Application of Discrete Periodic Wavelets to Measured Equation of Invariance

Yao-Wu Liu, Kenneth K. Mei, *Fellow, IEEE*, and Edward Kai-Ning Yung, *Senior Member, IEEE*

***Abstract*—Recently, the wavelet expansions have been applied in field computations. In the frequency domain, the application is focused on the thinning of matrices arising from the method of moment (MoM) [1]–[4]. The thinning of matrices can best be done by the measured equation of invariance (MEI) [5], which provides sparsity almost without sacrificing accuracy [6] in that the boundary equation it entails is convertible to that of the MoM. The real power of the wavelet expansions is to give high resolution in convolution integrals. High resolution is also needed in the process of finding the MEI coefficients, which are obtained via an integration process almost identical to that of the MoM. In this paper, it is shown that when the fast discrete periodic wavelets (FDPW) are used as metron currents in the MEI method, the resolutions of the MEI coefficients are improved at high-frequency computations or at geometric extremities. The level of sparsity of the MEI is much more favorable than that achievable by the thinning of MoM matrix using the wavelet expansions. The role of FDPW in the MEI happens to be more fitting than its place in the MoM.**

***Index Terms*—Measured equations of invariance, wavelets.**

I. Introduction

RECENTLY, wavelets have been used as base functions in the boundary integral equations, the objective is the thinning of the method of moments (MoM) matrices [1]–[4]. For the scattering problem of a perfect conduct circular cylinder with its diameter $d = 9\lambda$, the most favorable level of thinning so obtained is a sparsity of 11.5%, and it is done by a substantial sacrifice on the accuracy of the solution [7]. That level of thinning is still far behind the level of sparsity achievable by the measured equation of invariance (MEI), which, for a simple problem of the same circular cylinder, the sparsity of the MEI is 2.3%. For a much larger cylinder of diameter $d = 4,000\lambda$, it has been demonstrated [8] that the number of matrix element in the MEI is only 0.015% of a corresponding full matrix.

The real power of the wavelet expansions is in the improving of the resolution of the convolution integrals. So, its ability could be better utilized in the area of improving the accuracy of calculation rather than thinning the matrix. The MEI method is one where the matrix is already very sparse, but there is plenty of room to improve resolution of the matrix elements. Thus, the wavelet expansions can play a role fitting its ability in the MEI method.

Manuscript received November 15, 1996; revised May 23, 1997. This work was supported in part by the City University of Hong Kong under Strategic Research Grant 7000665–570, by the Hong Kong Research Grant Council CERG 9040237, and by the City University of Hong Kong Strategic Research Grant SRG 700–412 and 595–570.

The authors are with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong.

Publisher Item Identifier S 0018-926X(98)09732-4.

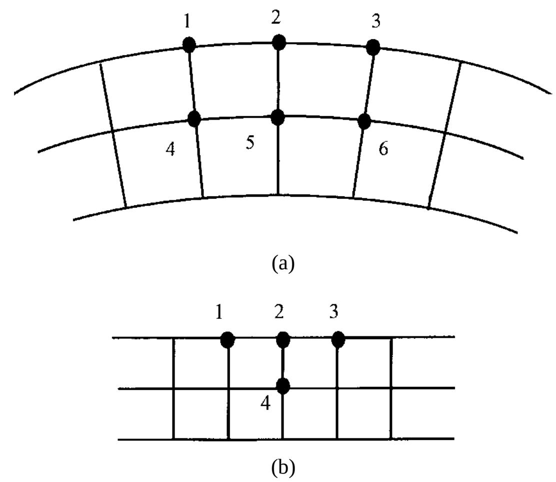

Fig. 1. (a) Six-node subsection mesh for circular cylinder. (b) Four-node subsection mesh for rectangular cylinder.

In this paper, the fast discrete periodic wavelets (FDPW) are used as metron currents in the MEI method. The effect is improved resolution of the MEI coefficients, which has direct impact on the accuracy of the calculated results. The improvement is most visible at high frequencies or at geometric extremities where some fine details of the results may be blurred due to insufficient resolution when the sinusoidal metron currents are used. However, for a cylinder of small radius or regular geometric scatterer, the FDPW metrons give almost same the results as the sinusoidal metrons.

II. The MEI Method

For the purpose of present discussion, we shall briefly describe the MEI procedures in the followings. The MEI is used at a mesh boundary to find the terminating equations, such as shown in Fig. 1. Instead of deriving the equation, the coefficients $a_i's$ of

$$\sum_{i=0}^{N-1} a_i\phi_i = 0 \tag{1}$$

are found by substituting ϕ_i with a set of solutions of the wave equations ϕ_i^n, which share the same geometry of the original problem. Those solutions ϕ_i^n are termed “measuring functions.” They are found by the following integrals:

$$\phi^n(\mathbf{r}) = -\frac{k\eta}{4}\int_c J^n(\mathbf{r})H_0^{(2)}(k|r-r'|)\,\mathrm{d}\ell' \tag{2}$$

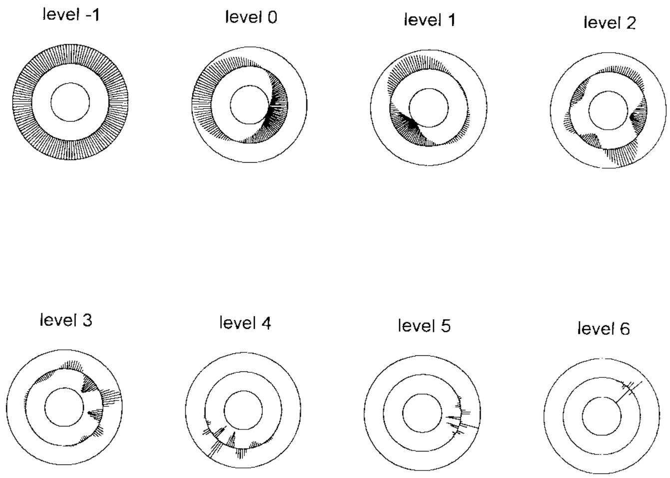

Fig. 2. Daubechies periodic wavelets on a discrete circle with node 2^7 and filter length 20.

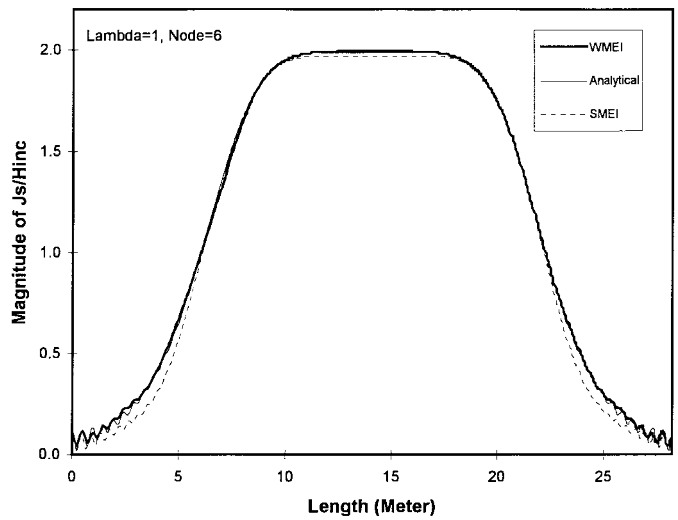

Fig. 3. TE surface current comparison for a circular cylinder of diameter $d = 9\lambda$.

where c is the surface of the target. J^n is the surface electric or magnetic currents of the target. The currents in (2) are called "metrons," and a linearly independent set of them should give a linearly independent set of measuring functions. $(N-1)$ or more metrons are sufficient to provide enough equations to solve for the MEI coefficients a_i. The question arises as to what functions are most appropriate to be metrons. In principle, any continuous function on c, which resembles an induced current density can be used as a metron. Without any further guidance, sinusoidal functions

$$J^n(l) = 1, \quad \cos\frac{2n\pi l}{L}, \quad \text{and} \quad \sin\frac{2n\pi l}{L} \qquad (n = 1, 2, 3, \cdots) \quad (3)$$

have been used as metrons, where l is the length along c measured from an arbitrarily chosen reference point, and L is the total length of the circumference of the target. Those are convenient functions for metrons, but they may not be the best. When the target is small the metrons of (3) serve quite well. But, when the size of the target increases, for example, a circular cylinder of diameter $d = 9\lambda$, the details of the induced current densities at the shadow side of the cylinder for the TE case, shown in Fig. 3, are blurred. One suggestion is that the inaccuracy is a disproof of the postulate of invariance [9] and another is that it is merely a lack of resolution of the equations that produces the MEI coefficients [10]. If the wavelets are able to improve the resolution of the integral of (2), they should be better candidates than the sinusoidal metrons of (3).

III. Fast Discrete Periodic Wavelets

There are many ways to generate wavelets [11]. For convenience, we have chosen the FDPW algorithm of Getz [12] and Vetterli [13] to generate orthogonal discrete periodic wavelets over the surface of a target for the MEI coefficients. In the following, we shall briefly describe the filter bank algorithms to create a set of the orthogonal and periodic wavelets.

To construct a level k, shift q wavelets $[\psi_n^{k,q,p}]$ (level ranges from 0 to $p-1$, shift ranges from 0 to $2^k - 1$), $n \in Z_p = \{0, 1, 2, \cdots, 2^p - 1\}$, at base level p one sets $w_n^k = \delta_{n,q}$, $n \in Z_p = \{0, 1, 2, \cdots, 2^p - 1\}$, with $f^k = 0_k$ $\{0_k$ denotes the zero vector in $\ell^\infty(Z_k)\}$, and $w^m = 0_m$ $\{0_m$ denotes the zero vector in $\ell^\infty(Z_m)\}$ for $m \in \{k+1, \cdots, p-1\}$ and perform the following inverse discrete wavelet transform:

$$\psi^{k,q,p} = L_p^* L_{p-1}^* \cdots L_{k+2}^* H_{k+1}^* w^k \quad (4)$$

where the filter sequences $[L_k^*]$ and $[H_k^*]$, $k = \{1, 2, \cdots, p\}$, denote the adjoints of low-pass filter $[L_k]$ and high-pass filter $[H_k]$.

Fig. 2 shows an example of the periodic discrete wavelets produced by Daubechies filters of length 20 and base level seven with shift zero.

It should be noticed that there exits a length matching problem between the wavelet support length 2^p and the target contour length n. It requires $n = 2^p$ for $\lambda/h \approx 20$, which is a ratio of the incident wave wavelength λ to the discrete step size h.

IV. Numerical Results

The induced current densities on the surface of a conducting circular cylinder of diameter $d = 9\lambda$ with $\lambda/h = 18$ (h is discrete step size), mesh layer $= 2$, and the MEI subsection node $N = 6$, are recalculated using the FDPW with Daubechies filters of length 20 as metrons and the results of an 0° incident TE plane wave are also shown in Fig. 3 with comparison to analytical result and sinusoidal MEI's result. It is clear that the details of the oscillation of the current at the back side of the cylinder is now accurately calculated.

Another case we have tested is the scattering of an 0° incident TE plane wave by a thin metal plate with 12.6λ length and 0.05λ width. It is shown that when the sinusoidal metrons are used, there are significant amount of errors, and when the FDPW with Daubechies filters of length 20 are used as metrons the improved resolution of the MEI equations almost eliminate all the errors, Fig. 4. The CPU time of the wavelet MEI in a SunSparc workstation is 12 s, which is identical to the CPU time of the sinusoidal MEI. The reason is only

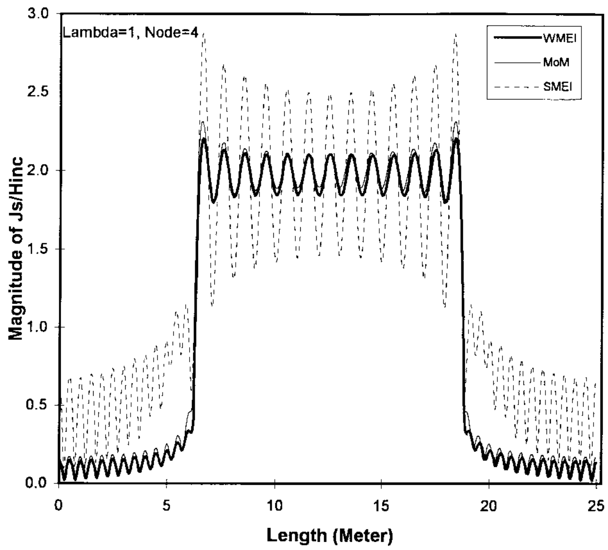

Fig. 4. TE surface current comparison for a thin plate.

five wavelets with the Daubechies filters of length 20 needs to be generated. So, comparing to total CPU time for solving whole problem, the time for generating five wavelets is almost negligible. However, the CPU time of the MoM to solve the same problem needs 82 s which is about seven times more than the wavelet MEI.

V. Conclusion

Sinusoidal functions as metrons serve quite well for the MEI method, but in several cases, especially when the objects are large or the geometry is extreme, they fail to achieve sufficient resolution to generate accurate MEI coefficients. The fast discrete period wavelets are shown to be able to provide the needed high resolution to overcome such difficulties.

References

[1] B. Z. Steinberg and Y. Leiviatan, "On the use of wavelet expansions in the method of moments," *IEEE Trans. Antennas Propagat.*, vol. 41, pp. 610–619, May 1993.

[2] K. F. Sabet and L. P. B. Katehi, "Analysis of integrated millimeter-wave and submillimeter-wave waveguides using othonormal wavelet expansions," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 2412–2422, Dec. 1994.

[3] J. C. Goswami, A. K. Chan, and C. K. Chui, "On solving first-kind integral equations using wavelets on bounded interval," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 614–622, June 1995.

[4] K. F. Sabet and L. P. B. Katehi, "An study of dielectric resonator based on two dimensional fact wavelet algorithm," *IEEE Microwave Guided Wave Lett.*, vol. 6, pp. 19–21, Jan. 1996.

[5] K. K. Mei, R. Pous, Z. Q. Chen, Y. W. Liu, and M. Prouty, "The measured equation of invariance: A new concept in field computation," *IEEE Trans. Antenna Propagat.*, vol. 42, pp. 202–214, Mar. 1994.

[6] Y. L. Chow, Y. W. Liu, K. K. Mei, K. M. Luk, and K. N. Yung, "The MEI method—A theoretical justification," in *URSI Int. Conf.*, Baltimore, MD, July 1996, p. 52.

[7] G. Wang, "A Hybrid wavelet expansion and boundary element analysis of electromagnetic scattering from conducting objects," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 170–178, Feb. 1995.

[8] Y. W. Liu, K. K. Mei, K. M. Luk, and K. N. Yung, "Numerical computation of scattering by very large cylinder," in *IEEE Antennas Propagat. Int. Symp.*, Baltimore, MD July 1996, pp. 946–949.

[9] J. Jevtic and R. Lee, "A theoretical and numerical analysis of the measured equation of invariance," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 1097–1105, Aug. 1994.

[10] K. K. Mei and Y. W. Liu, "Comments on 'A theoretical and numerical analysis of the measured equation of invariance'," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 1168–1171, Oct. 1995.

[11] I. Daubechies, *Ten Lectures on Wavelet*. Philadelphia, PA: SIAM, 1992.

[12] N. H. Getz, "A perfectly inevitable, fast, and complete wavelet transform for finite length sequences: The discrete periodic wavelet transform," *Proc. SPIE Int. Soc. Opt. Eng.*, vol. 2034, July 1993.

[13] M. Vetterli and C. Herley, "Wavelet and filter banks: Relationship and new results," in *Proc. Int. Conf. Acoust., Speech, Signal Processing*, Albuguerque, NM, Apr. 1990, pp. 1723–1726.

Yao-Wu Liu received the M.S. and Ph.D. degrees from the University of Electronic Science and Technology of China in 1981 and 1984, respectively.

He was a Research Associate with the Department of Electrical Engineering, University of Utah, Salt Lake City, from April to November 1986 and a Research Scientist with the Department of Electrical Engineering, State University of New York at Binghamton, from May to August in 1993. From 1986 to 1995, he was a Research Scientist with the Department of Electrical Engineering and Computer Science, University of California at Berkeley. Since March 1995 he has been a Research Assistant Professor at the Department of Electronic Engineering, City University of Hong Kong. His research interests include electromagnetic theory and numerical computational methods (time-domain finite-difference, finite element, integral equation methods, etc.) to solve electromagnetic problems such as scattering, antennas, microwave circuits, very large-scale integration (VLSI) interconnects, and multichip module packagings.

Kenneth K. Mei (S'61–M'63–SM'76–F'79) received the B.S.E.E., M.S., and Ph.D. degrees in electronic engineering from the University of Wisconsin, Madison, in 1959, 1960, and 1962, respectively.

He joined the faculty of the Department of Electrical Engineering and Computer Science of the University of California, Berkeley, in 1962, where he was a Professor from 1972 to 1994. Since 1994 he has been a Professor in the Department of Electronic Engineering, City University of Hong Kong. He began his research in electromagnetics in the area of computation, including the method of moments, finite element/difference, hybrid methods time-domain methods and most recently, the measured equation of invariance.

Dr. Mei received the Best Paper Award and Honorable Mention of the Best Paper Award in 1967 and 1974, respectively, from the IEEE Antennas and Propagation Society. He is a member of RSI/USNC.

Edward Kai-Ning Yung (M'85–SM'85) was born in Hong Kong. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from the University of Mississippi, University, MS, in 1972, 1974, and 1977, respectively.

After graduation, he worked briefly in the Electromagnetic Laboratory, University of Illinois at Urbana-Champaign. He returned to Hong Kong in 1978 and began his teaching career at the Hong Kong Polytechnic University, Hong Kong. He joined the newly established City University of Hong Kong, Kowloon, in 1984 and was instrumental in setting up a new Department of Electronic Engineering. He was promoted to Full Professor in 1989. He is currently the Head of Department of Electronic Engineering and the Director of the Telecommunications Research Center. He is the author of more than 130 papers in areas of antenna design, computational techniques, and microwave devices. He is actively engaged in research in microwave devices and antenna design for wireless communications.

Dr. Yung was awarded one of the first two personal chairs in the City University of Hong Kong in 1994.