Periodically Slotted Dielectrically Filled Parallel-Plate Waveguide as a Leaky-Wave Antenna: E-Polarization Case

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***Abstract*—The analysis of a periodically slotted dielectrically filled parallel-plate waveguide as a leaky-wave antenna for infinite and finite periodic structures for the E-polarization case is discussed. For the analysis of the infinite periodic structure, an integral equation in which Floquet's modes and the strip electric current satisfying the edge condition are used and Bloch wave analysis using the single-slot equivalent circuit parameters are used. Integro-differential equation with moment-method and transmission-line analysis with equivalent circuit representation are used to analyze the finite N-periodic structure. Results of interest such as complex propagation constant and the radiation pattern for the finite slot number case are observed to be in good agreement with those of the infinite geometry case for the appropriately chosen values of slot numbers, slot width, period, and waveguide height.**

***Index Terms*— Bloch wave, leaky wave, parallel-plate waveguide.**

I. Introduction

Because a periodically slotted dielectrically filled parallel-plate waveguide (often called metal-strip-loaded dielectric antenna) as a leaky-wave antenna has advantages such as unidirectional radiation, narrow beam, and a beam scanning property, the structure has been the subject of several investigations.

Jacobsen [1] considered both E and H polarization cases. His results, however, are restricted to strip widths small compared to a wavelength. His method of approach consists of numerically solving the approximated transcendental equation obtained by use of Hertzian potentials under the assumption that the strip width was narrow. Another approach to the problem was employed by Collin [2]. He obtained two forms of solutions, approximate and complete solutions, using the transverse resonance method and the approximate Green's function method, respectively. His results, however, apply only to the narrow slot case. Encinar [3] suggested a technique based on a mode expansion (for the finite thickness of the strip) and point matching (for the zero thickness of strip) formulation. The efficiency of his method for the zero thickness strip in searching for the eigenvalues is observed to become poorer as either the strip width or the slot width portion of a period becomes larger than the other. Recently, the dielectric-inset waveguide leaky-wave antenna was analyzed by use of a multimode network theory [4], which was developed in [5]–[7]. In this analysis, the strip-grating discontinuity was reduced to a multiport network, the elements of which were given in closed form. Thus, this method is very efficient for solving the infinite periodic structure.

The purposes of this paper are to propose another integral equation method that gives solutions without the above constraints and to obtain an approximate method that is based on the use of the equivalent circuit representation of the single slot. The methods proposed are summarized as: 1) the unknown surface current density on the strip is expanded into a product of a series of Chebyshev polynomials and a function satisfying the edge condition and the fields in each region (guide and half-space) are expressed as a summation of the space harmonics; boundary conditions are imposed at the interfaces and a homogeneous linear equation system is obtained from which the eigenvalue (complex propagation constant) is computed and 2) a single slot on the upper plate of the parallel-plate waveguide is replaced by the equivalent circuit parameters proposed in [8] and the equivalent network of the infinite periodic structure is then obtained by use of the single-slot equivalent circuit parameters. In this representation, the mutual coupling effects of the slots are not taken into account. Bloch wave analysis [9] is applied to this equivalent network and the complex propagation constant of this infinite periodic structure is obtained.

Up until now, most works have been devoted to the characteristics of the infinite geometry and so the feeding geometry and finite effects of the structure have not been taken into account. Hence, methods for solving the finite slot number case are needed. The main objective of this paper is to consider the analysis method for this case. For this case, two different methods are proposed. First, an integro-differential equation is set up for the unknown magnetic currents over the slots (for the case of fundamental TE wave incidence); piecewise sinusoidal functions are employed for expansion and testing in which case the integrations appearing in the calculation procedure for the admittance matrix elements and the total radiated power from the slots are given in closed form. Therefore, accurate results for the radiation pattern and complex propagation constant (in an average sense) are obtained. Second, the equivalent circuit

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of the finite structure is constructed by use of the single-slot circuit parameters and then transmission-line analysis is used to determine the complex propagation constant and radiation pattern. In this paper, only E polarization will be considered, but similar procedure can be applied to H polarization [13].

II. FORMULATION

A. Infinite Periodic Structure

For the infinite periodic structure shown in Fig. 1(a), the electromagnetic fields in each region can be expressed as a summation of space harmonics. The unknown surface current density on the strip can be expanded into a product of a series of Chebyshev polynomials and a function satisfying the edge condition as follows:

$$\underline{J}(x) = \hat{z}e^{-j\beta_0 x}\sum_{l=0}^{\infty} f_l \frac{T_l(2x/a)}{\sqrt{1-(2x/a)^2}}, \quad -\frac{a}{2} \le x \le \frac{a}{2} \qquad (1)$$

where f_l are unknown coefficients, $\beta_0 = \beta - j\alpha$ is complex propagation constant to be determined, and $T_l(2x/a)$ denotes the Chebyshev polynomial of the first kind. The continuity condition of the tangential electric field and continuity and jump condition of the magnetic field due to the strip electric current are imposed at dielectric-air interface ($y = 0$) and the tangential electric field is enforced to be zero on the conductor surfaces. The resultant equation for the electric field (vanishing condition on the strip) is multiplied by $e^{j\beta_0 x}T_i(2x/a)/\sqrt{1-(2x/a)^2}$ and integrated over the strip region. The equation for the magnetic field is multiplied by $e^{j2m\pi x/p}$ and integrated over one period. After considering the orthogonality property and some algebraic manipulations, the following homogeneous matrix equation is obtained:

$$\sum_{l=0}^{\infty} f_l Z_{il} = 0, \quad i = 0, 1, 2, \cdots. \qquad (2)$$

Here

$$Z_{il} = \frac{-k_0\eta_0}{p}\sum_{n=-\infty}^{\infty}\frac{H_{ln}^{*}H_{in}}{\gamma_n} \qquad (3)$$

where

$$\gamma_n = \frac{k_{yn}}{j\tan k_{yn}t} + k_{yn0}$$

$$H_{in} = \int_{-a/2}^{a/2}\frac{T_i(2x/a)}{\sqrt{1-(2x/a)^2}}e^{-j2n\pi x/p}\,dx$$

$$k_{yn0} = \sqrt{k_0^2 - \beta_n^2}, \quad k_{yn} = \sqrt{\epsilon_r k_0^2 - \beta_n^2}$$

$$\beta_n = \beta + \frac{2n\pi}{p} - j\alpha, \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}, \quad k_0 = \omega\sqrt{\mu_0\varepsilon_0}.$$

The asterisk $(*)$ stands for the complex conjugate. Since we are interested in nontrivial solutions, the determinant of (2) must be zero. Enforcing this condition, β and α are obtained.

The far-field radiation pattern of the grounded dielectric leaky-wave antenna is calculated from the known tangential electric fields over each slot and by using the equivalence

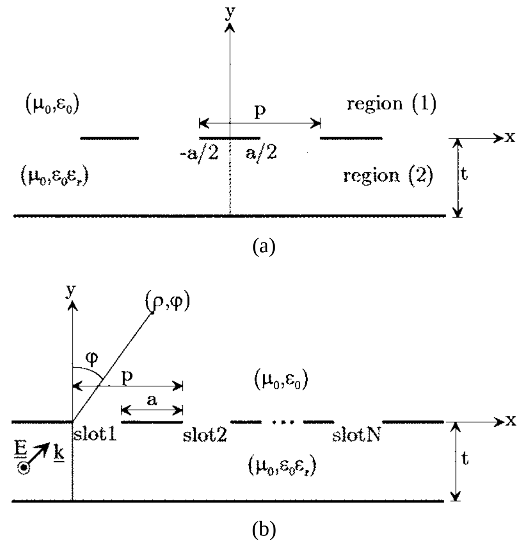

Fig. 1. Geometry of the problem. (a) Geometry of the infinite periodic structure. (b) Parallel-plate waveguide with N periodic slots.

principle. The far-zone electric field emanated from a finite section of infinite structure is given by

$$E_z \approx -\cos\varphi\left(jk_0 + \frac{1}{2\rho}\right)\sqrt{\frac{2k_0}{j\pi\rho}}e^{-jk_0\rho}\frac{\eta_0}{p}\sum_{n=-\infty}^{\infty}\sum_{l=0}^{\infty}f_l\frac{H_{ln}^{*}}{\gamma_n}$$
$$\times\frac{\sin\kappa_n d/2}{\kappa_n}e^{-j\kappa_n p/2}\frac{1-e^{-j\kappa_n pN_{\text{slot}}}}{1-e^{-j\kappa_n p}} \qquad (4)$$

where $\kappa_n = \beta_n - k_0\sin\varphi$, $d = p - a$, and N_{slot} is the total slot number in a finite section of infinite periodic structure.

B. Finite Periodic Structure

The finite geometry of the problem is shown in Fig. 1(b). A slotted parallel-plate waveguide for the TE case where the electric field is polarized in the z direction was previously reported by Chuang [10] and Cho [11]. Chuang employed piecewise sinusoidal functions for expansion and testing in the moment-method technique, and established a generalized matrix associated with the slot in closed form as opposed to numerical integration. We use Chuang's method [10] and extend his method to the N periodically slotted parallel-plate waveguide problem. $k_0\sqrt{\epsilon_r}t$ is assumed to be larger than π and smaller than 2π so that only the lowest TE mode can propagate and it is assumed that the field in an infinite parallel-plate waveguide is incident from the left onto the leaky-wave antenna and the right end of the structure is also assumed to be terminated in an infinite parallel-plate waveguide. Here, k_0 and ϵ_r have the usual meaning. In region (2), the electric field in an infinite parallel-plate waveguide incident from the left onto the leaky-wave antenna is

$$\underline{E}_{\text{inc}}(x,y) = \hat{z}e^{-j\beta_1^{\text{pp}}k_0 x}\sin\frac{\pi}{t}y \qquad (5)$$

where

$$\beta_1^{\text{pp}} = \sqrt{\epsilon_r - \left(\frac{\pi}{k_0 t}\right)^2}.$$

The desired integro-differential equation is obtained by enforcing continuity of tangential electromagnetic fields over each slot as follows:

$$\sum_{n=1}^{N} H_{x,n}^{(1)}(x;M_n) + \sum_{n=1}^{N} H_{x,n}^{(2)}(x;M_n) = H_{x,\text{inc}}(x) \tag{6}$$

where the superscript (1) and (2) refer to region (1) and (2), respectively, the subscript (x, n) refers to the x component of the magnetic field across the nth slot of total N slots and M_n $(=E_{z,n})$ is the equivalent magnetic current of the x-direction component. The two terms of the left side in (6) are given in [10]–[11].

Now expand the unknown equivalent magnetic current of the nth slot into the piecewise sinusoidal (PWS) basis function $M_n(x')$ as

$$M_n(x') = \sum_{i=1}^{M} \left\{ \frac{\sin k_0(x' - x_{n,i-1})}{\sin k_0 h} P_{n,i-1}(x') + \frac{\sin k_0(x_{n,i+1} - x')}{\sin k_0 h} P_{n,i}(x') \right\} V_{n,i} \tag{7}$$

where $P_{n,i-1}$ are pulse functions defined by

$$P_{n,i-1}(x') = \begin{cases} 1, & \text{if } x_{n,i-1} \le x' \le x_{n,i} \\ 0, & \text{otherwise} \end{cases}$$

the segment length h and the coordinate $x_{n,i}$ are given by $h = (p-a)/(M+1)$ and $x_{n,i} = (n-1)p + ih$, respectively, and $V_{n,i}$ are unknown constants. We apply Galerkin's method to (6). In Galerkin's method, the same expansion and testing functions are used. Therefore, we choose the following testing functions:

$$T_{l,j}(x) = \sin k_0(x - x_{l,j-1})P_{l,j-1}(x) + \sin k_0(x_{l,j+1} - x)P_{l,j}(x). \tag{8}$$

By substituting (7) into the integrals in (6) and multiplying (6) by each equation (8), and integrating the resultant equation over each slot, the following matrix equation is established:

$$\left[Y_{n,i}^{l,j}\right][V_{n,i}] = [I_{l,j}]. \tag{9}$$

The elements of $[I_{n,i}]$ for ith segment of nth slot can be easily evaluated as

$$I_{n,i} = 2k_0 \frac{\pi}{t} e^{-j\beta_1^{\text{PP}} k_0 x_{n,i}} \frac{\cos \beta_1^{\text{PP}} k_0 h - \cos k_0 h}{k_0^2 \left(1 - \beta_1^{\text{PP}2}\right)}. \tag{10}$$

The elements of $[Y_{n,i}^{l,j}]$ for the ith basis function of the nth slot and jth testing function of the lth slot are obtained and given by

$$Y_{n,i}^{l,j} = Y_{n,i}^{l,j,1} + Y_{n,i}^{l,j,2}. \tag{11}$$

Here

$$\begin{aligned} Y_{n,i}^{l,j,1} = \frac{k_0}{2j \sin k_0 h} \Big\{ & \left(x_{n,i}^{l,j} + 2h\right) H_1^{(2)}\left[k_0\left(x_{n,i}^{l,j} + 2h\right)\right] \\ & - 4\left(x_{n,i}^{l,j} + h\right) H_1^{(2)}\left[k_0\left(x_{n,i}^{l,j} + h\right)\right] \\ & \cdot \cos k_0 h + 2x_{n,i}^{l,j} H_1^{(2)}\left[k_0 x_{n,i}^{l,j}\right]\left(1 + 2\cos^2 k_0 h\right) \\ & - 4\left|x_{n,i}^{l,j} - h\right| H_1^{(2)}\left[k_0\left|x_{n,i}^{l,j} - h\right|\right] \\ & \cdot \cos k_0 h + \left|x_{n,i}^{l,j} - 2h\right| H_1^{(2)}\left[k_0\left|x_{n,i}^{l,j} - 2h\right|\right] \Big\} \end{aligned} \tag{12}$$

$$\begin{aligned} Y_{n,i}^{l,j,2} = & \frac{B_{n,i}^{l,j}}{\sin k_0 h} + \frac{k_0 t}{j\pi^2 \sin k_0 h} \sum_{m=1}^{\infty} \frac{C_m^2}{m^2 \beta_m^{\text{PP}}} \\ & \times \Big\{ e^{-j\beta_m^{\text{PP}} k_0 (x_{n,i}^{l,j} + 2h)} - 4\cos k_0 h e^{-j\beta_m^{\text{PP}} k_0 (x_{n,i}^{l,j} + h)} \\ & + 2e^{-j\beta_m^{\text{PP}} k_0 x_{n,i}^{l,j}} \left(1 + 2\cos^2 k_0 h\right) \\ & - 4\cos k_0 h e^{-j\beta_m^{\text{PP}} k_0 |x_{n,i}^{l,j} - h|} + e^{-j\beta_m^{\text{PP}} k_0 |x_{n,i}^{l,j} - 2h|} \Big\} \end{aligned} \tag{13}$$

where (see equation at the bottom of the page) and

$$f = \sqrt{\epsilon_r - 1} k_0 t \cot(\sqrt{\epsilon_r - 1} k_0 t)$$
$$d = -\frac{1}{2\sqrt{\epsilon_r - 1} k_0 t} \cot(\sqrt{\epsilon_r - 1} k_0 t) + \frac{1}{2} \csc^2(\sqrt{\epsilon_r - 1} k_0 t).$$

Once the unknown coefficients $M_{k,l}$ in the slots are known, the complex propagation constant can be found (in an average sense) by

$$\beta_{\text{av}} - j\alpha_{\text{av}} = (\angle V_1 - \angle V_N)/[p(N-1)] - j \ln \frac{|V_1|}{|V_N|} \Big/ [p(N-1)]. \tag{14}$$

Here, $V_{N(1)}$ is the magnetic current at the center of the last (first) slot.

The far-zone electric field, which can be calculated from the known tangential electric fields over the each slot in region

$$B_{n,i}^{l,j} = \begin{cases} 0, & \text{if } x_{n,i}^{l,j} \ge 2h \\ k_0 t \sin(k_0 h) d + \frac{1}{2k_0 t}[k_0 h \cos(k_0 h) - \sin(k_0 h)] f, & \text{if } x_{n,i}^{l,j} = h \\ -k_0 t \sin(2k_0 h) d - \frac{1}{2k_0 t}[2k_0 h - \sin(2k_0 h)] f, & \text{if } x_{n,i}^{l,j} = 0 \end{cases}$$

$$C_m = \left\{ 1 - (\epsilon_r - 1)\left(\frac{k_0 t}{m\pi}\right)^2 \right\}^{-1}$$

$$\beta_m^{\text{PP}} = \sqrt{\epsilon_r - \left(\frac{m\pi}{k_0 t}\right)^2}$$

$$x_{n,i}^{l,j} = |x_{n,i} - x_{l,j}|$$

(1) is given by

$$E_z \approx \sqrt{\frac{jk_0}{2\pi\rho}}\cos\varphi \frac{2k_0}{\sin(k_0 h)} \sum_{n=1}^{N}\sum_{m=1}^{M} V_{n,m} \times \frac{\cos(k_0 h \sin\varphi) - \cos(k_0 h)}{k_0^2 - (k_0 \sin\varphi)^2} e^{-jk_0(\rho - \sin\varphi x_{n,m})} \quad (15)$$

from which the radiation pattern is obtained.

Having determined the magnetic currents from (9), the time-averaged incident power ($P_{\rm inc}$), reflected power (P_r), coupled power to the guide beyond the slotted region (P_t), and radiated power from slots ($P_{\rm rad}$) per unit width can be calculated by the following expressions:

$$P_{\rm inc} = \frac{\beta_1^{\rm PP} t}{4\eta_0} \quad (16)$$

$$P_r = \frac{\beta_1^{\rm PP} t}{4\eta_0}\left(\frac{\pi}{k_0 t^2 \beta_1^{\rm PP}}\right)^2 \left| \sum_{n=1}^{N}\sum_{m=1}^{M} V_{n,m} F_{n,m}^{-}\left(\beta_1^{\rm PP} k_0\right)\right|^2 \quad (17)$$

$$P_t = \frac{\beta_1^{\rm PP} t}{4\eta_0}\left| 1 - \frac{\pi}{jk_0 t^2 \beta_1^{\rm PP}} \sum_{n=1}^{N}\sum_{m=1}^{M} V_{n,m} F_{n,m}^{+}\left(\beta_1^{\rm PP} k_0\right)\right|^2 \quad (18)$$

$$P_{\rm rad} = \frac{k_0}{4\pi\eta_0}\int_{-\pi/2}^{\pi/2} \left| \cos\varphi \sum_{n=1}^{N}\sum_{m=1}^{M} V_{n,m} F_{n,m}^{+}(k_0 \sin\varphi)\right|^2 d\varphi \quad (19)$$

where

$$F_{n,m}^{\pm}(\zeta) = \frac{2k_0}{\sin k_0 h} \frac{\cos(\zeta h) - \cos(k_0 h)}{k_0^2 - \zeta^2} e^{\pm j\zeta x_{n,m}}. \quad (20)$$

In (19), the radiated power is calculated by numerical integration over a semicircle of infinite radius in the upper half-space. This numerical integration decreases calculation efficiency because the integrand oscillates more rapidly as $x_{n,m}$ increases. Using the expressions for the conservation of complex power [12] to avoid this numerical difficulty, this integration can be expressed as

$$\frac{1}{2}\,{\rm Re}\left\{\int_{-\pi/2}^{\pi/2} \underline{E} \times \underline{H}^* \cdot \hat{\rho}\rho \, d\varphi\right\} = \frac{1}{2}\,{\rm Re}\left\{\sum_{n=1}^{N}\int_{S_n} \underline{M}_n \cdot \underline{H}^* \, dx\right\}. \quad (21)$$

The right side of (21) can be evaluated from the admittance matrix elements, i.e., $Y_{n,i}^{l,j,1}$. The radiated power can therefore be expressed in another form as

$$P_{\rm rad} = {\rm Re}\left\{\frac{1}{j2k_0\eta_0 \sin(k_0 h)} \sum_{n=1}^{N}\sum_{i=1}^{M} V_{n,i} \sum_{l=1}^{N}\sum_{j=1}^{M} V_{l,j}^{*} Y_{n,i}^{l,j,1*}\right\}. \quad (22)$$

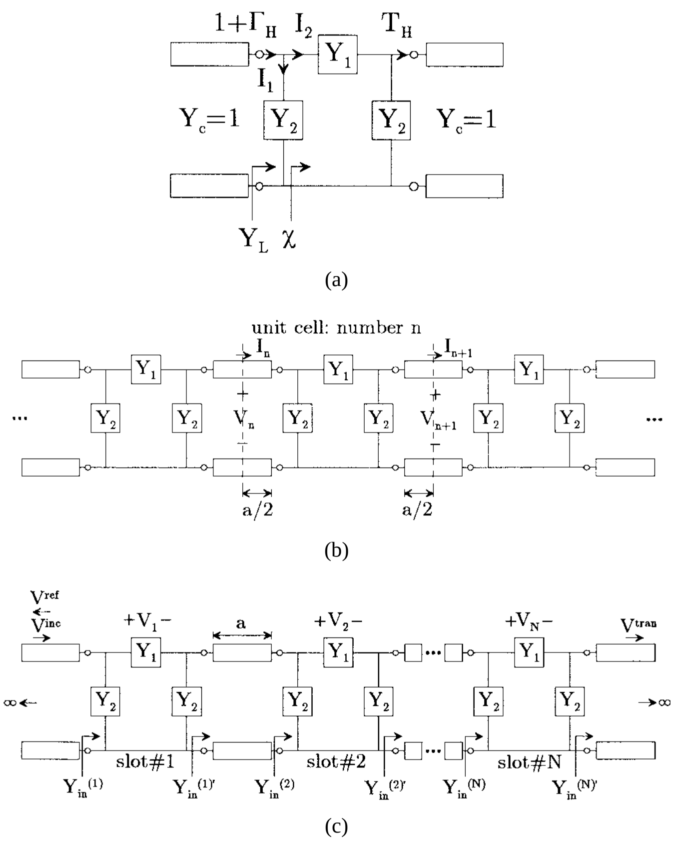

Fig. 2. (a) Equivalent circuit of the single slot. (b) Equivalent circuit of the infinite periodic structure. (c) Equivalent circuit of the finite N periodic structure.

C. Bloch Wave Analysis

The equivalent circuit of the single slot is obtained by the reflection and transmission coefficients of the dominant mode (in this paper, the lowest TE mode in the parallel-plate waveguide) [8]. Fig. 2(a) shows the equivalent circuit of a single slot on the upper plate of the parallel-plate waveguide. The normalized admittances of the circuit are expressed as [8]

$$\begin{aligned} Y_1 &= \frac{\chi[Y_2 + 1]}{Y_2 + 1 - \chi} \\ Y_2 &= \frac{1 + [\Gamma_H - T_H]}{1 - [\Gamma_H - T_H]} \\ Y_L &= \frac{1 + \Gamma_H}{1 - \Gamma_H} \\ \chi &= Y_L - Y_2. \end{aligned} \quad (23)$$

Here Γ_H and T_H are the reflection and transmission coefficients of the y component of magnetic field at the edge of the slot, respectively, and are given by

$$\Gamma_H(x) = \frac{\pi}{j\beta_1^{\rm PP} k_0 t^2} \sum_{n=1}^{N} V_n F_n^{-}\left(\beta_1^{\rm PP} k_0\right) e^{j2\beta_1^{\rm PP} k_0 x}$$

$$T_H(x) = \left[1 - \frac{\pi}{j\beta_1^{\rm PP} k_0 t^2} \sum_{n=1}^{N} V_n F_n^{+}\left(\beta_1^{\rm PP} k_0\right)\right] e^{-j2\beta_1^{\rm PP} k_0 x} \quad (24)$$

where N is the basis function numbers of the single slot. The infinite periodic structure can be represented by the equivalent

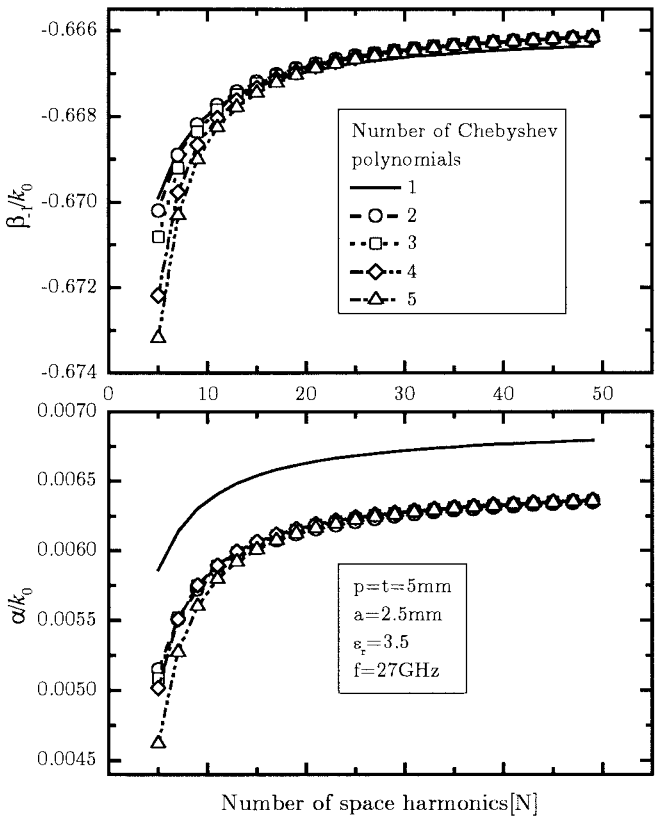

Fig. 3. Convergence of the complex propagation constant with number of space harmonics and Chebyshev polynomials.

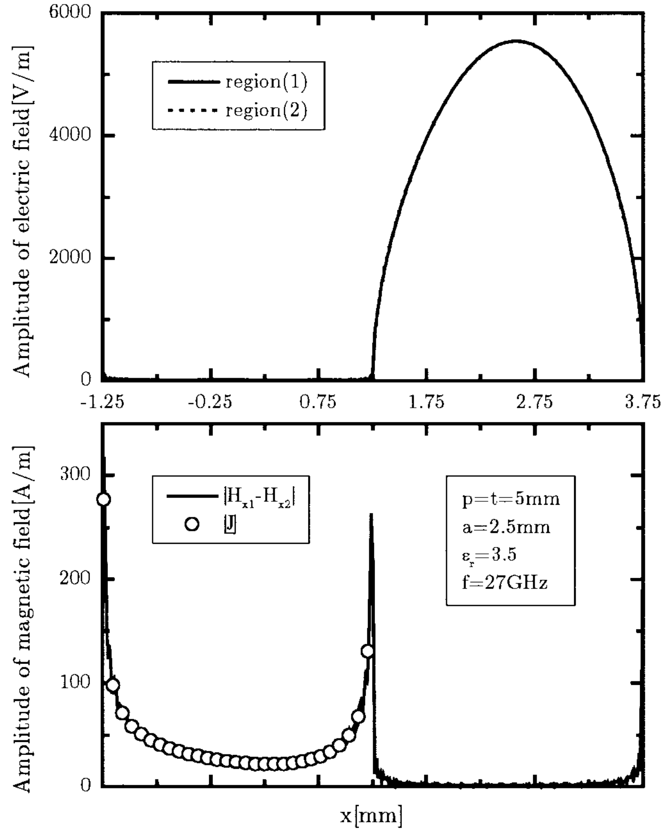

Fig. 4. Electromagnetic fields over one period.

network as shown in Fig. 2(b) by use of the parameters (23). Then the complex propagation constant of the periodic structure can be determined by solving the following:

$$e^{\alpha a}e^{j\beta a} = A \pm \sqrt{A^2 - 1} \tag{25}$$

where

$$\begin{aligned} A = &(1 + Y_2/Y_1)\cos\beta_1^{\text{PP}}a \\ &+ j/2\big(2Y_2 + Y_2^2/Y_1 + 1/Y_1\big)\sin\beta_1^{\text{PP}}a \end{aligned} \tag{26}$$

and Bloch wave analysis [9] has been used.

For the finite N-periodic slot case, the equivalent network can be established by use of the single slot circuit parameters (23), as shown in Fig. 2(c) and then the slot voltages can be calculated by transmission line analysis. The radiation pattern of the finite N periodic slot case can be determined by use of the slot voltages and equivalence principle and reflected, transmitted, and radiation powers also can be calculated.

III. Numerical Results and Discussion

To test the convergence of our numerical method a structure having $\epsilon_r = 3.5$, $p = 5$ mm, $a = 2.5$ mm, $t = 5$ mm, and $f = 27$ GHz is analyzed for an increasing number of space harmonics and Chebyshev polynomials. Fig. 3 shows the propagation constant $(\beta - j\alpha)$ versus the number of space harmonics for different numbers of Chebyshev polynomials, where the Newton–Raphson method is used to find the complex zero $\beta - j\alpha$. It is seen that 30 space harmonics for any number of Chebyshev polynomials gives results with excellent convergence.

To be certain that β and α converge to the right values, we calculate the tangential electric and magnetic fields at the interface $(y = 0)$ for the same structure and plot in Fig. 4 when 40 space harmonics and 5 Chebyshev polynomials are employed. The tangential electric fields are identical on both sides of the interface and vanish on the metal strip surface. The tangential magnetic field is continuous over the slot region and is discontinuous over the strip region due to the strip surface current density, as shown in figure. Fig. 4 shows that the boundary conditions at the interface are satisfied and the problem is solved in the right way.

A structure for the case of $\epsilon_r = 15$, $t/p = 0.8$, and $a/p = 0.1$ is now considered. The $k - \alpha$ and $k - \beta$ diagrams obtained by the method are plotted and compared with the results given by [1] in Fig. 5, which shows good agreement.

For the finite periodic structure in Fig. 1(b), in the case of $p = 5$ mm, $t = 5$ mm, $\epsilon_r = 3.5$, $f = 27$ GHz, and $N = 60$, the slot electric fields can be found by solving (9). The magnetic currents (slot electric fields) at the center of each slot are shown in Fig. 6 with slot width as parameter. In Fig. 6, as expected, the amplitude of the magnetic currents are observed to decay more rapidly as slot width increases, which means that the attenuation constant α increases as slot width increases.

The complex propagation constants determined by each method as a function of slot width are plotted in Fig. 7, which shows good agreement between the different methods. The

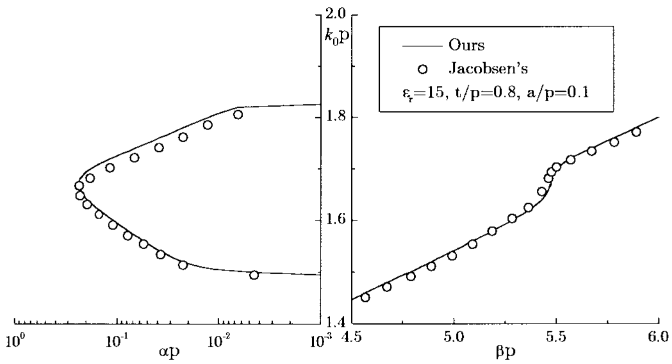

Fig. 5. $k-\beta$ and $k-\alpha$ diagrams.

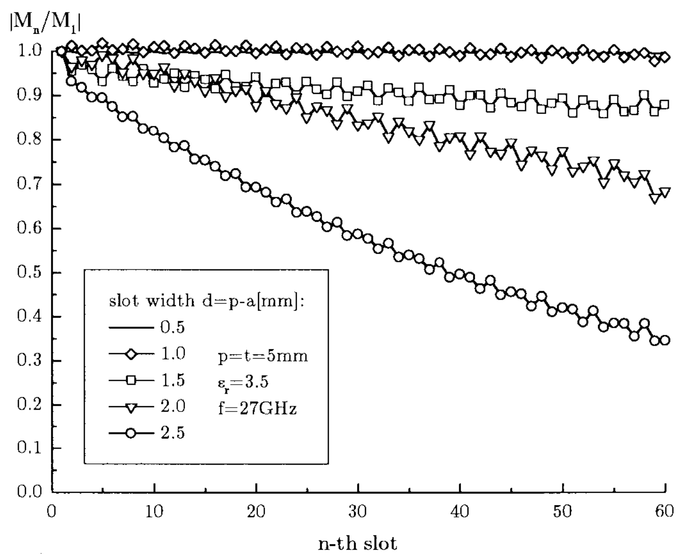

Fig. 6. Variation of the magnetic currents at the center of the each slot.

phase constants (β) calculated by each method are almost same for all slot widths. However, the attenuation constants (α) agree well for narrow slot widths but not for a wide slot width, which may be the result of neglecting mutual coupling. In Fig. 7, for small slot width β does not change appreciably from the fundamental TE wave propagation constant $\beta_1^{\rm PP}$, but an attenuation constant α is produced that accounts for the leakage of power out of the guide. In this structure, the leaky-wave mode is established in the waveguide and half-space regions. The radiation characteristics of this kind of leaky-wave antenna for a finite periodic structure can be determined from $\beta_{\rm av}$ and $\alpha_{\rm av}$.

The radiation characteristics of the leaky-wave antenna are directly obtained from $\mathrm{Re}(\beta_{-1})$ and α when only the $n=-1$ space harmonic is radiating. The angle of maximum radiation φ_m is obtained from $\mathrm{Re}(\beta_{-1})$, i.e., $\varphi_m = \sin^{-1}[\mathrm{Re}(\beta_{-1})/k_0]$, and the beamwidth from α. Fig. 8 shows the variation of the attenuation constant and radiation angle φ_{-1} with the frequency. It is seen that the radiation angle changes almost linearly, while the attenuation constant has less variation with frequency below 28 GHz and rapid change with maximum value above 28 GHz. In this figure, the results of Bloch wave analysis are well matched with those of the integral equation up to 29 GHz, which means that the Bloch wave analysis has failed for a wide slot width.

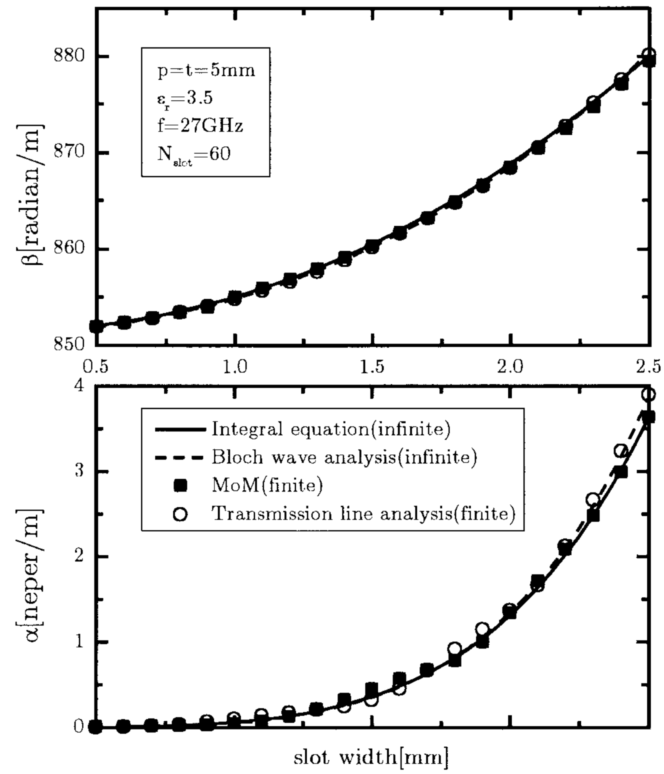

Fig. 7. Comparison of the complex propagation constant between infinite and finite periodic structure.

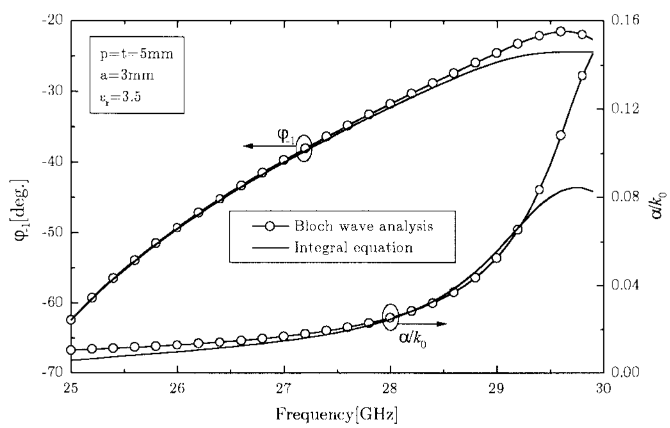

Fig. 8. Frequency dependence of the -1th radiation angle and the attenuation constant.

The variation of the complex propagation constant with the total slot number for the finite periodic structure is shown in Fig. 9. In Fig. 9, the phase constant β, which determines the leaky-wave radiation angle is already almost same (within 0.22%) with the value of the infinite case for a slot number of ten. However, the attenuation constant, which is related to the beamwidth converges when the slot number is larger than 30. This explains that the maximum radiation angle is almost independent of the slot number when it is compared with the infinite case. The converged values obtained by each method are slightly different but it makes no difference in practice.

The radiation pattern calculated by (15) is compared with that obtained by (4) in Fig. 10 assuming the slot field is the

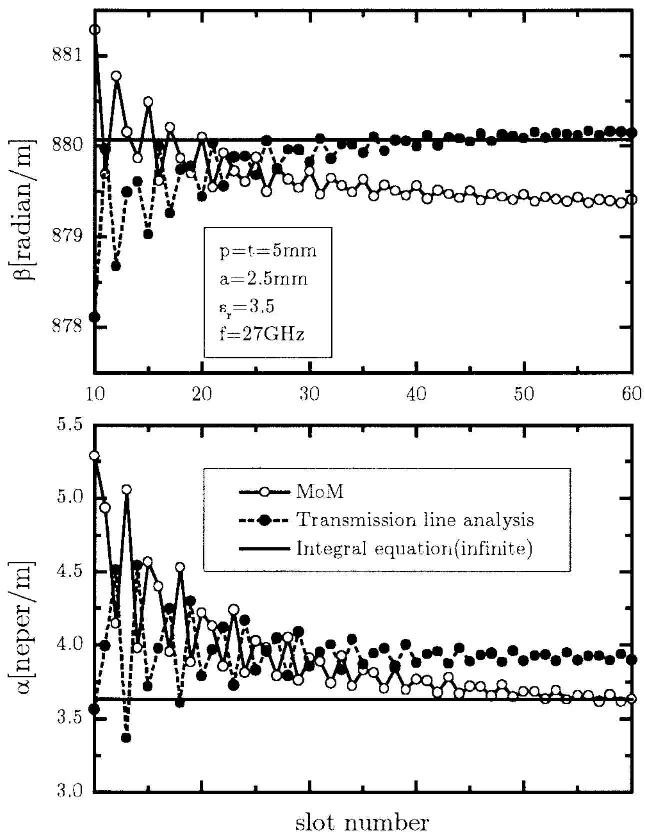

Fig. 9. Variation of the complex propagation constant with the total slot number.

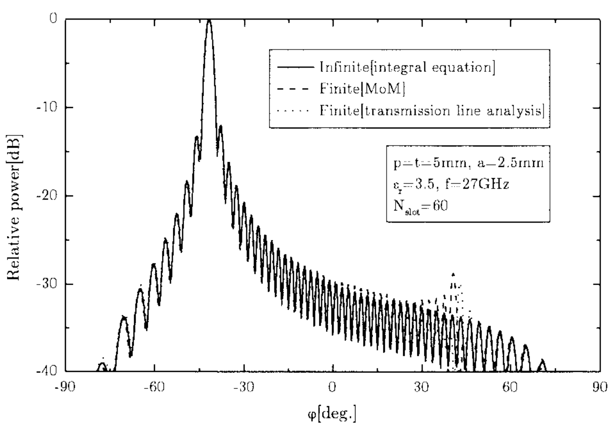

Fig. 10. Radiation pattern.

same as for the infinite geometry, where the main beam angle and beamwidth are almost the same as those for the infinite geometry case, but some discrepancies (though small) between the finite and infinite cases are observed in the range 36°–48°. This difference can be explained by the fact that the leaky wave, which is traveling in the positive x direction, radiates at a maximum beam angle $\varphi_m \approx -42°$, while the backward reflected wave forms a (considerably weaker) beam in the direction $\varphi = -\varphi_m$.

The accuracy of the numerical results in this study is assured by a check of the power conservation law

$$P_r + P_t + P_{\rm rad} = P_{\rm inc}. \tag{27}$$

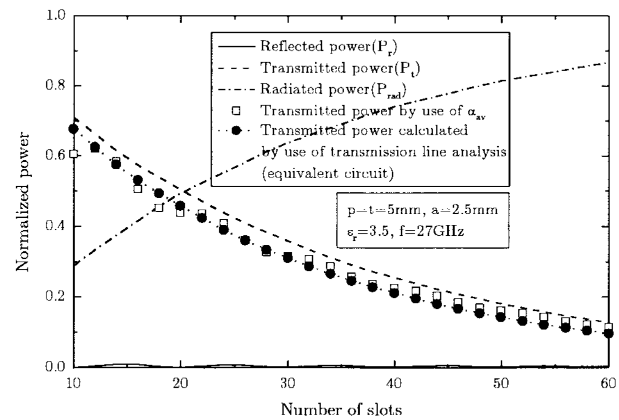

Fig. 11. Normalized powers P_r, P_t, and $P_{\rm rad}$ versus total slot number.

In Fig. 11, we show the dependence of P_r, P_t, and $P_{\rm rad}$ on the slot number N. In the case of $N = 60$, 86.1% of the incident power is radiated, 0.4% is reflected, 13.5% is transmitted to the guide beyond the slotted region under the assumption that both conductor wall loss and dielectric loss is negligible.

With incident power $P_{\rm inc}$ normalized to unity, P_t is expressed as

$$P_t = e^{-2\alpha_{\rm av} l_t} \tag{28}$$

where l_t means the total length of slotted section. Values of P_t obtained by use of (28) are also plotted with the □ symbols and compared with those obtained by direct integration over all slots in the guide region in Fig. 11, which shows good agreement for large slot numbers.

IV. Conclusion

The analysis methods for the infinite periodically slotted dielectrically filled parallel-plate waveguide as a leaky-wave antenna, and the solving methods for the leaky wave from the parallel-plate waveguide with N periodic slots excited by a TE incident wave are proposed. In this paper, rigorous (integral equation and integro-differential equation) and approximate (Bloch wave analysis and transmission-line analysis using equivalent circuit representation) methods for infinite and finite periodic leaky-wave structures are considered. Results of interest such as complex propagation constant and radiation pattern for the finite slot number case are observed to be in good agreement with those of the infinite geometry case for appropriately chosen values of slot number, slot width, period, and waveguide height. It is found that the Bloch wave analysis and transmission line analysis are the very efficient methods at the rough design stage where the leaky-wave radiation angle is concerned.

The methods considered here are thought to be useful in predicting the performance of the infinite geometry case at the design stage of the practically finite leaky-wave structure.

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