A Novel Exact Two-Point Field Equation (2PFE) for Solving Electromagnetic Scattering Problems

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***Abstract*— The finite-difference (FD) method is a basic technique for solving differential equation. The disadvantage of it for electromagnetic (EM) problems of an open region is that the mesh needs to be terminated with the application of a proper boundary condition. In this paper, a novel exact two-point field equation (2PFE) is derived from rigorous analysis of the radiation field and it is proposed to be used as the termination boundary condition (BC) for solving EM scattering problems in the open region by the iterative FD method. This 2PFE-BC approaches its exact solution through the iteration process and, at the same time, the scattered field and the induced current density approach their exact solutions. The novel 2PFE is simple in concept and easy to apply. The validity of the 2PFE and the iterative FD method has been tested. Several two-dimensional (2-D) scattering problems have been successfully solved. The results agree very well with those obtained by method of moments (MoM) or measured equation of invariance (MEI).**

***Index Terms*— Boundary condition, finite-difference methods, mesh termination.**

I. Introduction

THE NUMERICAL simulation of electromagnetic (EM) scattering and radiation from a two- or three-dimensional (2-D or 3-D) object has been established [1]–[10] and it has been playing an increasingly important role in EM field theory and applications. Most of the numerical methods are based on mathematical model of differential equations or integral equations. To solve these equations, the space is mapped onto a grid, and the solution is sampled at the grid points. Then a numerical solution is found that represents the exact solution as accurately as possible.

Finite difference (FD) and finite element (FE) are two basic methods in the analysis of EM wave propagation problems. By FD or FE, the unbounded spatial domain needs to be terminated by an artificial boundary in order to make the computational domain finite; and boundary conditions are required for the termination points. In [11], an exact boundary condition at the outermost boundary is developed that imposed the radiation condition in a rigorous manner. But this boundary condition is a nonlocal integral representation that relates the field variables at each point on the object's surface to those at the every other point on the same surface. Therefore, it generates a full matrix, which spoils the sparsity of the FE matrix and, hence, increases the computational cost. In [12], absorbing boundary conditions (ABC's) were proposed. Localized partial differential boundary operators were employed to keep the sparsity of the matrix equation. However, unlike the global boundary condition of [11], the distance between the object's surface and the absorbing boundary must be large enough to reduce the spurious reflection of the propagating wave. This requirement results in an increase of the computational domain. A number of different forms of ABC's and numerical implementation were shown in [12]–[14]. In [15], numerical ABC's (NABC's) are analytically discussed.

Another mostly notable and widely used method for mesh termination is measured equation of invariance (MEI) [16]–[18]. The basic form of the MEI is the FD [19]. The MEI was introduced by Mei *et al.* in 1992, as a mesh termination boundary condition for the FD method. In this method, many metrons (reasonable sources) are used to find the MEI coefficients. These MEI coefficients fix the field relation of an exterior layer point with the fields of its neighboring points. In [20]–[24], some progress on the MEI and discussions about the MEI in solving scattering or radiation problems are reported.

It is desirable to find a boundary condition simple in concept and easy to use to terminate the mesh in order to reduce the size of computational domain. Recently, Sarkar *et al.* [24]–[26] proposed an exact method for simulating boundary condition for mesh termination in FD techniques. Much effort has been made in their research. The main idea of this method, as expressed in [24], is as follows. At the beginning of the iteration, the potential is assumed to be zero on the termination mesh. Then, using this potential as a boundary condition, the potential at interior mesh points can be solved. The charge density distribution can be computed from the potential of interior mesh point. And this charge density distribution is used to evaluate the new potential on the termination mesh and now the iterative process continues. This method seems to be very desirable. But actually it cannot always work well. Although the potential on the termination mesh being used as the termination boundary conditions are generated from the Green's function, which is used to enforce the radiation condition, the iteration process can't be forced to converge to the exact solution in many cases.

In this paper, a novel and exact two-point field equation (2PFE) is derived and proposed to be used as termination boundary condition for FD mesh. We first start in Section II with the rigorous analysis of a radiation problem to define the 2PFE for two arbitrary points in space using a coefficient which we called 2PFE coefficient. The fields are obtained by the integration of the source over the radiator's surface, using the property of Green's function. In general, the 2PFE

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is exact for arbitrary two points in space and it sets up for these two points, a simple and clear relation between: 1) the source distribution; 2) the geometrical information of the radiator; and 3) the radiation fields. Second, the 2PFE is proposed to be used as the boundary condition to terminate the FD mesh in an exact manner when solving a scattering problem. For a scattering problem, the induced current density and the scattered field are unknown. So is the 2PFE coefficient. But actually, the induced current density, the scattered field and the 2PFE coefficient can be obtained by an iterative process simultaneously. Section III demonstrates the iterative process and the FD analysis of two-dimensional (2-D) scattering problems with the use of the 2PFE as the mesh-termination boundary condition. An initial source value needs to be set for the first iteration and the results are used as a new trial value. This process is repeated until the result reaches the required accuracy. Through the iterative process, the 2PFE boundary condition approaches to its exact solution and, simultaneously, the induced current density and the scattered field of the scattering problem approach their exact solutions.

The 2PFE can be applied to arbitrary two points in space so it can terminate the mesh at a position very close to the object's surface. This advantage results in the reduction of unknown numbers. In Section III, several scattering problems have been solved in order to test the validity of the 2PFE and the iterative FD method. Corresponding results by method of moment (MoM) [3] or the measured equation of invariance (MEI) are given for comparison. Very good agreement can be observed from these results.

II. The Two-Point Field Equation

Let's consider the boundary value problem of a vector EM field $\vec{\Phi}(\vec{r})$ in open space domain $\mathcal{D}$ bounded by $(S_o + S_e)$. Let $\mathcal{L}$ be an operator, and the mathematical model for this problem can be expressed as

$$\mathcal{L}\Big(\vec{\Phi}(\vec{r})\Big) = 0 \qquad \vec{r} \in \mathcal{D} \tag{1a}$$

$$\mathcal{B}\Big(\vec{\Phi}(\vec{r})\Big) = \begin{cases} \vec{f}(\vec{r}') & \vec{r}' \in S_o \\ 0 & \vec{r} \to \infty,\, S_e. \end{cases} \tag{1b}$$

S_o is the object boundary contour, S_e is the exterior boundary contour. $\mathcal{B}$ is also an operator and $f(\vec{r}')$ is the source or excitation term. Equation (1b) is the boundary condition which may also be a differential equation such as Dirichlet, Neumann, or radiation condition. For the case of EM radiation (or scattering) problem, $\vec{\Phi}(\vec{r})$ is the field radiated by a current source $\vec{J}(\vec{r}')$ on the object. Equation (1a) can usually be rewritten as

$$(\nabla^2 + k^2)\vec{\Phi}(\vec{r}) = 0 \qquad \vec{r} \in \mathcal{D}. \tag{2}$$

Here we consider only the basic case of scalar field. Let $\Phi(\vec{r}_1)$ and $\Phi(\vec{r}_2)$ be the scalar fields at two arbitrary points $\vec{r}_1$ and $\vec{r}_2$ in space $\mathcal{D}$, respectively, as shown in Fig. 1. These two scalar fields can be expressed with the source $J(r')$ as

$$\Phi(\vec{r}_1) = \int_c J(\vec{r}')G(\vec{r}_1,\, \vec{r}')\, dl' \tag{3a}$$

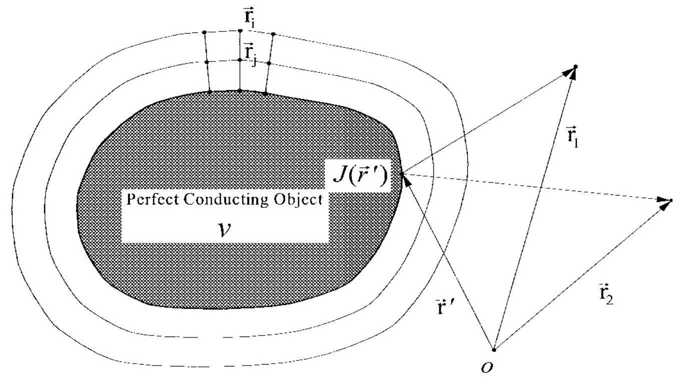


Fig. 1. Geometry of EM scattering problem and conformal FD mesh.

and

$$\Phi(\vec{r}_2) = \int_c J(\vec{r}')G(\vec{r}_2,\, \vec{r}')\, dl'. \tag{3b}$$

$G(\vec{r},\, \vec{r}')$ is the Green function. And a scalar coefficient $A(\vec{r}_1,\, \vec{r}_2,\, \vec{J})$ is used to specify the field relation of these two points as

$$\Phi(\vec{r}_1) + A(\vec{r}_1,\, \vec{r}_2,\, \vec{J})\Phi(\vec{r}_2) = 0 \tag{4a}$$

or

$$A(\vec{r}_1,\, \vec{r}_2,\, \vec{J}) = -\frac{\Phi(\vec{r}_1)}{\Phi(\vec{r}_2)} = -\frac{\int_c J(\vec{r}')G(\vec{r}',\, \vec{r}_1)\, dl'}{\int_c J(\vec{r}')G(\vec{r}',\, \vec{r}_2)\, dl'} \qquad (\text{if } \Phi(\vec{r}_2) \neq 0). \tag{4b}$$

Equation (4) is designated as the 2PFE. In general, the coefficient $A(\vec{r}_1,\, \vec{r}_2,\, \vec{J})$ depends on: 1) the observation points; 2) the geometry of the scatterer; and 3) the excitation source. From (3) and (4), it can seen that $A(\vec{r}_1,\, \vec{r}_2,\, \vec{J})$ is exact if $\Phi(\vec{r}_1)$ and $\Phi(\vec{r}_2)$ are exact.

When solving a scattering problem, the 2PFE is used to express the relation of scattered field at the exterior layer points with its immediate inner neighboring points. Therefore, it can be used to terminate the FD mesh. The coefficient $A(\vec{r}_1,\, \vec{r}_2,\, \vec{J})$ can be obtained by an iterative process. In the next section, the 2PFE is applied to solve 2-D scattering problems. The convergence of coefficient $A(\vec{r}_1,\, \vec{r}_2,\, \vec{J})$ is also studied.

III. Application in FD Solution of 2-D Problems

The scattering and radiation problems are two of the most interesting problems in EM field theory and applications. When an incident wave strikes on a metallic object, it causes an induced current to flow on the object's surface and, in turn, this current radiates a scattered wave. The induced current density and the scattered wave are the unknowns to be solved. In this section, the 2PFE is applied to solve the scattering problems of some 2-D metallic bodies.

A. The Field Formulation

s consider the scalar problem of computing the induced current density on the perimeter of a 2-D perfectly conducting z-directed cylinder which is illuminated by an uniform plane wave travelling in the direction of $\vec{k}$ in free-space. Two cases of TM$_z$ and TE$_z$ incident waves are considered here

$$\begin{cases} \vec{E}^{inc} = \hat{a}_z E_0 \exp\{-i\vec{k}\cdot\vec{r}\}, & \text{TM}_z \text{ wave} \\ \vec{H}^{inc} = \hat{a}_z H_0 \exp\{-i\vec{k}\cdot\vec{r}\}, & \text{TE}_z \text{ wave} \end{cases} \tag{5}$$

where the time factor $e^{j\omega t}$ is suppressed. $\vec{k} = k_o(\hat{a}_x \cos\varphi + \hat{a}_y \sin\varphi)$ is wave number vector $k_o = 2\pi/\lambda$, $\vec{r} = \hat{a}_x x + \hat{a}_y y$ is a point in space, $(\hat{a}_x, \hat{a}_y)$ are the two unit vectors, and φ is the angle between the traveling direction $\vec{k}$ of incident wave and the x axis. Each of these two incident waves will induce a current distribution on the surface of the cylinder. For convenience, we use $\phi(\vec{r})$ to express the unknown z-directed scattered field for these two incident wave cases as

$$\phi(\vec{r}) = \begin{Bmatrix} E_z^{sc}(\vec{r}) \\ \eta_o H_z^{sc}(\vec{r}) \end{Bmatrix} = \begin{cases} \oint_c J_s(\vec{r}')G(\vec{r}\,|\,\vec{r}')\,d\ell', & \text{TM}_z \text{ wave} \\ \oint_c M_s(\vec{r}')G(\vec{r}\,|\,\vec{r}')\,d\ell, & \text{TE}_z \text{ wave} \end{cases} \tag{6}$$

where $\eta_o = \sqrt{\mu_o/\varepsilon_o}$ is the intrinsic impedance of free space. The superscript "sc" stands for scattering field. J_s is the induced electric current density on the surface of the cylinder for TM$_z$ incident wave case, whereas M_s is the induced magnetic current density on the surface of the cylinder for TE$_z$ incident wave case. The integration is taken over the contour C of the cylinder which is the cross-sectional boundary of the cylinder. $d\ell'$ is the arc length of the current filament on C. The Green's function for the 2-D radiation problems in free-space is

$$G(\vec{r}\,|\,\vec{r}') = \frac{k_0\eta_0}{4} H_0^{(2)}(k_0|\vec{r} - \vec{r}'|) \tag{7}$$

where $H_0^{(2)}(x)$ is the Hankel's function of second kind and of order zero. $\phi(\vec{r})$ must satisfy (1) or (2). That is

$$(\nabla_t + k^2)\phi(\vec{r}) = 0 \tag{8a}$$

$$\frac{\partial\phi(\vec{r})}{\partial r} + jk\phi(\vec{r}) = 0 \qquad \text{when } \vec{r} \to \infty \tag{8b}$$

$$\vec{E}_{\tan}(\vec{r}') = \vec{E}_{\tan}^{inc}(\vec{r}') + \vec{E}_{\tan}^{sc}(\vec{r}') = 0 \tag{8c}$$

where $\nabla_t = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplace operator. Equation (8b) is the Sommerfeld radiation condition for scattered field at the infinity ($\vec{r} \to \infty$). Equation (8c) is the boundary condition on the surface of the cylinder. Let E and H represent the total electric and magnetic field intensity, respectively. Therefore,

$$\begin{cases} E = E_z^{inc}(\vec{r}) + \phi(\vec{r}), & \text{TM}_z \text{ wave} \\ H = H_z^{inc}(\vec{r}) + \dfrac{\phi(\vec{r})}{\eta_o}, & \text{TE}_z \text{ wave.} \end{cases} \tag{9}$$

With the total E and H in space domain $\mathcal{D}$, the induced current densities on the object surface are therefore expressed as

$$\vec{J}(\vec{r}') = \begin{cases} -\dfrac{\hat{a}_z}{jk_0\eta_0}\dfrac{\partial E(\vec{r}')}{\partial n} \\ \quad = -\dfrac{\hat{a}_z}{jk_0\eta_0}\dfrac{\partial}{\partial n}\left[E_z^{inc}(\vec{r}') + \phi(\vec{r}')\right], & \text{TM}_z \text{ wave} \\ \hat{a}_t H(\vec{r}') = \hat{a}_t\left[H_z^{inc}(\vec{r}') + \dfrac{\phi(\vec{r}')}{\eta_0}\right], & \text{TE}_z \text{ wave.} \end{cases} \tag{10}$$

Here $\hat{a}_t$ is the unit vector tangential to the surface of the cylinder.

B. The FD Matrix Equation

To use FD method to solve the scattered field problem, the space around the scatterer is discretized as FD mesh, as shown in Fig. 1. For the points between the cylinder surface and the exterior layer, (8a) is employed and is sampled into a standard five-point FD equation. For points on the object's surface, the boundary condition (8c) is applied. And (4) is applied for exterior points on the termination mesh layer. Let $\vec{r}_i$ be a point on the exterior mesh layer and $\vec{r}_j$ is its immediate inner mesh point. The 2PFE for these two points is

$$\Phi(\vec{r}_i) + A(\vec{r}_i, \vec{r}_j, \vec{J})\Phi(\vec{r}_j) = 0. \tag{11}$$

Finally, using the FD equations, boundary conditions on the surface of the cylinder, and (11) for the exterior layer points, the unknown scattered fields at every mesh point can be summarized into a complex linear sparse matrix equation as

$$[M][\Phi^{sc}] = [s]. \tag{12}$$

In (12), the vector $[\Phi^{sc}]$ is the unknown scattered field term, which represents E^{sc} fields for TM$_z$ case or $\eta_o H^{sc}$ field for TE$_z$ case, respectively. The vector $[s]$ contains the source terms of excitation, whose elements are usually zero except for points on the object's surface. The constructed matrix $[M]$, related to the whole EM problem, is a very sparse matrix. Most of the elements in $[M]$ are zero elements. The nonzero elements are mainly concentrated in the diagonal band.

If the 2PFE coefficient $A(\vec{r}_1, \vec{r}_2, \vec{J})$ at each exterior point is known, the unknown scattered field for each point could be obtained only by solving (12). But in fact, same as the scattered field $[\Phi^{sc}]$ and the induced current density $J(\vec{r}')$, the coefficient $A(\vec{r}_1, \vec{r}_2, \vec{J})$ for the termination layer is still unknown up to now. Next, we will discuss how to obtain the exact solution of $J(\vec{r}')$, $[\Phi^{sc}]$ and $A(\vec{r}_1, \vec{r}_2, \vec{J})$ simultaneously by an iterative process.

C. The Iteration Process

To find the solutions of $J(\vec{r}')$, $[\Phi^{sc}]$, and $A(\vec{r}_1, \vec{r}_2, \vec{J})$ of a scattering problem, an iterative process is used. This process starts with setting a trial value for the induced current density $J^0(\vec{r}')$ for the first iteration. Let $J^p(\vec{r}')$ be the value of the current density obtained after p iteration, then the corresponding 2PFE coefficient $A^p(\vec{r}_1, \vec{r}_2, \vec{J})$ is calculated

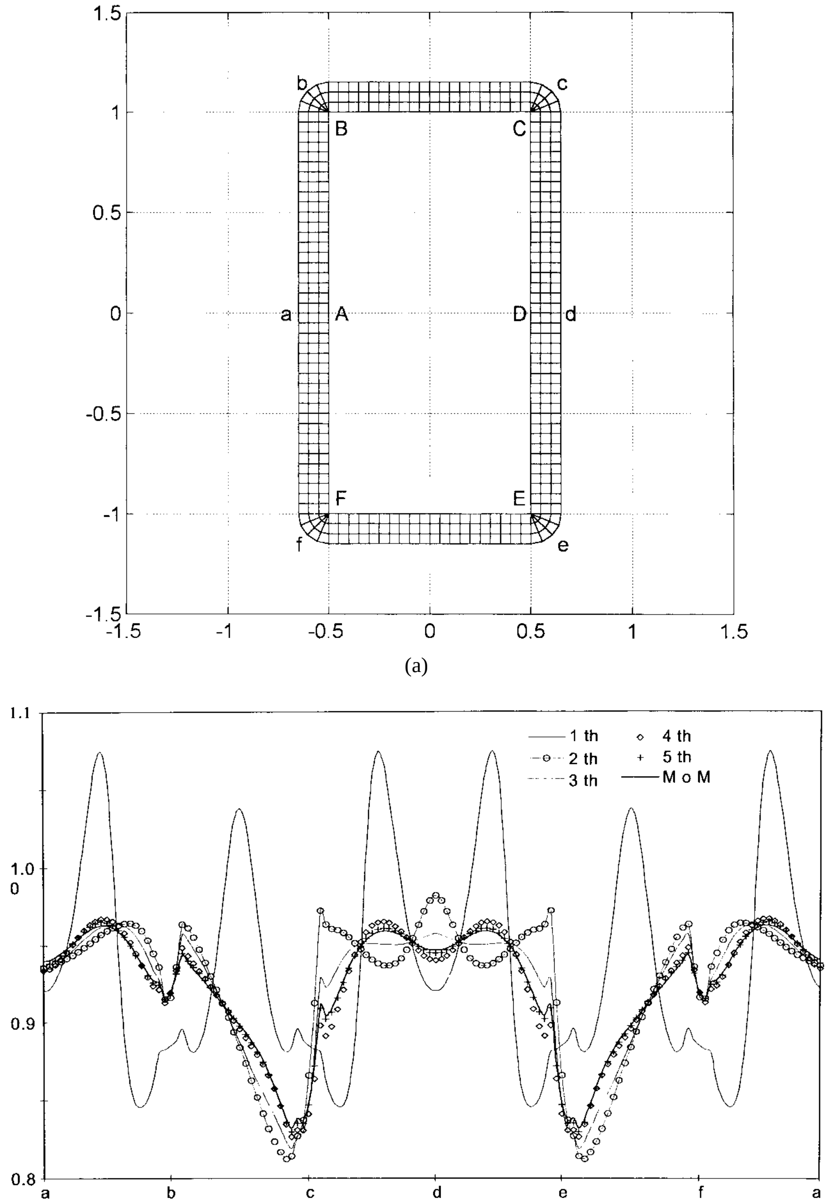


Fig. 2. Convergence of $A(\vec{r}_1, \vec{r}_2, \vec{J})$ and $\vec{J}$ obtained by 2PFE and iteration method. The scatterer is a conducting rectangular cylinder with dimensions $1\lambda\times\ 2\lambda$ and $\varphi = 180°$. (a) FD mesh. (b) Real part of $A(\vec{r}_1, \vec{r}_2, \vec{J})$.

from (4) as

$$A^p(\vec{r}_1, \vec{r}_2, \vec{J}) = -\frac{\Phi^p(\vec{r}_1)}{\Phi^p(\vec{r}_2)} = -\frac{\oint_c J^p(\vec{r}')G(\vec{r}_1 \,|\, \vec{r}')\, d\ell'}{\oint_c J^p(\vec{r}')G(\vec{r}_2 \,|\, \vec{r}')\, d\ell'} \tag{13}$$

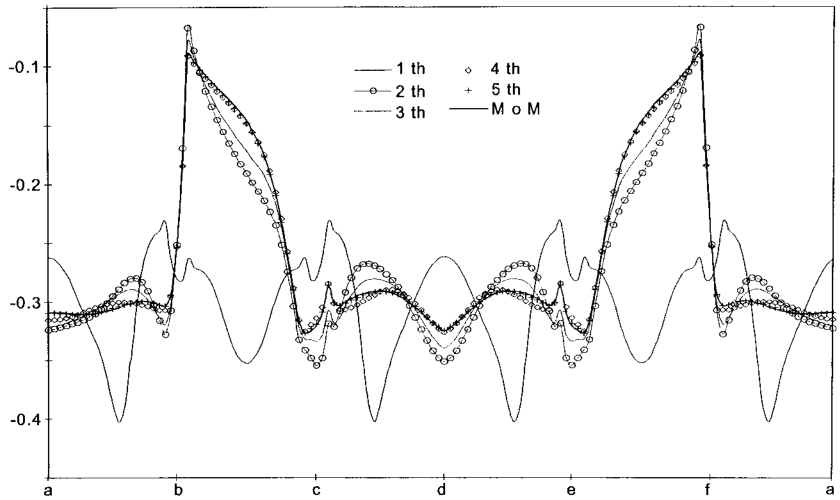
which is used as a new boundary condition for the exterior layer points to determine the field value $\phi(\vec{r})$ in the $(p+1)$th iteration as

$$\Phi^{p+1}(\vec{r}_1) + A^p(\vec{r}_1, \vec{r}_2, \vec{J})\Phi^{p+1}(\vec{r}_2) = 0. \tag{14}$$

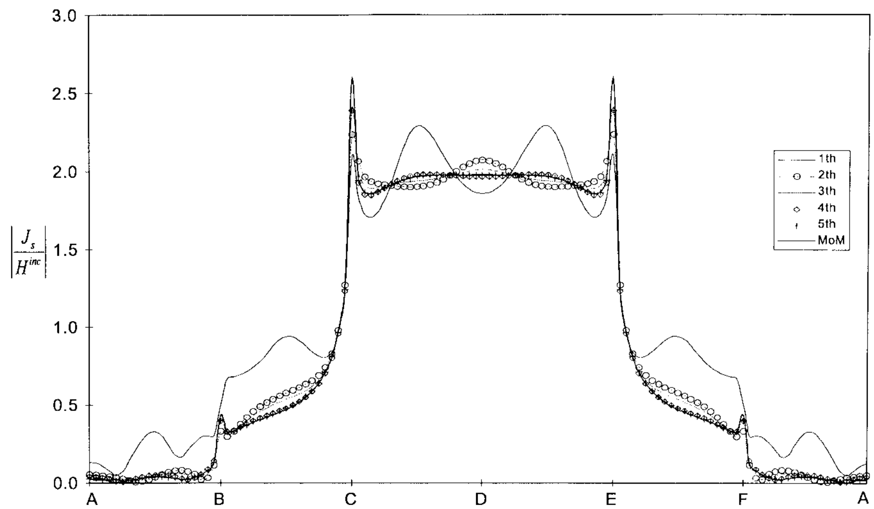
Then the FD matrix equation (12) can be set up for the $(p+1)$th iteration process. Solving this FD equation, a new value of scattered field $\phi^{p+1}(\vec{r})$ is obtained. Consequently, a new value $J^{p+1}(\vec{r}')$ can be obtained from (10). Using $J^{p+1}(\vec{r}')$ as a new trial current and repeating the above process iteratively, a steady solution can finally be obtained. The termination of the iterative process depends on the required accuracy.

Unlike [24]–[26], the termination boundary condition involves two points and the coefficient $A(\vec{r}_1, \vec{r}_2, \vec{J})$ in (4) is expressed as a ratio of the fields at these two points so that it makes the field relation rigorously, not merely directly using the field value calculated by integration on the termination mesh layer as a termination boundary condition. This ratio of fields filters out the ill-manner effects due to amplitude or phase of the trial source.

It is helpful to explain more about the 2PFE boundary condition and the iterative FD method. $\phi(\vec{r})$, $J(\vec{r}')$ and



(c)



(d)

Fig. 2. (*Continued.*) Convergence of $A(\vec{r}_1, \vec{r}_2, \vec{J})$ and $\vec{J}$ obtained by 2PFE and iteration method. The scatterer is a conducting rectangular cylinder with dimensions $1\lambda \times 2\lambda$ and $\varphi = 180^\circ$. (c) Imaginary part of $A(\vec{r}_1, \vec{r}_2, \vec{J})$. (d) The induced current density $\vec{J}$.

$A(\vec{r}_1, \vec{r}_2, \vec{J})$ are actually the three related parameters of a scattering problem. From (4), it can be seen that the 2PFE coefficient $A(\vec{r}_1, \vec{r}_2, \vec{J})$ is related to the source $J(\vec{r}\,')$ or the field $\phi(\vec{r})$. So for a scattering problem solved by the iterative FD method, approaching of the exact solution of $J(\vec{r}\,')$ or $\phi(\vec{r})$ is equivalent to approaching the exact solution of $A(\vec{r}_1, \vec{r}_2, \vec{J})$. From above it can be seen that in every iteration: 1) the information of the scatterer's geometry and the boundary condition is transferred to $\phi(\vec{r})$ by (12); 2) the information of incident wave and $\phi(\vec{r})$ is transferred to $J(\vec{r}\,')$ by (10); and 3) the information of $J(\vec{r}\,')$ and the scatterer's geometry is also transferred to $A(\vec{r}_1, \vec{r}_2, \vec{J})$ by (3) and (4). The Green's function is applied in the integration for fields to enforce the radiation condition. Therefore, $\phi(\vec{r})$, $J(\vec{r}\,')$, and $A(\vec{r}_1, \vec{r}_2, \vec{J})$ are adjusted toward their exact solutions through the iterative process by repeatedly using the incident wave, the boundary condition on the object's surface and the current integration. One may observe that the calculation of $A(\vec{r}_1, \vec{r}_2, \vec{J})$of $\Phi(\vec{r}_1)$ and $\Phi(\vec{r}_2)$ from (4b) may be impossible for points in space where one (or both) of the fields because zero. This problem can be solved by adjusting the locations of the grid points.

As an example, Fig. 2 shows the convergence behavior when the 2PFE boundary condition and the iterative FD method are applied to solve a scattering problem. The scatterer is a rectangular perfectly-conducting cylinder with dimension of $1\lambda \times 2\lambda$. The TM_z incident wave comes from normal direction. Fig. 2(a) shows the mesh of the cylinder. This FD mesh has three conformal layers with node step size of 0.05λ. The initial value of current density for the first iteration is taken as one everywhere. Fig. 2(b) and (c) shows the real part and the imaginary part of the 2PFE coefficient $A(\vec{r}_1, \vec{r}_2, \vec{J})$ in each iteration. Fig. 2(d) shows the induced current density resulted from the iterative process. The corresponding MoM solutions are also obtained for comparison. From Fig. 2(b)–(d), it can be seen that after four iterations, the 2PFE coefficient

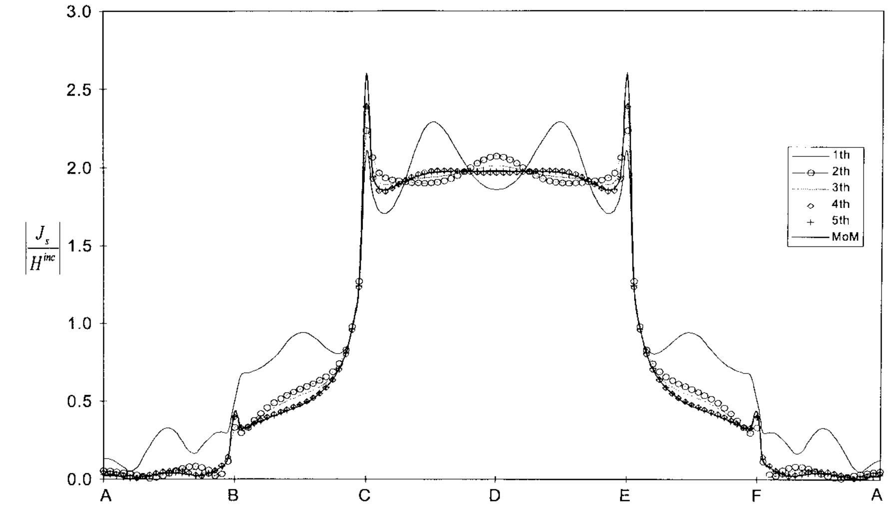


Fig. 3. Induced electric current distributions on a 2-D conducting circular cylinder (radius $=1\lambda$; normal incidence $\varphi = 180°$).

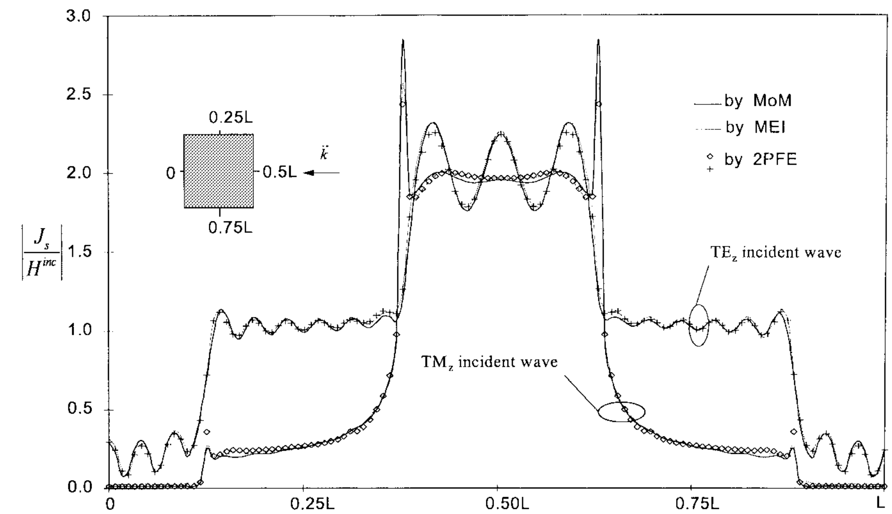


Fig. 4. Induced electric current distributions on a 2-D conducting square cylinder (dimensions: $3\lambda \times 3\lambda$; normal incidence, $\varphi = 180°$).

$A(\vec{r}_1, \vec{r}_2, \vec{J})$, and the induced current density $J(\vec{r}')$ converge to steady-state values which are almost the same as the MoM solutions.

It should be pointed out that neither (4) or (14) is an artificial termination boundary condition for the termination FD mesh layer; it is an exact field relation. No approximation has been made on deriving it. Next, we will examine the validity of this 2PFE and the iterative FD method by solving some 2-D scattering problems.

D. Numerical Results

In this section, numerical results based on the 2PFE and the iterative FD method are reported. We have tested this novel method by applying it to solve a variety of scattering problems involving 2-D perfectly conducting scatterers. Both cases of TM$_z$ and TE$_z$ wave incidence are taken into account. The induced current densities on the surfaces of scatterers with different cross-sectional shapes and different dimensions have been calculated by the iterative FD method with the use of the 2PFE as a mesh-termination boundary condition. The initial value of the current density for the first iteration is taken as one everywhere. Of course, other nonzero current density may be used without affecting the final solution. Fig. 3 shows the calculated induced current densities on a circular conducting cylinder with radius $r = 1\lambda$. $L = 2\pi\lambda$ is the perimeter of the cylinder. Fig. 4 shows the calculated induced current densities on a square-conducting cylinder of dimensions $3\lambda \times 3\lambda$. Normal incidence ($\varphi = 180°$) is studied in this case. Fig. 5 shows the calculated induced current densities on a rectangular conducting cylinder of dimensions $1\lambda \times 3\lambda$. Oblique incidence ($\varphi = 225°$) is studied in this case. Fig. 6(a) and (b) shows, respectively, the calculated induced current density for the TM$_z$ and TE$_z$ incident waves on a cavity-like conducting cylinder with dimensions $a = 4.4\lambda$, $b = 2.4\lambda$, $c = 1.7\lambda$, $d = 1.2\lambda$, $r_1 = 0.2\lambda$, and $r_2 = 0.5\lambda$. Normal incidence ($\varphi = 180°$) is studied in this case.

To check the results obtained by the 2PFE and the iterative FD method, corresponding results by MoM and MEI are also

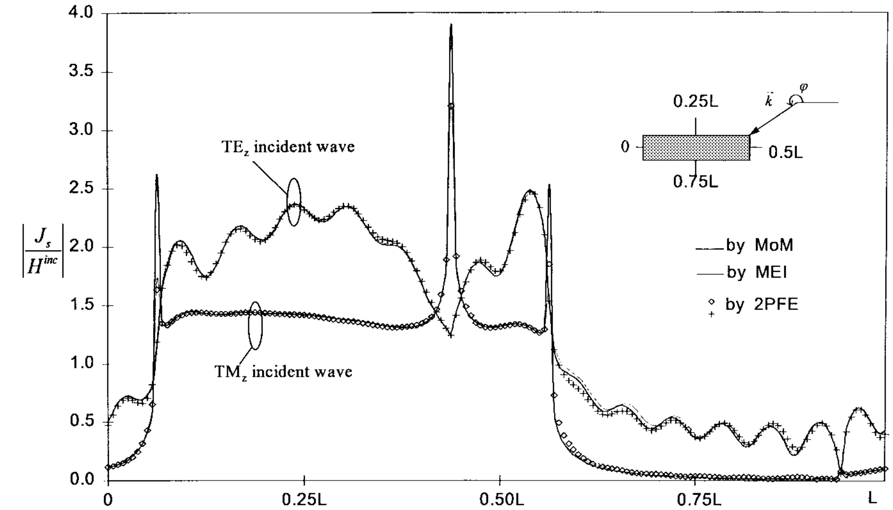
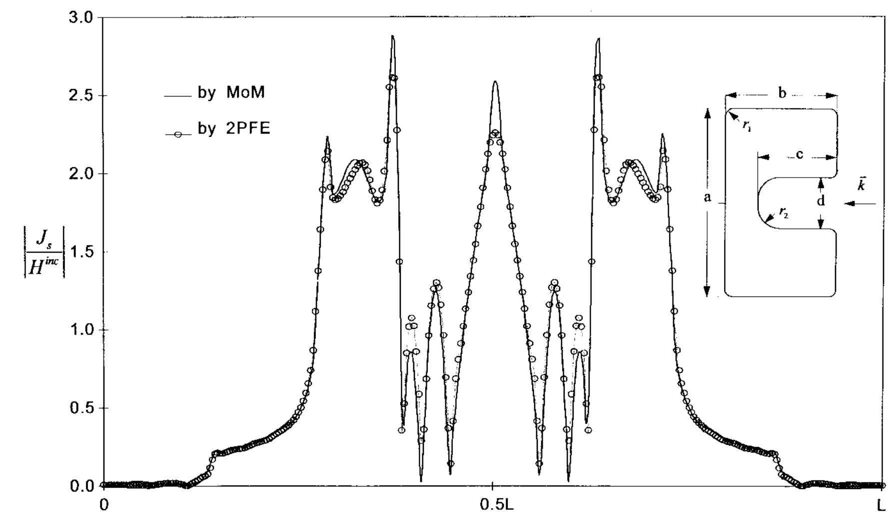
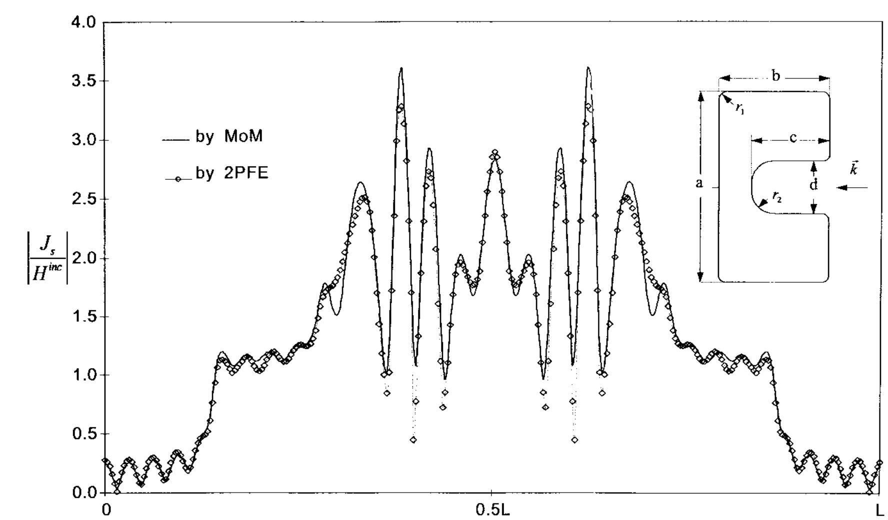


Fig. 5. Induced electric current distributions on a 2-D conducting rectangular cylinder (dimensions: $3\lambda \times 1\lambda$; oblique incidence, $\varphi = 225°$).



(a)



(b)

Fig. 6. Induced electric current distributions on a 2-D cylinder scatterer with a cavity-like indentation (dimensions: $a = 4.4\lambda$, $b = 2.4\lambda$, $c = 1.7\lambda$, $d = 1.2\lambda$, $r_1 = 0.2\lambda$, and $r_2 = 0.5\lambda$; normal incidence, $\varphi = 180°$). (a) TM$_z$ wave case. (b) TE$_z$ wave case.

calculated for comparison. It can be seen that these results agree very well. This indicates that the new method is valid for solving 2-D scattering problems. All the results given in this paper are obtained under such a mesh step size of 0.05λ and three conformal mesh layers and are carried out on a Pentium personal computer.

E. Computation Time

For MoM, the dominant part of computation time is of order $O(N^3)$ and it is spent mainly on calculating the inversion of its full matrix. Here, N is the number of unknowns. For MEI, the dominant part of computation time is on the integration process for measuring functions. The time spent on a single-metron integration is of order $O(N^2)$ [18]. But the total time of integration should take the number of metrons into account.

The proposed method has the advantage of producing sparse matrices. The dominant part of computation time is on the integration process for determining the coefficient $A(\vec{r}_1, \vec{r}_2)$. For each iteration, the major computation time is of order $O(N^2)$. The total computation time for this new method should be the time for each iteration multiplies the number of iteration required. It can be seen that the computation time for this new method is of the same order of the computation time of MEI.

IV. Conclusion

The 2PFE is simple in concept and easy to use. It is an exact field relation and can be used as the termination boundary condition for solving scattering problems by iterative FD method. The virtue of the application of the 2PFE in solving a scattering problem is that the mesh can be limited to the vicinity of the scatterer in a simple and exact manner. So the size of computational domain and the number of unknowns can be reduced and, at the same time, the iteration process could be converged easily. Furthermore, the whole problem is inferred to a sparse matrix equation, not a full matrix equation. This results in saving in both computation time and computer memory.

The 2PFE is different from the theory in [24]–[26] because it does not directly use any field value as the termination boundary condition. The 2PFE is also different from the MEI because it does not employ any postulates. Furthermore, the 2PFE involves only two points, the corresponding FD matrix is more sparse than that of MEI. The initial value of induced current density for the first iteration can be any value other than zero. The 2PFE has been used successfully for solving many 2-D scattering problems. Further study is on the extension to three-dimensional problems.

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Shiu-Ming Shum, for a photograph and biography, see p. 333 of the March 1998 issue of this TRANSACTIONS.