Scattering from Complex Bodies Using a Combined Direct and Iterative Technique

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***Abstract*— An iterative technique is developed for frequency-domain plane wave scattering from electrically large composite bodies. An electric field integral equation (EFIE) formulation is employed in which the submatrices of the moment-method matrix are uncoupled and the current on each geometrically separable region of the composite body is solved independently using a direct method. The currents on the various subcomponents of the body are then recalculated within an outer iterative loop. The technique is applied to the case of a multiwavelength body of revolution (BOR) with two flat-plate attachments. This composite body iterative technique is shown to preserve the simplicity and attractiveness of an isolated BOR while maintaining current continuity across the structure without the use of additional junction currents. This new formulation also allows simple suppression of interior resonance effects normally associated with large closed conducting bodies.**

***Index Terms*— Electromagneric scattering, iterative methods, large bodies. method of moments.**

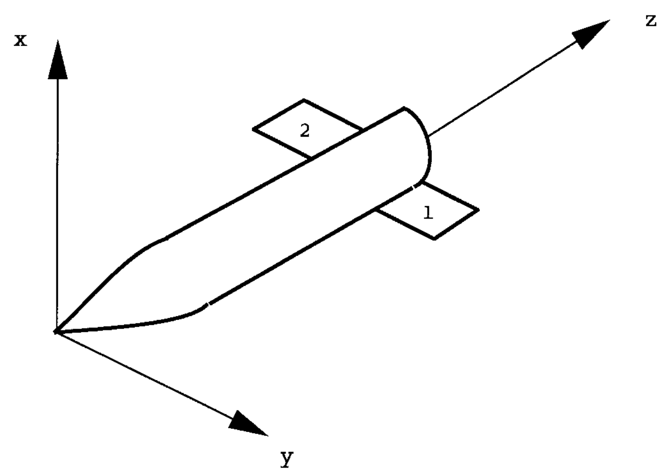

Fig. 1. Body of revolution with two flat-plate attachments.

I. Introduction

SINCE the publication of Harrington's classic book on the method of moments (MoM) [1] almost 30 years ago, surface integral equations in conjunction with the MoM have been used extensively for the numerical solution of scattering from perfectly conducting bodies. This approach, however, is computationally intensive and requires significant computer storage for even moderately sized objects. In this paper, we develop a generalized method to efficiently analyze plane wave scattering from electrically large complex bodies using a MoM formulation, where the MoM matrix equation is solved using a combination of direct and iterative techniques. The particular configuration considered is a long (with respect to wavelength) slender body of revolution (BOR) with two flat-plate attachments (Fig. 1). It is shown that the efficient solution of the isolated BOR is maintained in this composite body iterative formulation.

As with the MoM, electromagnetic scattering from BOR's is also a relatively old subject. Numerical solutions by Andreasen [2] appeared as early as 1965. Shortly thereafter, Mautz and Harrington used the MoM to solve the BOR problem [3], [4] and recently, very large BOR's have been solved using a combined field integral equation approach [5]. An extension of the early BOR work was investigated by Albertsen *et al.* [6] for a composite BOR and thin wire configuration using a coupled magnetic field integral equation (MFIE) and electric field integral equation (EFIE) formulation. Shaeffer and Medgyesi-Mitschang [7], [8], looked at the problem of radiation and scattering from thin wires attached to BOR's using an EFIE formulation. More recently, investigators have looked at the problem of scattering from structures consisting of BOR's with arbitrary surface attachments. In work by Durham and Christodoulou [9], [10], the arbitrarily shaped attachments were modeled using triangular surface patches. This allowed greater flexibility in choosing the shape of the attachments.

As is well known in the BOR formulation, when the current is expressed as a Fourier series in azimuthal angle, each mode of the current can be evaluated separately. An unfortunate consequence of the composite body formulation is the loss of the block diagonal BOR MoM matrix. Instead, the composite body matrix equation is partial block diagonal. The added complexity of the impedance matrix occurs regardless of whether the attachment is a thin wire or an arbitrary surface. In all the work mentioned previously [7]–[10], the composite body matrix was solved using partitioning [11]. This method takes advantage of the partial block diagonal nature of the impedance matrix to reduce the size of the largest matrix that must be inverted, but requires cumbersome matrix operations. A new scheme, with general utility, is developed here that preserves the simplicity and attractiveness of the original BOR formulation. In this approach, the fields incident on the BOR are modeled as the usual incident plane wave plus the fields radiated by the induced currents on the surface attachment, while the fields incident on the attachment are

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modeled as the incident plane wave plus the fields radiated by the induced currents on the BOR. Each of these two problems is solved independently, using the currents of one to update the fields incident on the other. Iterations are continued until convergence is achieved. While somewhat indirect, this procedure maintains the block diagonal efficiency associated with isolated BOR's.

The iterative approach presented here is conceptually similar to that described in [12], but differs in implementation and detail. In this recent work by Hodges and Rahmat-Samii [12], an iterative method coupling the MFIE and the EFIE was developed for radiation and scattering problems, where the integral equations were applied to geometrically distinct regions of a complex structure in a manner similar to what was done in [6]. The application of the MFIE to certain regions of a complex body, however, necessarily restricts those regions to be closed. In the work presented here, an EFIE formulation is used. The restriction to closed bodies is, therefore, not imposed as in this other work [6], [12]. For the example given in [12], a generalized triangular surface patch representation was used to model the current on the conducting BOR, rather than the highly efficient specialized BOR currents we use. The method of [12] was also shown to be generally more efficient for radiation problems rather than scattering problems because the entire iterative procedure has to be repeated for each incidence angle. In this paper, we show that due to the "partially direct" nature of the composite body iterative technique, our approach is ideally suited for both radiation and backscatter calculations. Finally, the issue of interior resonances, which was not discussed in the previously referenced work on composite bodies [6]–[10], [12], will be addressed here.

In Section II, the composite body iterative technique is presented. The technique is validated for a simple composite body configuration. Convergence of the method is demonstrated and results are compared with traditional MoM solutions. In Section III, the iteration technique is applied to a composite body consisting of a BOR with two flat-plate attachments. In lieu of a direct MoM solution comparison, convergence results are given. Conclusions are given in Section IV.

II. Composite Body Iteration Technique

A generalized technique is presented to reduce the large body scattering problem into smaller, subcomponent level scattering problems. This technique is similar to the multiple reflection approach described in [13]. Consider the infinitesimally thin, perfectly conducting flat plates in the $y-z$ plane shown in Fig. 2. The upper plate represents a single large body. The lower plates, which can be attached or separated, constitute a composite body. When attached, the composite body has the same overall dimensions as the large plate. In the composite body formulation, the fields incident on body 1 are modeled as the usual incident plane wave plus the fields radiated by the induced currents on body 2, while the fields incident on body 2 are modeled as the incident plane wave plus the fields radiated by the induced currents on body 1. Each of these two problems is solved separately with a standard MoM procedure [14], using the currents of one body as a source term to update the fields incident on the other body. Iterations are continued until convergence is achieved.

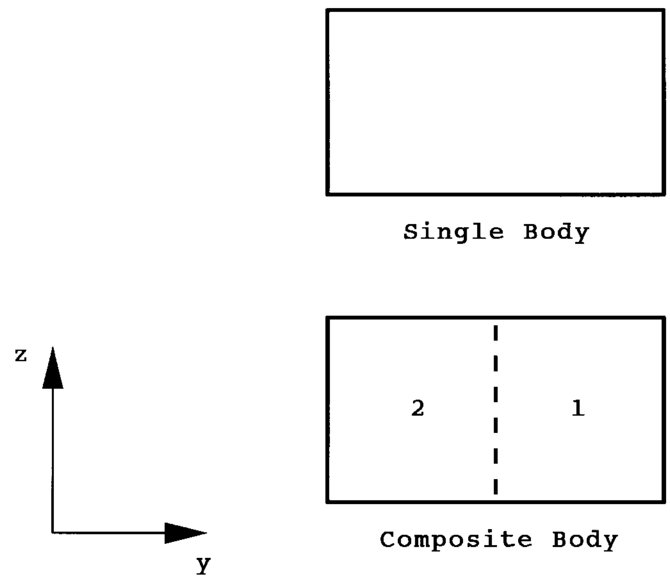

Fig. 2. Flat-plate geometry for a single and composite body.

Notice that for the composite body, a nonphysical interior edge has been formed. This is illustrated in the figure by the dashed line. This is a mathematical artifact that results from modeling a single large body by two or more smaller bodies. This nonphysical edge can introduce errors into the formulation because the boundary condition states that current flowing normal to an edge must go to zero at that edge, but these edge currents are not necessarily zero. We can, however, take advantage of the iteration technique by using the neighboring currents that are computed at each iteration to estimate the boundary currents normal to the attachment point. If the current is expanded in terms of two-dimensional pulse functions, and the y directed current coefficients on body 1 and 2 in Fig. 2 are given, respectively, as $(J_y^1)_{j,k}$ and $(J_y^2)_{j,k}$, where $j = 1, \cdots, N$, and $k = 1, \cdots, N$, the boundary currents normal to the interior attachment point can be expressed as $(J_y^1)_{1,k}$ and $(J_y^2)_{N,k}$ for $k = 1, \cdots, N$. To estimate the interior boundary currents, the procedure is as follows. When solving for the current on body 2, assume

$$
\begin{aligned}
(J_y^1)_{1,k} &= \frac{(J_y^1)_{2,k} + (J_y^2)_{N,k}}{2} \\
(J_y^2)_{N,k} &= (J_y^1)_{1,k}
\end{aligned}
\tag{1}
$$

where the current terms on the right-hand side of the first equation are obtained from the previous iteration. For the next iteration, when solving for the current on body 1, assume

$$
\begin{aligned}
(J_y^2)_{N,k} &= \frac{(J_y^2)_{N-1,k} + (J_y^1)_{1,k}}{2} \\
(J_y^1)_{1,k} &= (J_y^2)_{N,k}
\end{aligned}
\tag{2}
$$

where the current terms on the right-hand side of the first equation are again obtained from the previous iteration. In this manner, the boundary currents normal to the attachment point are recomputed after each iteration. The z directed currents at the attachment point are treated as unknowns and solved for in the usual manner since these currents are parallel to the interior edge. As can be seen by the first equation in (1) and (2), the currents normal to the interior edge are calculated by taking the average value of the current from either side, while the second equation simply enforces current continuity across the fictitious edge. It is important to recognize that this manipulation of the normal interior edge boundary current is the key to the effectiveness of the composite body iterative solution. This will be demonstrated in the following example.

The single body and composite body configurations of Fig. 2 are used as a test case. The large plate has dimensions of $4\lambda \times 2\lambda$. The smaller plates have dimensions of $2\lambda \times 2\lambda$ and they are attached. The upper section in Fig. 3 shows the normalized y-directed current density on the large plate using an "exact" MoM formulation for a plane wave incident in the x-z plane at $\theta^{\rm inc} = 135°$, where θ is measured from the z axis. To make the currents somewhat asymmetric, the plane wave electric field had both a θ and ϕ component. For this solution, the entire plate was modeled in the usual manner (see [14]), and the solution was obtained using a direct method. Also shown in the lower section of the figure is the iterative solution, where the large plate was modeled as two smaller plates and the boundary currents at the attachment point were recomputed after each iteration using (1) and (2). Ten iterations were used in the iterative solution. As can be seen in the figure, the iterative solution appears to have converged quite nicely to the exact solution, particularly at the attachment point. (The attachment point corresponds to the $y = 1$ coordinate line in the iterative solution figure.) Although not shown (see [15] for details), using the composite body iteration technique and enforcing current continuity at the attachment point produced very good backscatter results, even at the difficult grazing angles.

III. BOR Flat-Plate Composite Body

In this section, the application of the iteration technique to a BOR flat-plate composite body is considered. The particular configuration is show in Fig. 1. This configuration, consisting of a BOR with two flat-plate attachments, represents a generic missile shape. The EFIE for this body can be written as

$$L(\vec{J}) = \vec{E}^{\rm inc}_{\rm tan} \tag{3}$$

where L is the well-known integrodifferential operator [3], [7]. For the composite body considered here

$$\vec{J} = \vec{J}^{\rm BOR} + \vec{J}^{P} \tag{4}$$

and

$$\vec{J}^{P} = \vec{J}^{1} + \vec{J}^{2} \tag{5}$$

where $\vec{J}^{\rm BOR}$ represents the usual BOR surface current expanded in terms of a Fourier series in azimuthal angle and $\vec{J}^{P}$ represents the surface currents on the two flat-plate attachments. Substituting (4) into (3), and using the linearity of the operator L gives

$$L(\vec{J}^{\rm BOR}) + L(\vec{J}^{P}) = \vec{E}^{\rm inc}_{\rm tan}. \tag{6}$$

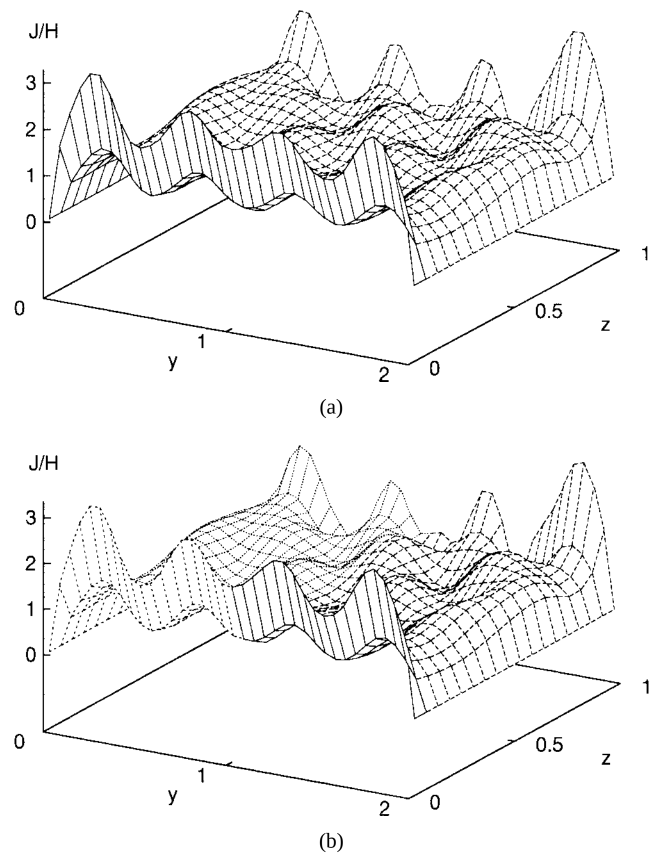

Fig. 3. Normalized y directed current density on a 4λ by 2λ flat plate for a plane wave incident in the $x - z$ plane at $\theta^{\rm inc} = 135$ degrees. Comparison between (a) direct MoM solution and (b) composite body iteration technique. Incident electric field has both a θ and ϕ component.

Notice the governing equation has been divided into two major subcomponents: the BOR part and the flat-plate part. The solution for the BOR has been well documented [3], [4] and the flat-plate problem was solved in the previous section. Testing with the complex conjugates of the BOR basis functions on the BOR surface and using razor blade testing functions on the flat plate attachments [14], the integral equation in (6) can be reduced to a matrix equation given by

$$\left[\begin{array}{ccccc|c} \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \cdots & Z^{BB}_{-1} & 0 & 0 & \cdots & Z^{BP}_{-1} \\ \cdots & 0 & Z^{BB}_{0} & 0 & \cdots & Z^{BP}_{0} \\ \cdots & 0 & 0 & Z^{BB}_{1} & \cdots & Z^{BP}_{1} \\ \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hline \cdots & Z^{PB}_{-1} & Z^{PB}_{0} & Z^{PB}_{1} & \cdots & Z^{PP} \end{array}\right] \cdot \left[\begin{array}{c} \vdots \\ J^{B}_{-1} \\ J^{B}_{0} \\ J^{B}_{1} \\ \vdots \\ \hline J^{P} \end{array}\right] = \left[\begin{array}{c} \vdots \\ E^{B}_{-1} \\ E^{B}_{0} \\ E^{B}_{1} \\ \vdots \\ \hline E^{P} \end{array}\right] \tag{7}$$

where

$$Z^{PP} = \begin{bmatrix} Z^{11} & Z^{12} \\ Z^{21} & Z^{22} \end{bmatrix} \tag{8}$$

and

$$J^P = \begin{bmatrix} J^1 \\ J^2 \end{bmatrix}, \quad E^P = \begin{bmatrix} E^1 \\ E^2 \end{bmatrix}. \tag{9}$$

The self-impedance terms for the BOR are represented by Z_n^{BB}, where the subscript n represents the nth Fourier mode of the current expansion function [4]. The new terms in (7), Z_n^{BP} and Z_n^{PB}, describe the mutual interactions between the BOR and flat plates. The column vector on the right-hand side of (7) contains the usual information about the plane wave excitation of the structure and the column vector on the left-hand side contains the current coefficients for the BOR and flat plates. Matrices similar to (7) were developed in all the previously mentioned references dealing with BOR's in conjunction with wires and arbitrary surfaces [7]–[10]. These matrix equations were solved using partitioning [11] to take advantage of the partial block diagonal nature of the impedance matrix. Using this method, the largest matrix that must be inverted is the larger of Z^{PP} or Z_n^{BB}. It is this feature that allows analysis of much larger structures than would be possible using a generalized surface-patch representation of the entire composite body [9].

At this point, the composite body iteration technique could be used to solve (7) in lieu of performing the extensive matrix operations from partitioning. Either method would be premature at this time, however, because the effect of interior resonances on the solution has yet to be addressed. For electromagnetic scattering problems, the MoM solution of a surface integral equation can produce inaccurate surface currents at frequencies associated with the interior cavity resonance problem [16]. In the earlier work [6]–[10], [12], resonances were not treated because only low frequency cases were examined and, as long as the excitation stayed below cutoff frequency, resonances did not become a problem. In this investigation, electrically large bodies are considered, so additional constraints on the problem must be imposed in order to suppress the resonance effects. An extremely robust method to suppress interior resonances which fits in well with the current EFIE formulation is given by Canning [17]. In this method, supplemental boundary conditions of zero total electric field are enforced at multiple interior locations. In MoM terminology, this amounts to introducing additional testing points in the interior of the closed conducting body. This results in an over determined system of equations which can be solved using a least squares solution. Excellent results have been obtained using this technique for a variety of shapes and frequencies [15].

To compliment the generalized MoM matrix equation $[Z][J] = [E]$, a supplemental equation utilizing the null interior field boundary condition can be written as

$$[C][J] = [V] \tag{10}$$

where $[C]$ is an $M \times N$ matrix whose elements are found by introducing M additional testing functions in the body interior. $[V]$ is a vector of length M whose elements represent the inner product of the incident field and the interior testing functions and $[J]$ is a vector of length N whose elements represent the current expansion coefficients. The least squares solution to this over determined system is given by [17]

$$\{[Z]^H[Z] + [C]^H[C]\}[J] = [Z]^H[E] + [C]^H[V] \tag{11}$$

where $[Z]$ is the usual MoM impedance matrix and $[E]$ represents the usual incident plane wave excitation. The H superscript indicates taking the Hermitian. Due to the orthogonality of the testing functions, the isolated BOR MoM matrix was said to be block diagonal, where each mode of the current was solved independently. Using similar testing functions, the BOR supplemental matrix equation (10) is also block diagonal. Substitution of the BOR matrix equation and the supplemental matrix equation into (11) produces a matrix equation, which is also block diagonal so the "resonance free" current can still be solved one mode at a time. What remains now is to determine what happens to the composite body matrix equation (7) when the supplemental equations are included in the formulation.

For the composite body considered here (Fig. 1), the supplemental equations can be expressed in matrix form as

$$\left[\begin{array}{ccccc|c} \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \cdots & C_{-1}^{BB} & 0 & 0 & \cdots & C_{-1}^{BP} \\ \cdots & 0 & C_0^{BB} & 0 & \cdots & C_0^{BP} \\ \cdots & 0 & 0 & C_1^{BB} & \cdots & C_1^{BP} \\ \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \end{array}\right] \cdot \begin{bmatrix} \vdots \\ J_{-1}^B \\ J_0^B \\ J_1^B \\ \vdots \\ \hline J^P \end{bmatrix} = \begin{bmatrix} \vdots \\ V_{-1}^B \\ V_0^B \\ V_1^B \\ \vdots \end{bmatrix} \tag{12}$$

where C_n^{BB} and V_n^B represent the impedance terms and the excitation vectors of the interior testing body, respectively. The C_n^{BP} submatrices represent the interactions of the flat-plate currents and the interior testing body. The left-hand side column vector is the same as in (7). As stated previously, (12) results from the imposition of the null interior field boundary condition. Using shorthand notation, (7) can be expressed as

$$\left[\begin{array}{c|c} Z^{BB} & Z^{BP} \\ \hline Z^{PB} & Z^{PP} \end{array}\right] \begin{bmatrix} J^B \\ \hline J^P \end{bmatrix} = \begin{bmatrix} E^B \\ \hline E^P \end{bmatrix} \tag{13}$$

where Z^{BB}, Z^{BP}, and Z^{PB} represent block diagonal, block column, and block row matrices, respectively, and (12) can be expressed as

$$[C^{BB} \mid C^{BP}] \begin{bmatrix} J^B \\ \hline J^P \end{bmatrix} = [V^B] \tag{14}$$

where C^{BB} and C^{BP} represent block diagonal and block column matrices, respectively. Using (11), it can be shown that the least squares solution to the over determined system in (13) and (14) produces a matrix equation that is *not* partial block diagonal [15]. This is due to the $[Z]^H[Z]$ term in (11). The least squares impedance matrix [denoted by the curly braces in (11)] is, in general, extremely large, containing all the impedance submatrices for the two flat plates and the BOR. Partitioning is of no use in this instance because it would require taking the inverse of the now completely filled matrix in the upper left quadrant [see (7)]. For all practical purposes then, the governing matrix equation given by the least squares solution can no longer be solved in-core using a direct method. Even indirect methods such as Gauss–Seidel [18] or conjugate gradient [19] may require out-of-core storage of the entire composite body-least squares impedance matrix. A modified form of the iteration technique is needed to efficiently handle this case. If the submatrices in (13) and (14) are multiplied through, the following coupled set of equations result:

$$\begin{aligned}[Z^{BB}][J^B] + [Z^{BP}][J^P] &= [E^B]\\ [Z^{PB}][J^B] + [Z^{PP}][J^P] &= [E^P]\\ [C^{BB}][J^B] + [C^{BP}][J^P] &= [V^B]. \end{aligned} \quad (15)$$

If the current on the flat-plate attachments is initially given by physical optics (PO), the BOR currents can be evaluated using the first and third equation in (15)

$$\begin{aligned}[Z^{BB}][J^B] &= [E^B] - [Z^{BP}][J^P] = [E^B]'\\ [C^{BB}][J^B] &= [V^B] - [C^{BP}][J^P] = [V^B]'. \end{aligned} \quad (16)$$

These equations are identical to the isolated BOR block diagonal matrix equations except that the right-hand side vectors are slightly modified to include the effects of the flat-plate currents. Using the procedure described earlier, the BOR currents $[\cdots J^B_{-1}\ J^B_0\ J^B_1 \cdots]^T$ can be evaluated one mode at a time. The flat-plate currents $[J^1\ J^2]^T$ can then be found by utilizing the second equation in (15) and the BOR currents that were just computed

$$[J^P] = [Z^{PP}]^{-1}\{[E^P] - [Z^{PB}][J^B]\}. \quad (17)$$

Since the BOR attachments are open surfaces, it is guaranteed that $[Z^{PP}]$ is stable and well conditioned. Equations (16) and (17) are solved repeatedly, where the most recently computed currents are used to update the right-hand sides. At convergence, all three equations in (15) are satisfied simultaneously.

Current continuity can be applied at the attachment points between the BOR and the two flat plates. As shown in Fig. 1, plate 1 is attached to the BOR in the $\phi = 90°$ plane, while plate 2 is attached to the BOR in the $\phi = 270°$ plane. To examine the flow of currents near the attachment point, a magnified subsectional view is provided in Fig. 4. Notice that the t-directed current on the BOR is parallel to the z-directed current on the flat plate. Using the notation from the previous section, the boundary current normal to the attachment point on the two plates can be approximated by

$$\begin{aligned}(J^1_y)_{1,k} &= J^{\rm BOR}_\phi(\rho, z, \phi = 90 - \delta)\\ &\quad - J^{\rm BOR}_\phi(\rho, z, \phi = 90 + \delta)\\ (J^2_y)_{N,k} &= J^{\rm BOR}_\phi(\rho, z, \phi = 270 + \delta)\\ &\quad - J^{\rm BOR}_\phi(\rho, z, \phi = 270 - \delta) \end{aligned} \quad (18)$$

where δ is an arbitrarily small constant. It is understood that the k index on the flat plate (running in the z direction), and the (ρ, z) coordinate on the BOR correspond to the same physical location. The z-directed current at the boundary is solved as before.

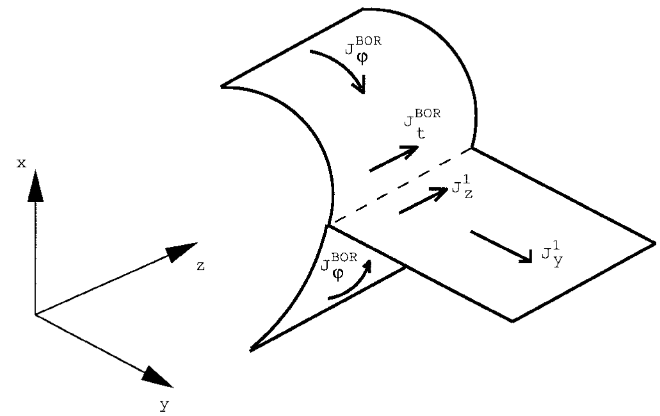

Fig. 4. Subsectional view of the current flow at the attachment point between BOR and flat plate.

The BOR current is the sum of all the Fourier modes. In order to obtain the necessary fidelity in the BOR current near the attachment point, a sufficient number of Fourier modes are required, even for the case of axial incidence. To test the validity of the composite body iteration technique, the effect of the total number of Fourier modes on solution convergence was examined. The number of iterations versus convergence was also examined. The composite body in Fig. 1 was illuminated with a ϕ-polarized plane wave at $\theta^{\rm inc} = 90°$ and $\phi^{\rm inc} = 0°$ (broadside incidence). The length of the BOR was 10λ and the diameter was 1λ. Each flat-plate attachment was 1 $\lambda\times$ 1 λ and the BOR nose was elliptically shaped. The BOR generating curve was discretized into $\lambda/10$ subsections, and the flat plate subdomains had dimensions of $\lambda/10\ \times\lambda/$ 10. In the two remaining figures, the current is plotted against the normalized BOR arc length coordinate t. Fig. 5 is a plot of the t-directed current along the $\phi = 90°$ coordinate of the BOR portion of the composite body for different values of the maximum Fourier mode number MAX. These results were computed for eight iterations and show that the solution converges for $MAX = 6$. The influence of the flat plates can be seen in the current by the two spikes in the data where plate 1 would be attached. While not plotted for the sake of brevity (see [15] for details), the current on the fins (flat plates) was shown to converge for $MAX = 3$. The same configuration was run again, but with $MAX = 6$—and the number of iterations varied. Results showed the current to be fairly well converged after eight iterations. For this case, each additional iteration required 0.25 s of CPU on a CRAY-YMP or an increase of 0.2% in CPU time per iteration. Obviously, the cost of iterating is minimal compared with the initial computations. The normalized current on the BOR described above, with and without the flat-plate attachments, is shown in Fig. 6 for a ϕ-polarized plane wave at $\theta^{\rm inc} = 180°$ (axial incidence). The total composite body solution is denoted by the curve "BOR+FINS," and the isolated BOR solution is denoted "BOR." The t-directed current along the side of the BOR ($\phi = 90°$) is plotted. Also shown for reference is the z-directed current, "FIN," on plate 1 of the composite body

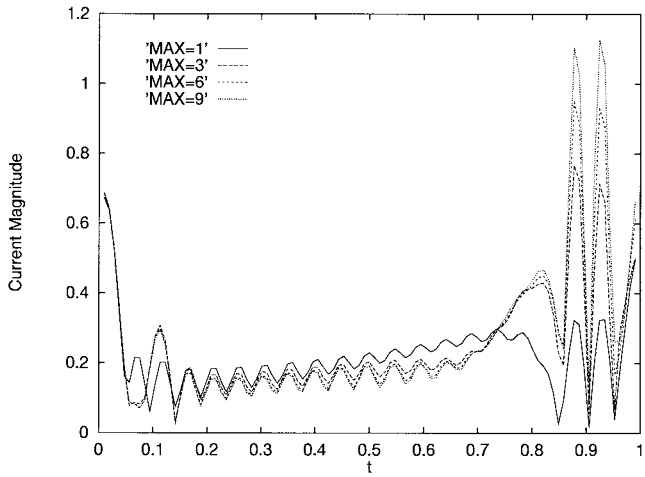

Fig. 5. Normalized t-directed current density, J_t^{BOR} (see Fig. 4) on BOR portion of a 10λ-long composite body (see Fig. 1) for a ϕ-polarized plane wave at $\theta^{\mathrm{inc}} = 90^\circ$ and $\phi^{\mathrm{inc}} = 0^\circ$ (broadside incidence). Current plotted along the $\phi = 90^\circ$ coordinate of the BOR.

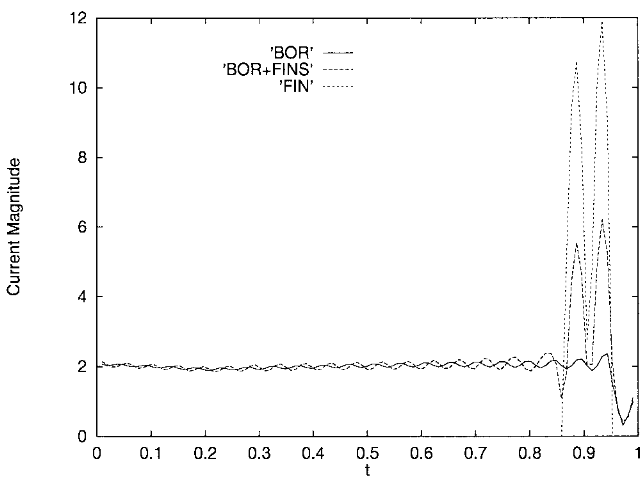

Fig. 6. Normalized t-directed current density, J_t^{BOR} (see Fig. 4), on isolated BOR and on BOR portion of a 10λ-long composite body (see Fig. 1) for a ϕ-polarized plane wave at $\theta^{\mathrm{inc}} = 180^\circ$ (axial incidence). Current plotted along the $\phi = 90^\circ$ coordinate of the BOR. Also shown is the z-directed current density J_z^1 (see Fig. 4) on plate 1 at the attachment point.

at the attachment point. As can be seen, the spikes in the current line up properly on the BOR and flat-plate portions of the composite body. The magnitude of the peaks are less important because they are a function of the subdomain size of the current expansion functions. Again, the effects of the flat-plate current on the BOR at and near the attachment point is clearly seen in the composite body curve. For the composite body solution, the maximum Fourier mode was nine. For the isolated BOR, only the ± 1 modes needed to be computed for this case of axial incidence.

While the original intent for developing the composite body iterative technique was for modeling intractable, electrically large complex structures where the current has largely local dependence, results have shown that subcomponents of the complex body can have dimensions on the order of a single wavelength. It is by enforcing current continuity during the iteration process via (1), (2), and (18) that makes the technique so effective for these smaller bodies.

Along with these size considerations, shadowing effects on the body were also examined. Several cases were run where the fin shadowed the rear section of the BOR, and other cases were run where the BOR shadowed an entire fin. The shadowing effects were clearly seen in all cases and there was no problem with solution convergence.

IV. Conclusions

A robust generalized technique has been developed to model large complex bodies with several smaller simpler bodies using the composite body matrix formulation. In this technique, the submatrices of the system impedance matrix are multiplied through and the current on various subcomponents of the body are evaluated separately. Taking advantage of the "iterative" nature of the solution process, the computed currents adjacent to the attachment point can be used to better estimate the normal boundary currents and enforce current continuity across the structure. On the other hand, the "partially direct" nature of the solution algorithm leads to efficient backscatter calculations. This is evidenced in (16) and (17), where it can be seen that the inverse of the self-impedance terms for the various subcomponents of the complex structure are computed once and then stored in memory. The composite body iteration technique was validated for a simple flat-plate test case. The general development of the iteration technique was then applied to the specific case of a BOR with two flat-plate attachments (our generic missile model). Traditionally, this problem has been solved using a method based on partitioning. This method takes advantage of the partial block diagonal nature of the composite body MoM matrix to reduce the size of the largest matrix that must be inverted. It was observed, however, that the composite body matrix equation was no longer partial block diagonal when the null field interior boundary conditions were included in the matrix formulation. For this case, where the method of partitioning provided limited utility, the composite body iteration technique preserved the block diagonal matrix formulation of the original BOR scheme. The flat-plate results showed that the iteration technique provided traditional MoM quality results (while running in half the time [15]). The BOR flat-plate results showed that an efficient in-core solution was still possible for this electrically large complex structure.

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