{J}_{sV}$ can be written in the form

$$\boldsymbol{J}_{sU}(U, V) = \frac{(a_{U0} + a_{U1}U)(\boldsymbol{r}_U + \boldsymbol{r}_{UV}V)}{(1 + \beta U + \alpha V)S}$$
$$\boldsymbol{J}_{sV}(U, V) = \frac{(a_{V0} + a_{V1}V)(\boldsymbol{r}_V + \boldsymbol{r}_{UV}U)}{(1 + \beta U + \alpha V)S} \tag{15}$$

where a_{U0}, a_{U1}, a_{V0}, and a_{V1} are arbitrary coefficients. If they satisfy

$$a_{U1} = a_{U0}\beta - a_{V0}\alpha = -a_{V1} \tag{16}$$

the total current over the bilinear surface is obtained as

$$\boldsymbol{J}_{s\,\mathrm{tot}}(U, V) = \boldsymbol{J}_{sU}(U, V) + \boldsymbol{J}_{sV}(U, V) = \frac{a_{U0}\boldsymbol{r}_U + a_{V0}\boldsymbol{r}_V}{S}. \tag{17}$$

It is seen that above distribution does not depend on local u and v coordinates. It means that rooftop basis functions (exact formulation) satisfy property P4 (constant current distribution).

The charge distribution over the above flat quadrilateral is the same for all four doublets, given by the second of (14). It is seen that property P5 (constant charge distribution) is satisfied only for rectangles and rhomboids, i.e., for $\alpha, \beta = 0$. In the case when $\alpha, \beta \neq 0$, undesirable variations of charge distribution are obtained. Since the shapes that produce greater deviation of charge distribution are less desirable, the shape-quality factor can be defined as a ratio of minimal and maximal charge density

$$Q = \frac{\min\{\rho_s(U,V)\}}{\max\{\rho_s(U,V)\}} = \frac{2 - |\alpha| - |\beta|}{2 + |\alpha| + |\beta|}. \tag{18}$$

TABLE II
VARIOUS TYPES OF DOUBLETS AND THEIR PROPERTIES

type of doublet	mathematical expression	P1	P2	P3	P4	P5
quadrilateral doublet (approximate formulation)	$\boldsymbol{J}_s(u,v) = C\frac{f(u,v)}{\lvert\boldsymbol{a}_u\rvert\lvert\boldsymbol{a}_v\rvert}\boldsymbol{a}_u$	X	O	O	O	O
quadrilateral doublet (exact formulation)	$\boldsymbol{J}_s(u,v) = \frac{f(u,v)}{\lvert\boldsymbol{a}_u \times \boldsymbol{a}_v\rvert}\boldsymbol{a}_u$	X	X	X	–	–
quadrilateral rooftop (approximate formulation)	$\boldsymbol{J}_s(u,v) = \frac{C}{\lvert\boldsymbol{a}_u\rvert\lvert\boldsymbol{a}_v\rvert}\boldsymbol{\rho}$	X	O	O	O	O
quadrilateral rooftop (exact formulation)	$\boldsymbol{J}_s(u,v) = \frac{1}{\lvert\boldsymbol{a}_u \times \boldsymbol{a}_v\rvert}\boldsymbol{\rho}$	X	X	X	X	O
triangular rooftop	$\boldsymbol{J}_s(u,v) = \frac{1}{2S}\boldsymbol{\rho}$	X	X	X	X	X

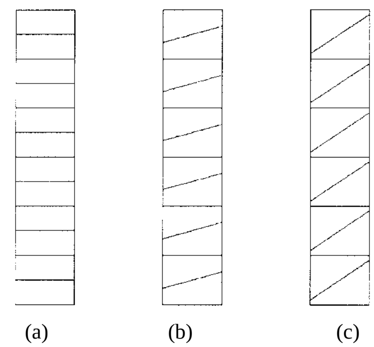

Fig. 2. Strip scatterer modeled by flat quadrilaterals that have three different shape quality factors. (a) $Q = 1$. (b) $Q = 1/2$. (c) $Q = 1/10$.

V. COMPARISON OF VARIOUS TYPES OF DOUBLETS

Various types of doublets and their properties are listed in Table II. ("X" means that property is satisfied, "O" means that property is not satisfied. In the case when the fulfillment of some property depends on the choice of function, $f(u, v)$ "–" is used.) It is seen that

1) exact formulation of doublets is superior to the approximate formulation;
2) quadrilateral doublets (exact formulation) show the same good properties as triangular doublets, except that they cannot provide constant charge distribution (P5).

In order to examine the influence of the property P5 to the overall solution, let us consider current and charge distribution of half wavelength strip scatterer, shown in Fig. 2. (The length and the width of the scatterer are $l = \lambda/2$ and $w = l/5$. The scatterer is excited normally by an incident plane wave, which is polarized along a scatterer length.) The scatterer is subdivided into $n = 12$ patches, in such a manner that the shape quality factor is 1) $Q = 1$; 2) $Q = 1/2$; and 3) $Q = 1/10$. Fig. 3(a)–(c) shows three-dimensional (3-D) graphs of current distributions along the scatterers shown in Fig. 2(a)–(c), respectively. All these results are obtained by using quadrilateral rooftop basis functions (exact formulation). Fig. 3(d) shows the same results as Fig. 3(c) except that order of current approximation along each patch coordinate is increased by one, i.e., the higher order approximation is used. Fig. 4 shows 3-D graphs of charge distribution that correspond to the currents shown in Fig. 3. It is seen that current distribution does not depend much on the quadrilateral shape. On the other side, corresponding charge distributions can show large variations, as expected according to the second of (14). Finally, undesirable local variations of charge distribution can

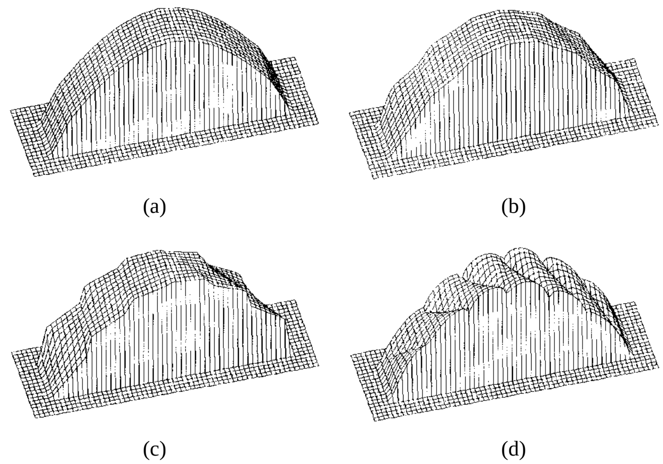

Fig. 3. Current distribution over strip scatterer shown in Fig. 2 (the scatterer length and width are $l = \lambda/2$ and $w = l/10$. The scatterer is excited normally by an incident plane wave, which is polarized along a scatterer length). Rooftop basis functions are applied in the cases (a) $Q = 1$, (b) $Q = 1/2$, and (c) $Q = 10$. Higher order approximation is applied in the case (d) $Q = 1/10$.

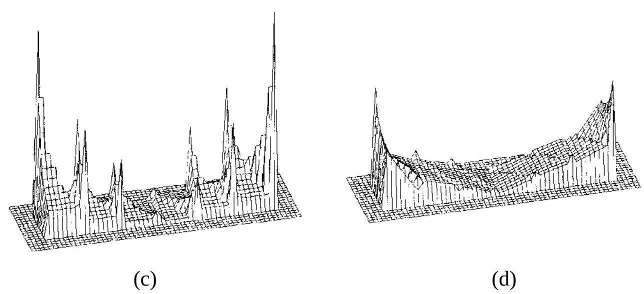

Fig. 4. Charge distributions that correspond to current distributions given in Fig. 3.

be avoided by using higher orders of approximation.

Let us consider radar cross section (RCS) of the same scatterer except that its width is $w = l/50$. Fig. 5 shows bistatic RCS (normalized by maximum value) in a plane $\varphi = 0°$ versus θ. Fig. 5(a) shows the results obtained by using rooftop quadrilateral basis functions (exact formulation) for $Q = 1/2$ and for different number of patches $n = 12$, 24, 48. (Almost the same results are obtained for $Q = 1$.) Fig. 5(b) shows the same results as Fig. 5(a) except that approximate formulation is used. Fig. 5(c) shows the same results as Fig. 5(a), except that $Q = 1/10$. Having this and many other examples in mind, it can be concluded that acceptable results can be obtained even by using quadrilaterals of very low Q, e.g., $Q = 1/100$. However, the number of unknowns needed for accurate analysis increases for higher Q, especially if approximate formulation is used instead of exact one. In the case when exact formulation is used, the optimal geometrical model should contain patches for which $Q > 1/2$.

In order to compare the efficiency of rooftop basis functions (exact formulation) and triangle doublets, two complex examples are given. As a first example, let us consider the square plate scatterer of size $3\lambda\times\ 3\lambda$ with two triangular holes, as shown in the inset of Fig. 6. The scatterer is situated in xOy plane with y axis along the longest hole edge. Incident electric field has only θ component. Fig. 6 shows monostatic RCS in the plane $\varphi = 90°$ versus angle θ. Results obtained by rooftop basis (980 unknowns) are compared with the measured and the theoretical results (1848 unknowns) from [13]. It is seen that the results obtained by quadrilateral modeling are closer to the experimental results than the results obtained by triangular modeling. As a second example, let us consider the scatterer in the form of spherical radome. Inner and outer radii of random, dielectric constant, and operating frequency are $a = 0.5$ m, $b = 0.6$ m, $\varepsilon_r = 4$, and $f = 0.3$ GHz. The scatterer is centered at the origin and is illuminated by an x polarized, z traveling plane wave. Geometrical model of the shell is given in the inset of Fig. 7. (A quarter of the model is omitted in order that inner surface can be inspected.) Fig. 7 shows bistatic RCS in the plane $\varphi = 0°$ versus angle θ. Results obtained by rooftop basis (768 unknowns) are compared with the exact and the theoretical results (1848 unknowns) from [14]. It can be seen that the results obtained by quadrilateral modeling are closer to the exact results than the results obtained by triangular modeling. Having this and some other examples in mind, it can be concluded that triangular modeling requires almost twice more unknowns than quadrilateral modeling based on exact formulation.

VI. Conclusions

The exact formulation of doublets and particularly rooftop basis functions, suitable for subdomain modeling is presented. When compared with classical (approximate) formulation, the exact formulation has the following advantages:

1) it satisfies continuity equation locally, i.e., at each point of the interconnection;
2) current component normal to the interconnection is constant.

In the particular case of rooftop basis functions, this formulation has an additional advantage: it enables modeling of constant current distribution.

Hence, in the general case, exact formulation of doublets enables higher accuracy and faster convergency of the results than approximate formulation.

This formulation does not enable modeling of the constant charge distribution. Moreover, local variations of charge distribution depends only on quadrilateral shape. Hence, the shape quality factor Q is defined as a ratio of minimal and maximal charge density over the quadrilateral. It is shown by numerical experiment that acceptable overall solution can be obtained by using quadrilaterals of very low Q (e.g., $Q = 1/100$). However, in order to minimize the number of unknowns needed in the analysis quadrilaterals of relatively high Q should be used (e.g., $Q > 1/2$). If such quadrilaterals are used for modeling of general structures, the number of unknowns is almost halved when compared with modeling by triangular doublets.

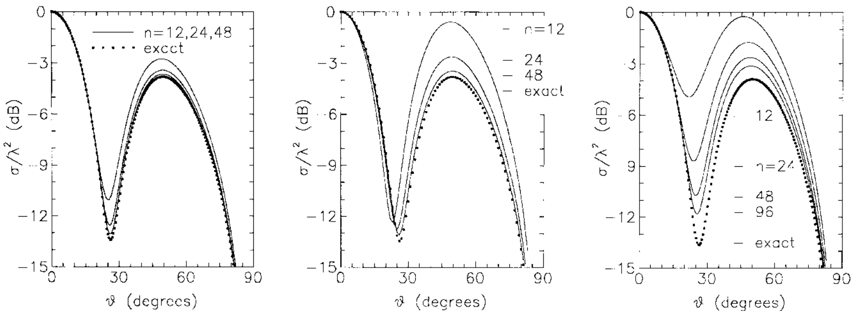

Fig. 5. Bistatic RCS of strip scatterer (normalized with maximum value) in the plane $\varphi = 0°$ versus angle θ. (A long scatterer edge of length $l = 3\lambda/2$ coincides with z axis and a short scatterer edge of length $w = l/50$ coincides with y axis. The scatterer is excited normally by an incident plane wave, which is polarized along a scatterer length.) Results are shown for different number of patches n for both formulations and for two shape quality factors: (a) exact formulation $Q = 1/2$; (b) approximate formulation $Q = 1/2$; and (c) exact formulation $Q = 1/10$.

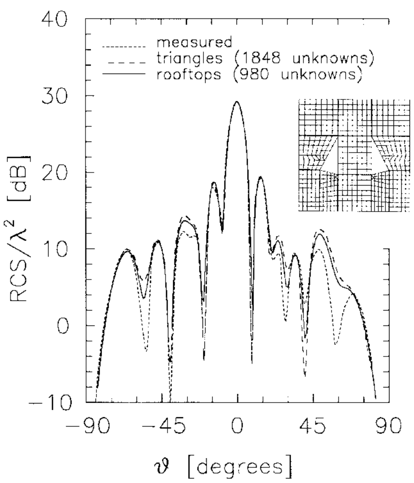

Fig. 6. Monostatic RCS of scatterer shown in the inset (normalized by wavelength squared) in the plane $\varphi = 90°$ versus angle θ. (The square plate scatterer of size $3\lambda\times$ 3λ with two triangular holes is situated in xOy plane with y axis along the longest hole edge. Incident electric field has only θ component.) Results obtained by proposed basis functions are compared with measured and theoretical results taken from [13].

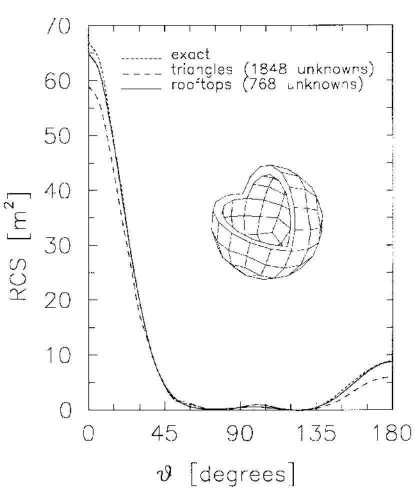

Fig. 7. Bistatic RCS of the radome shown in the inset in the plane $\varphi = 0°$ versus angle θ. (Inner and outer radii of random, dielectric constant, and operating frequency are $a = 0.5$ m, $b = 0.6$ m, $\varepsilon_r = 4$, and $f = 0.3$ GHz.) The scatterer is centered at the origin and is illuminated by an x-polarized z-traveling plane wave. Results obtained by proposed basis functions are compared with exact and theoretical results taken from [14].

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