Gaussian Rough Surfaces and Kirchhoff Approximation

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***Abstract*— Electromagnetic scattering is often solved by applying Kirchhoff approximation to the Stratton–Chu scattering integral. In the case of rough surfaces, it is usually assumed that this is possible if the incident electromagnetic wavelength is small compared to the mean radius of curvature of the surface. Accordingly, evaluation of the latter is an important issue. This paper generalizes the groundwork of Papa and Lennon [1] by computing the mean radius of curvature for Gaussian rough surfaces with no restriction on its correlation function. This is an interesting extension relevant to a variety of natural surfaces. Relations between the surface parameters and the mean radius of curvature are determined and particular attention is paid to the relevant small slope regime.**

***Index Terms*—Electromagnetic scattering, Kirchoff approximation, rough surfaces.**

I. Introduction

The problem of the electromagnetic scattering from natural surfaces is a matter of great relevance from both theoretical and application points of view [2]. This problem is of interest in many research areas, including remote sensing of the environment, medical imaging, sonar, optics, and astronomy.

A popular and effective approach calls for the surface height description by means of random functions, usually Gaussian, and for the evaluation of the mean scattered field, or density power, by means of an approximation of the scattering integral in terms of local boundary conditions. This allows the use of Fresnel plane wave reflection coefficients and is consistent with Kirchhoff approximation. This procedure is reasonable whenever the mean radius of curvature R of the surface is much greater than the incident electromagnetic wavelength λ.

It is customary to model the surface height by means of a zero-mean Gaussian random function whose height correlation belong to a restricted class of functions [1]. In this paper, we generalize the groundwork of Papa and Lennon [1] to the case of Gaussian surface height whose first and second derivatives at the same point can be correlated, i.e., no restriction to the height correlation is requested. This is relevant for a certain class of height correlation functions [1], [3] and for the cases of certain fractal surfaces [4], [5].

The mean radius of curvature R is expressed in terms of the usual statistical parameters, i.e., the characteristics of the surface profile. In the case of uncorrelated height first and second derivatives at the same point, the mean radius of curvature R is an integral (hypergeometric) function of surface parameters [1]

$$R = \left[\frac{\sigma''}{2\sqrt{\pi}\sigma'} U\big(0.5,\, 0,\, 0.5/\sigma'^2\big)\right]^{-1} \tag{1}$$

wherein σ'^2 and σ''^2 are the variances of the height surface first and second derivatives, respectively, and $U(\cdot)$ is the confluent hypergeometric function of the second kind [6].

For the relevant case of small slope regime [7], [8] the expression of R given by (1) can be analytically evaluated in a closed form [1].

Let us emphasize that derivation of (1) is possible if the assumed height correlation function is differentiable, stationary and rotationally invariant with odd-order derivatives that vanish at the origin [1]. This implies that $\langle z'(x)z''(x)\rangle$ is equal to zero, $z'(\cdot)$ and $z''(\cdot)$ being the height function first and second derivatives and $\langle\cdot\rangle$ the ensemble average [1]. Such a result is a key element in the simplification operated in deriving (1). In the following, it will be referred to as the inchoerence theorem.

In this paper, we show that it is possible to evaluate the mean radius of curvature also for correlated height first and second derivatives at the same point, i.e., for any height correlation function. This is done by extending the approach given in [1]. We show that the general solution exhibits an extra term compared to the classical case. A full discussion on such a new general result is presented. In particular, the small slopes regime is examined in detail. For this case, a simple and readable analytical expression is obtained and discussed.

Physical motivation of this work is that there are surface models widely employed in electromagnetic scattering from natural surfaces that do not comply with Papa and Lennon's paper [1]. Two cases are of relevance and merit to be referred to as surfaces whose correlation is not differentiable at the origin and fractal surfaces.

The first case is widely illustrated in pertinent literature (see [3, Appendix 2B]) which reads "$\cdots$ bell-shaped curve generated by the Gaussian correlation function does not occur very often. Instead, many angular scattering coefficient curves appear to follow an exponential shape generated by the expo-

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nential correlation function $\cdots$. This is a correlation function used very often in practice." As a matter of fact, the more useful function in practical application is a combination of an exponential and a Gaussian function which, again, does not fulfill the incoherence theorem [3].

The second case pertains to fractal surfaces. Electromagnetic scattering from natural surfaces modeled by fractal surfaces has provided remarkable results [4], [5]. These surfaces show a correlated height first and second derivatives at the same point. A physical motivation is given by the well-known persistence and antipersistence behavior [9], [10]. As a consequence, the results illustrated in this paper are preliminary to the case of rough surface modeled in terms of fractal surfaces [4].

The paper is organized as follows. In Section II, the general solution to the problem is determined and a comparison with previous relevant results is accomplished. In Section III, the general solution is specified under some usually verified hypothesis; the small slope regime is investigated in detail and compared to what is illustrated in [1]. Finally, in Section IV, conclusions are reported.

II. General Solution

In this section, we determine the expression of the surface mean radius of curvature as defined in [1]

$$R = 1/\langle |K| \rangle \tag{2}$$

wherein $\langle |K| \rangle$ is the mean curvature [1], [11]:

$$\begin{aligned}\langle |K| \rangle &= \left\langle \left| \frac{z''}{[1+(z')^2]^{3/2}} \right| \right\rangle \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{|z''|}{[1+(z')^2]^{3/2}}\, p(z', z'')\, dz'\, dz''. \end{aligned}\tag{3}$$

In (3), $p(z', z'')$ is the joint probability density function (pdf) of first and second derivatives of the height profile $z(x)$[1] at zero displacement.[2]

In the classical case, evaluation of (3) is made possible by assuming that the joint pdf $p(z', z'')$ can be factorized by postulating indipendency of $z'(x)$ and $z''(x)$ [1]. Unfortunately, this is not always the case [3], [5], even in situations of interest for the applications [3]–[8]. If the mutual correlation between $z'(x)$ and $z''(x)$ is zero, this does not imply in general their independency, but for the Gaussian case, where also $z'(x)$ and $z''(x)$ are Gaussian as well. In the following, we restrict ourselves to the Gaussian case and generalize computation of the mean curvature (3) to the case of correlated $z'(x)$ and $z''(x)$. This is, for example, the case of some fractal surfaces.

To evaluate the integral of (3), we exploit an appropriate linear transformation that factorizes the joint (Gaussian) pdf's of (3). We define the mutual correlation coefficient ρ between $z'(x)$ and $z''(x)$ as follows:

$$\rho = \frac{r_{z'z''}}{\sigma'\sigma''} \tag{4}$$

[1]We consider the one-dimensional (1-D) case in accordance to what examined in [1].

[2]Evaluation of the mean radius of curvature at a given point is of interest. Accordingly, the joint correlation function in the same point, i.e., at zero displacement [1], [11] must be used in (3).

and apply the following transformation:

$$\begin{cases} \eta = z' \\ \zeta = z'' - \rho \dfrac{\sigma''}{\sigma'} z' = z'' - \kappa z' \end{cases} \tag{5}$$

with $\kappa = \rho\sigma''/\sigma'$. Note that generally the quantities appearing in (4) may depend on x for the nonhomogenous case. The case of [1] is recovered by letting ρ equal to zero.

The two new Gaussian processes η and ζ are independent. In fact, their mutual correlation coefficient $\overline{\rho}$ is given by

$$\overline{\rho} = \frac{\langle \eta(x), \zeta(x) \rangle}{\sigma_\eta \sigma_\zeta} = \frac{1}{\sigma_\eta\sigma_\zeta}(\langle z'(x), z''(x)\rangle - \rho\sigma'\sigma'') = 0 \tag{6}$$

where σ_η and σ_ζ are the standard deviation of η and ζ, respectively. As a consequence, the random processes η and ζ are uncorrelated and, being Gaussian, they are also independent. Their pdf's are completely characterized by knowledge of their variances

$$\begin{cases} \sigma_\eta^2 = \sigma'^2 \\ \sigma_\zeta^2 = \sigma''^2(1-\rho^2). \end{cases} \tag{7}$$

We make use of the transformation of (5) in (3) and we get

$$\begin{aligned}\langle |K| \rangle = \frac{1}{2\pi\sigma_\zeta\sigma_\eta} \int_{-\infty}^{\infty} \frac{1}{[1+\eta^2]^{3/2}} \exp\left(-\frac{\eta^2}{2\sigma_\eta^2}\right) \\ \cdot \int_{-\infty}^{\infty} |\zeta + \kappa\eta| \exp\left(-\frac{\zeta^2}{2\sigma_\zeta^2}\right) d\zeta\, d\eta \end{aligned}\tag{8}$$

the Jacobian of the transformation being unitary.

Although the transformation of variables of (5) allows factorization of the joint pdf $p(\eta, \zeta)$, it does not allow to factorize the integral of (8), at variance of the classical case [1]. This makes evaluation of the mean curvature by far less immediate.

To proceed further, we first evaluate the inner integral in the variable ζ

$$\begin{aligned}&\int_{-\infty}^{\infty} |\zeta+\kappa\eta| \exp\left(-\frac{\zeta^2}{2\sigma_\zeta^2}\right) d\zeta \\ &= \int_{-\kappa\eta}^{\infty} (\zeta+\kappa\eta) \exp\left(-\frac{\zeta^2}{2\sigma_\zeta^2}\right) d\zeta - \int_{-\infty}^{-\kappa\eta} (\zeta+\kappa\eta) \\ &\quad \cdot \exp\left(-\frac{\zeta^2}{2\sigma_\zeta^2}\right) d\zeta \\ &= 2\sigma_\zeta^2 \exp\left[-\frac{(\kappa\eta)^2}{2\sigma_\zeta^2}\right] + \sqrt{2\pi}\kappa\eta\sigma_\zeta \operatorname{erf}\left(\frac{\kappa}{\sqrt{2}\sigma_\zeta}\eta\right) \end{aligned}\tag{9}$$

wherein

$$\operatorname{erf}(\chi) = \frac{2}{\sqrt{\pi}} \int_0^\chi \exp(-s^2)\, ds \tag{10}$$

is the error function [12].

The first term of (9) is of Gaussian type and, therefore, formally similar to the one encountered in the classical case [1]. It turns out to be dependent on ρ via σ_ζ and κ, see (5) and (7), and it reduces to the classical case only and if only $\rho = 0$. The second term in (9) is a new one which vanishes for $\rho = 0$.

Substituting (9) in (8) and evaluating the outer integral in the η variable, we finally get

$$\langle |K| \rangle = \frac{\sigma''}{2\sigma'}\sqrt{\frac{1-\rho^2}{\pi}}\,U\!\left(0.5,\,0,\,\frac{0.5}{\sigma'^2(1-\rho^2)}\right) + \frac{\rho\sigma''}{\sigma'^2}\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\eta}{[1+\eta^2]^{3/2}}\exp\left(-\frac{\eta^2}{2\sigma'^2}\right) \cdot \mathrm{erf}\left(\frac{\rho}{\sqrt{2}\sigma'\sqrt{1-\rho^2}}\,\eta\right)d\eta. \tag{11}$$

Equation (11) is the main result of the present work: it is a generalized expression for the mean curvature and it can be numerically and even analytically evaluated in some appropriate slope (σ'^2) regimes [1].

Equation (11) is an extension of the classical result [1]. In fact, (11) allows to evaluate the conditions of validity of the Kirchhoff approximation even for a nonclassical Gaussian rough surface, whose first and second derivatives at the same point are not statistically independent. This implies that (11) fully extends results reported in [1] to a much wider class of random surfaces. In particular, (11) applies to the relevant scattering surfaces illustrated in the introduction.

The first term of (11), i.e., the term containing the hypergeometric confluent function of second type $U(\cdot)$, is identical to the one present in the classical case, but for the presence of ρ; if ρ is equal to zero, this term reduces to the expression described in [1] and reported under (1). The presence of ρ is due to the lack of restricting assumption on the height correlation function. The second term appearing in (11) is a totally new one, not predicted by the classical analysis conducted in [1]. Notwithstanding, the complexity of (11) is on the same footing of the classical one [see (1)] because numerical evaluation of $U(\cdot)$ requires computation of a 1-D integral.

Equation (11) may be satisfactory on the speculative viewpoint, but it is of limited practical comprehension because its dependence on surface parameters is rather involved. This is also true for the first term only, therefore for the classical formulation.

Comparison of (11) with the classical case (1) is in order. This is accomplished by numerically solving the integrals appearing in the formulation. Fig. 1 shows the ratio $\langle|K|\rangle/\langle|K|\rangle_{\rho=0}$ versus ρ parametrized for different values of σ' from 0.025 up to 10.0. Such values have been chosen in order to best represent the different slope regimes. Note that 0.25 is usually considered as the limit value for the small slope regime [8], [13].

This numerical analysis shows that in the small slope regime the classical approach overestimates the mean radius of curvature whereas in the large slope regime it underestimate it. This result is emphasized for large ρ values. This behavior of $\langle|K|\rangle/\langle|K|\rangle_{\rho=0}$ at small and large regimes is congruent with the appropriate asymptotic expansions [6] of (11). These expansions benefit of formula 13.1.8 and 13.5.11 of [6] and of the limiting behavior of the Gaussian function in the second term of (11). In particular, if $\sigma' \to 0$, we have

$$\frac{\langle|K|\rangle}{\langle|K|\rangle_{\rho=0}} = 1+\rho^2 \tag{12}$$

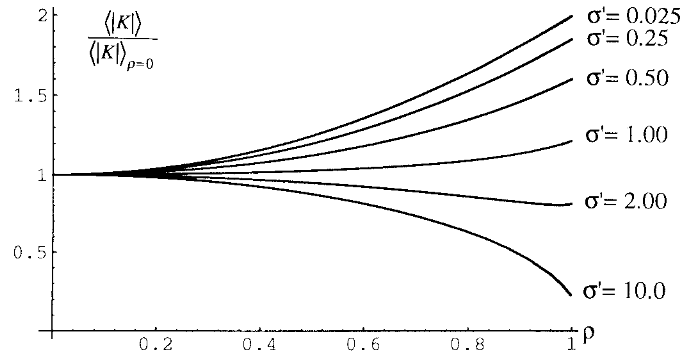

Fig. 1. Graph of $\langle|K|\rangle/\langle|K|\rangle_{\rho=0}$ versus ρ parametrized for different values of σ'. From top to bottom σ' is equal to: 0.025, 0.25, 0.50, 1.00, 2.00, 10.0.

and for $\sigma' \to \infty$

$$\frac{\langle|K|\rangle}{\langle|K|\rangle_{\rho=0}} = \sqrt{1-\rho^2}. \tag{13}$$

As final comments to (11) and Fig. 1, note that an increase (or decrease) of the mean radius of curvature implies a similar increase (or decrease) of the upper electromagnetic wavelength consistent with the Kirchhoff approximation. Examination of Fig. 1 shows that the small slope regime is the most critical because the classical approach overestimates the mean radius of curvature, thus, wrongly suggesting applicability of Kirchhoff approximation. For this and other reasons illustrated in the following, in Section III we detail (11) to the relevant case of the small slope regime [1], [13].

III. Special Solution: The Small Slope Regime

In this section, we specify the general result expressed by (11) to the small slope regime [1]. This has three main motivations. First, rough surfaces satisfying the small slope regime are of relevance in remote sensing as quoted in [13]. Second, in the small slope regime PO (physical optic) applies and scattering can be evaluated in a closed analytical form [1], [13]. This allows to relate statistics of the scattered field to surfaces parameters [2]. Third, in the small slope regime a simple and readable analytical expression of the mean curvature can be obtained.

Expansion of the two terms appearing in (11) for small values of σ'^2 is in order. For the first term by using formula 13.5.2 of [6] we get

$$\frac{\sigma''}{2\sigma'}\sqrt{\frac{1-\rho^2}{\pi}}\,U\!\left(0.5,\,0,\,\frac{0.5}{\sigma'^2(1-\rho^2)}\right) = \sigma''\frac{(1-\rho^2)}{\sqrt{2\pi}}\left\{\sum_{n=0}^{N}\frac{(1/2)_n(3/2)_n}{n!}\left[-2\sigma'^2(1-\rho^2)\right]^n + o\left[2\sigma'^2(1-\rho^2)\right]^{N+1}\right\} \tag{14}$$

where

$$(p)_n = p(p+1)\cdots(p+n-1), \qquad (p)_0 = 1$$

and $o(\cdot)$ is the Landau symbol.

In the classical case, a similar expression has been derived [1] and can be retrieved from (14) by setting $\rho = 0$ and

$N = 1$. Accordingly, (14) is much more general. Note that the presence of the factor $1 - \rho^2 \leq 1$ in the reminder of the series generally improves validity of its truncation compared to the classical case.

The second term of (11) can be evaluated, in the same hypothesis, by noting that for small value of σ'^2 the Gaussian term in the integral is concentrated around the origin. Accordingly, we can expand the irrational term in the integral of (11) around $\eta = 0$

$$\frac{\eta}{[1+\eta^2]^{3/2}} \approx \eta - \frac{3}{2}\eta^3 + \frac{15}{8}\eta^5 + o(\eta^7) \tag{15}$$

thus getting

$$\begin{aligned} &\frac{\rho\sigma''}{\sigma'^2}\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\eta}{[1+\eta^2]^{3/2}} \exp\left(-\frac{\eta^2}{2\sigma'^2}\right) \\ &\quad\cdot \operatorname{erf}\left(\frac{\rho}{\sqrt{2}\sigma'\sqrt{1-\rho^2}}\,\eta\right) d\eta \\ &\approx \frac{\rho\sigma''}{\sigma'^2}\sqrt{\frac{2}{\pi}}\int_0^\infty \left(\eta - \frac{3}{2}\eta^3 + \frac{15}{8}\eta^5\right) \exp\left(-\frac{\eta^2}{2\sigma'^2}\right) \\ &\quad \operatorname{erf}\left(\frac{\rho}{\sqrt{2}\sigma'\sqrt{1-\rho^2}}\eta\right) d\eta. \end{aligned} \tag{16}$$

By making use of the result [12]

$$\begin{aligned} &\int_0^\infty \operatorname{erf}(\beta x)\exp(-\mu x^2)x\,dx = G(\beta,\mu) = \frac{\beta}{2\mu\sqrt{\mu+\beta^2}} \\ &\operatorname{Re}(\mu) > -\operatorname{Re}(\beta^2), \quad \operatorname{Re}(\mu) > 0 \end{aligned} \tag{17}$$

with the identifications

$$\mu = \frac{1}{2\sigma'^2}, \qquad \beta = \frac{\rho}{\sigma'\sqrt{2(1-\rho^2)}} \tag{18}$$

we have for the first term in (16)

$$\begin{aligned} &\frac{\rho\sigma''}{\sigma'^2}\sqrt{\frac{2}{\pi}}\int_0^\infty \eta\exp\left(-\frac{\eta^2}{2\sigma'^2}\right)\operatorname{erf}\left(\frac{\rho}{\sqrt{2}\sigma'\sqrt{1-\rho^2}}\,\eta\right) d\eta \\ &\quad = \sqrt{\frac{2}{\pi}}\,\sigma''\rho^2. \end{aligned} \tag{19}$$

For evaluating the subsequent terms, we differentiate both members of (17), thus obtaining

$$\begin{aligned} &\int_0^\infty \operatorname{erf}(\beta x)\exp(-\mu x^2)x^3\,dx \\ &\quad = -\frac{\partial G(\beta,\mu)}{\partial\mu} = \frac{\beta}{4}\frac{3\mu+2\beta^2}{\mu^2(\mu+\beta^2)^{3/2}} \end{aligned} \tag{20}$$

and we get

$$\begin{aligned} &-\frac{3}{\sqrt{2\pi}}\rho\frac{\sigma''}{\sigma'^2}\int_0^\infty \eta^3\exp\left(-\frac{\eta^2}{2\sigma'^2}\right) \\ &\quad\cdot\operatorname{erf}\left(\frac{\rho}{\sqrt{2}\sigma'\sqrt{1-\rho^2}}\,\eta\right) d\eta = \frac{3}{\sqrt{2\pi}}\,\sigma''\sigma'^2(\rho^4 - 3\rho^2). \end{aligned} \tag{21}$$

Similarly,

$$\begin{aligned} &\int_0^\infty \operatorname{erf}(\beta x)\exp(-\mu x^2)x^5\,dx \\ &\quad = \frac{\partial^2 G(\beta,\mu)}{\partial\mu^2} = \frac{\beta}{8}\frac{15\mu^2+20\mu\beta^2+8\beta^4}{\mu^3(\mu+\beta^2)^{5/2}} \end{aligned} \tag{22}$$

and

$$\begin{aligned} &\frac{15}{4\sqrt{2\pi}}\rho\frac{\sigma''}{\sigma'^2}\int_0^\infty \eta^5\exp\left(-\frac{\eta^2}{2\sigma'^2}\right)\operatorname{erf}\left(\frac{\rho}{\sqrt{2}\sigma'\sqrt{1-\rho^2}}\eta\right) d\eta \\ &\quad = \frac{15}{4\sqrt{2\pi}}\sigma''\sigma'^4(3\rho^6 - 10\rho^4 + 15\rho^2). \end{aligned} \tag{23}$$

This procedure could be continued, thus generating a series expansion in σ'^2 of the integral of (16).

We are now in a position to present the expansion of the mean curvature (11) in powers of σ'^2: it is sufficient to collect equal order terms in the expansions of the first term (14) and of the second term (19), (21), (23) and add them together.

To the zero-order (to σ'^2) we get

$$\langle|K|\rangle^{(0)} = \frac{\sigma''}{\sqrt{2\pi}}(1+\rho^2) \tag{24}$$

which reduces to the mean curvature zero-order approximation of the classical case as ρ is set equal to zero [1].

Equation (24) generalizes the result presented in [1] to surface profiles whose first- and second-order derivatives (at the same point) are correlated. In this case, the presence of ρ give rises to an extra factor quadratic in ρ. The resulting surface mean curvature increases with ρ. In the limit of $\rho^2 = 1$, the mean curvature is magnified by a factor 2 with respect to the case of uncorrelated (at the same point) first and second derivatives of $z(\cdot)$. As a consequence, at least in principle, the higher ρ the more questionable is the applicability of Kirchhoff approximation in the small slope regime due to the increase of $\langle|K|\rangle$.

To the first-order in σ'^2 we get

$$\begin{aligned} \langle|K|\rangle^{(1)} &= \frac{\sigma''}{\sqrt{2\pi}}\left[(1+\rho^2) - \frac{3}{2}(1+4\rho^2-\rho^4)\sigma'^2\right] \\ &= \frac{\sigma''}{\sqrt{2\pi}}\left\{\mathcal{A} - \frac{3}{2}\mathcal{B}\sigma'^2\right\} \end{aligned} \tag{25}$$

wherein $\mathcal{A}$ and $\mathcal{B}$ are functions of ρ only and turn out to be both unitary whenever $\rho = 0$. Hence, for the classical case ($\rho = 0$) we again have a mean curvature approximation in total accordance with known results [1]. We note also that in the classical case the zero-order approximation for $\langle|K|\rangle$ can be tolerated whenever $3\sigma'^2/2 \ll 1$. Conversely, the presence of $\mathcal{A}$ and $\mathcal{B}$ modifies this latter conclusion. In fact, we have now that the zero-order approximation for $\langle|K|\rangle$ can be tolerated whenever $3\mathcal{B}\sigma'^2/2\mathcal{A} \ll 1$.

To the second-order in σ'^2 we get

$$\begin{aligned} \langle|K|\rangle^{(2)} &= \frac{\sigma''}{\sqrt{2\pi}}\left[(1+\rho^2) - \frac{3}{2}(1+4\rho^2-\rho^4)\sigma'^2 + \frac{45}{8}\right. \\ &\qquad \left.\cdot\left(1+7\rho^2-\frac{11}{3}\rho^4+\rho^6\right)\sigma'^4\right] \\ &= \frac{\sigma''}{\sqrt{2\pi}}\left\{\mathcal{A} - \frac{3}{2}\mathcal{B}\sigma'^2 + \frac{45}{8}\mathcal{C}\sigma'^4\right\} \end{aligned} \tag{26}$$

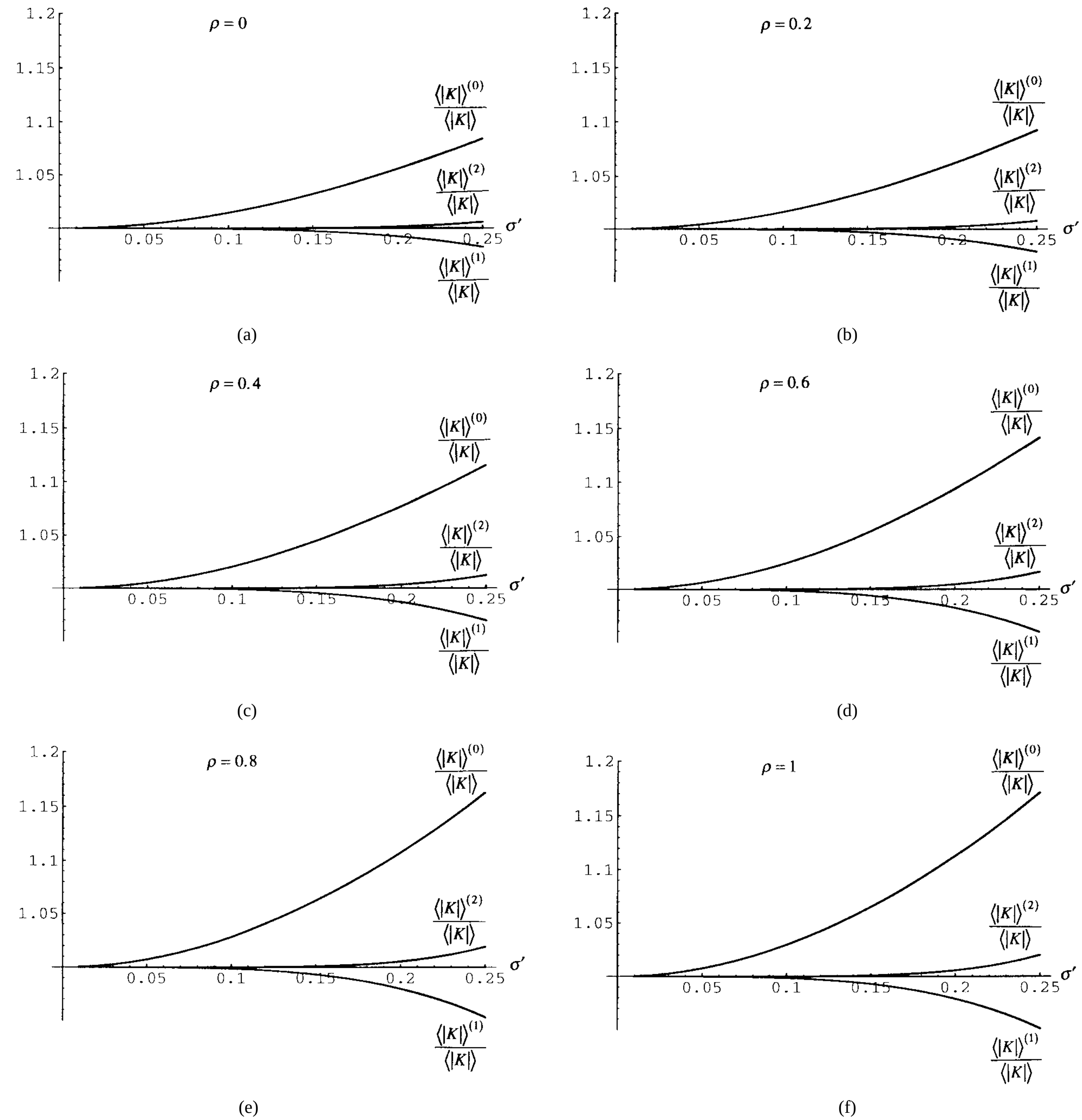

Fig. 2. Graphs of $\langle|K|\rangle^{(0)}/\langle|K|\rangle$, $\langle|K|\rangle^{(1)}/\langle|K|\rangle$, $\langle|K|\rangle^{(2)}/\langle|K|\rangle$ versus σ' for different values of ρ. The σ' values range from 0 to 0.25 (small slope regime). Curves from (a)–(f) are parametrized in ρ from zero to one with a 0.2 step. The curves relevant to $\langle|K|\rangle^{(0)}/\langle|K|\rangle$ are always the top curves whereas the curves relevant to $\langle|K|^{(1)}/\langle|K|\rangle$ are always the bottom curves. The intermediate ones corresponds to $\langle|K|\rangle^{(2)}/\langle|K|\rangle$.

wherein $\mathcal{C}$ is also a functions of ρ only which turns out to be unitary whenever $\rho = 0$.

As a first general comment note that in the small slope regime, we get to an explicit analytical expression of the mean curvature which is also simple and readable. Complexity of the obtained expressions of the mean curvature is on the same footing of the classical results [1]. In fact, (24)–(26), obviously polynomial in σ'^2 show (not obviously) coefficients polynomial in ρ^2. This is by no means straightforward since (11) has an involved dependence on ρ. Similarly to the classical results we have now a dependence on σ', σ'', and on the additional parameter ρ only. As a consequence, although the height correlation is general, in order to evaluate the mean curvature we only require the extra knowledge of one simple parameter.

Let us now further examine the behavior of the different approximations expressed by (24)–(26). Accordingly, in Fig. 2 we have parametrized the plots of $\langle|K|\rangle^{(0)}/\langle|K|\rangle$, $\langle|K|\rangle^{(1)}/\langle|K|\rangle$, $\langle|K|\rangle^{(2)}/\langle|K|\rangle$ versus σ' for different values of ρ. The σ' values always range from 0 to 0.25. Note that the limiting value has been chosen according to what advocated in literature [7], [12] and implies the first-order term in σ'^2 to be at least one order of magnitude smaller than the zero-order term. Curves are parametrized in ρ; Fig. 2(a)–(f) is relevant to values of ρ from zero to one in steps of 0.2. The curves relevant to $\langle|K|\rangle^{(0)}/\langle|K|\rangle$ are always the top curves,

TABLE I
THIS TABLE IS RELEVANT TO THE SMALL SLOPE REGIME. THE COLUMN $e_{1\%}^{(o)}$ REPORTS THE UPPER σ' VALUES SUCH THAT THE ZERO-ORDER APPROXIMATION FOR $\langle|K|\rangle$ EXHIBITS AN ERROR NOT LARGER THAN 1%. THE OTHER TWO COLUMNS REFER TO THE FIRST AND SECOND-ORDER APPROXIMATION, RESPECTIVELY

Table I:

ρ	$e_{1\%}^{(0)}$	$e_{1\%}^{(1)}$	$e_{1\%}^{(2)}$
0	0.08	0.20	> 0.25
0.2	0.08	0.20	> 0.25
0.4	0.07	0.19	0.25
0.6	0.06	0.19	0.24
0.8	0.06	0.17	0.23
1	0.06	0.17	0.22

whereas the curves relevant to $\langle|K|\rangle^{(1)}/\langle|K|\rangle$ are always the bottom curves. The intermediate ones corresponds to $\langle|K|\rangle^{(2)}/\langle|K|\rangle$. As a result, we always have that $\langle|K|\rangle^{(0)}$ and $\langle|K|\rangle^{(2)}$ overestimate $\langle|K|\rangle$ whereas $\langle|K|\rangle^{(1)}$ underestimates it. Estimation of the mean curvature improves as soon as we move to higher order approximations. When ρ and/or σ' increase all approximations achieve poorer results.

Assessment of validity of results is given in Table I, which presents the σ' value for which the considered N order approximation results in a 1% error (absolute value) with reference to the exact value numerically evaluated by means of (11). In the zero-order approximation, this always happens for very small σ' values, thus confirming the need to move to higher order approximations. In particular, the second-order approximation reaches the 1% error (absolute value) always around or beyond the small slope limit value.

As a final comment we note that successive approximations of $\langle|K|\rangle$ appear as an alternate series converging to the true $\langle|K|\rangle$ of (11). To get a safe condition on the mean radius of curvature we must consider an approximation which underestimates R. The zero-order approximation has been shown to be too poor and we had better to move to the second-order one.

IV. CONCLUSIONS

A generalization of the conditions for the validity of Kirchhoff approximation in the electromagnetic scattering from rough surfaces is presented. Kirchhoff approximation is verified for surfaces whose radius of curvature is large compared to the electromagnetic wavelength of the incident field. In order to be useful this latter relationship must be expressed in terms of some meaningful parameters characterizing the surface profile, thus leading to appropriate conditions of validity. In this paper, this has been accomplished for the general case of a Gaussian surface with general correlation function, thus not ensuring applicability of the incoherence theorem [1]. This is relevant to surface models widely employed in electromagnetic scattering. This includes surfaces not satisfying Papa and Lennon's hypothesis reported by Fung [3] as well as fractal surfaces.

For any slope regime, the new general analytical expression of the mean curvature exhibits a complexity not higher than the classical one [see (1), (11)] and, similarly to classical results, the mean curvature depends on σ' and σ'' and on the additional parameter ρ [see (4)]. Numerical results show that significant differences with respect to the classical results may be encountered. In particular, it has been shown that in the small slope regime classical results are critical because they overestimate of the mean radius of curvature. For this reason and for its relevance to applications the small slope regime has been studied in detail.

The result is a polynomial expression in both σ'^2, the variance of the derivative of the surface profile, and ρ^2, the square of the correlation coefficient. This is by no means straightforward due to the rather involved dependence on ρ in the general formulation that encompasses a confluent hypergeometric-related function, an irrational one, and an integral without a primitive. Quite surprisingly, the small slope regime expressions for the mean curvature exhibit again, a complexity not higher than that encountered in the classical case. The mean curvature expression to the zero-order in σ'^2 is too poor, whereas the first-order expression leads to overestimate R. Accordingly, in order to get a safe condition on R we must move to the second-order approximation, which is rather good in general and, in any case, always underestimates the true mean radius of curvature.

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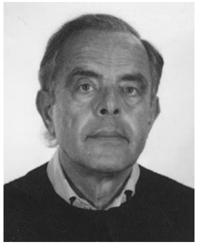

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