Superdirective Radiation from Finite Gratings of Rectangular Grooves

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***Abstract*—In this paper, the superdirective property of arrays comprising a finite number of rectangular grooves is studied by using the modal approach, which is a simple but powerful technique for analyzing these gratings. Numerical results show that when a specific characteristic mode of the structure referred to as the π mode is excited, the intensity of the field scattered in the specular direction exhibits a maximum, which becomes sharper and narrower as the number of grooves is increased. The far-field patterns exhibit superdirectivity at these resonant frequencies as evidenced by their beamwidths that are narrower than those expected from apertures of comparable size. The model-based parameter estimation (MBPE) technique has been employed to help locate extremely narrow resonances that are characteristic of superdirective arrays and its usefulness has been demonstrated.**

***Index Terms*—Gratings, superdirectivity.**

I. Introduction

It is well known that antenna arrays formed by a finite number of active elements can exhibit superdirective properties [1]. The phenomenon of superdirectivity has attracted the attention of many authors [2] since it appears to be promising for possible applications in high-gain antenna design. However, despite a considerable amount of effort invested in the investigation of these systems, they have been found to be difficult to realize in practice, since they are extremely sensitive to small changes in both the phase and amplitude distributions of the sources and this sensitivity increases with the increase in the directivity. As a consequence, a highly precise control of these quantities is needed and this is technically very difficult to achieve.

The use of passive elements in antenna arrays enables us to circumvent the difficulties associated with the excitation. Passive superdirective antennas are characterized by a very high level of stored electromagnetic energy in the vicinity of the antenna elements [3], [4]. The enhancement of the fields in the near region is associated with the excitation of high Q resonances in the system [5].

In early works, the modal method has been widely applied to the problem of diffraction by infinite periodic gratings formed by cavities with rectangular [6], triangular [7], and semicircular [8] shapes. Recently, it has been extended by Li [9] to handle infinite gratings with arbitrarily shaped grooves. However, until quite recently, the application of the modal approach to finite gratings had been limited only to rectangular cavities [10], and the extension to the problem of finite gratings with cavities of arbitrary cross section was carried out only very recently [11].

In an earlier paper, the authors have demonstrated [12] that the scattered field of a structure formed by a finite number of slotted cylinders exhibits superdirective characteristics at the resonant frequencies of the system. The occurrence of superdirectivity can be attributed to the excitation of modes that induce a phase reversal in the adjacent scatterers. It has also been shown that the nature of field distribution in the interior of the cavities plays an important role in the generation of superdirectivity, since this phenomenon is highly sensitive to the variations in the field amplitude. In this paper, we investigate a structure formed by a finite array of rectangular grooves in an attempt to address the question whether this property is a common attribute of systems formed by coupled resonant elements. This geometry has a distinct advantage over the slotted cylinder array, because the spacing between the adjacent grooves can be made small arbitrarily. This, in turn, enables us to increase the coupling between adjacent cavities, and thereby improve the superdirectivity of the system.

In Section II, we outline the modal approach used for the calculation of the scattered field from a surface with a finite number of rectangular grooves of arbitrary widths and depths. Numerical examples for finite arrays with 3, 5, 7, and 15 grooves are shown in Section III. The frequency responses of the above structures are presented together with the distribution of the fields in the interior regions of the cavities, as well as the far-field patterns at the resonant frequencies of the system. In Section IV, we illustrate the use of the model-based parameter estimation (MBPE) technique to search for the resonant frequencies. We show that this algorithm is very useful for locating extremely narrow peaks that are otherwise likely to be skipped if the frequency scan were carried out in a conventional manner. Finally, some concluding remarks that summarize the findings of this work are presented in Section V.

II. The Modal Method

We consider the problem of scattering by a perfectly conducting surface with a finite number of arbitrarily spaced grooves of rectangular shape, as shown in Fig. 1. The structure as well field variables are invariant in the z direction, and the

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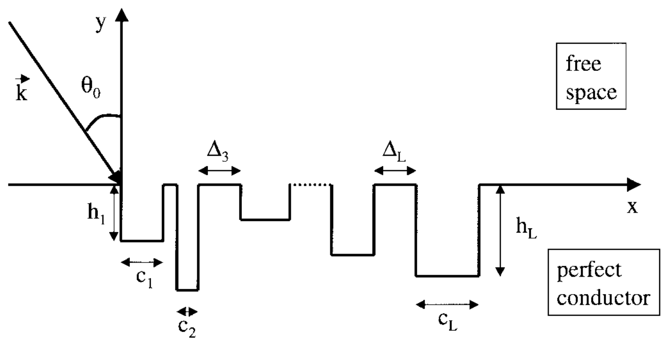


Fig. 1. Cross section of groove arrays with rectangular cavities.

problem reduces to a two-dimensional one. The groove widths and heights are c_l and h_l, respectively, and their spacing is Δ_l. This is a generalized version of the geometry investigated earlier by the authors [10], where all the grooves had identical depths.

Let an H-polarized (magnetic field parallel to the z direction) plane wave be incident at an angle θ_0 from the y axis. Assuming an $\exp(j\omega t)$ time dependence, we can derive the following wave equation satisfied by H_z:

$$(\nabla^2 + k^2)H_z(x,y) = 0 \tag{1}$$

where k is the free-space wavenumber. The field in the free-space region must satisfy (1) together with the radiation condition at infinity and the boundary conditions on the surface of the conductor. Next, we subdivide the domain of analysis into two: region-I $(y > 0)$, i.e., the upper half-space and region-II $(-h < y < 0)$, i.e., interior of cavities. The field in region-I is expressible as a sum of three terms as follows:

$$H_z^I(x,y) = H_z^{\text{inc}}(x,y) + H_z^{\text{spec}}(x,y) + H_z^{\text{scatt}}(x,y) \tag{2}$$

where $H_z^{\text{inc}}(x,y)$ represents the incident plane wave

$$H_z^{\text{inc}}(x,y) = e^{-j(\alpha_0 x - \beta_0 y)} \tag{3}$$

$H_z^{\text{spec}}(x,y) = e^{-j(\alpha_0 x + \beta_0 y)}$ is the plane wave reflected specularly by a flat surface and $H_z^{\text{scatt}}(x,y)$ represents the field scattered by the indentations of the surface. Our interest in this work is to investigate the last term (see [10]).

The scattered field admits a spectral domain representation

$$H_z^{\text{scatt}}(x,y) = \int_{-\infty}^{\infty} R(\alpha) e^{-j(\alpha x + \beta y)}\, d\alpha \tag{4}$$

where the complex function $R(\alpha)$ is the unknown Rayleigh function; $\beta^2 = k^2 - \alpha^2$ and $\alpha_0 = k\sin(\theta_0)$ and $\beta_0 = k\cos(\theta_0)$ are the x and y components of the incident wave vector $\vec{k_0}$, respectively.

Next, the fields inside the cavities are expanded in terms of the waveguide modes associated with each groove. For instance, inside the lth groove we can write the H_z^l field as

$$H_z^l(x,y) = \sum_{m=0}^{\infty} a_{ml} \cos\left[\frac{m\pi}{c_l}(x - x_l)\right] \cos[\mu_{ml}(y + h_l)] \tag{5}$$

where

$$\mu_{ml}^2 = k^2 - \left(\frac{m\pi}{c_l}\right)^2$$

and a_{ml} are the unknown modal coefficients. It is evident that the modal functions satisfy the boundary conditions at the walls of the grooves, viz.,

$$\frac{\partial H_z(x,y)}{\partial \hat{n}} = 0, \text{ on the perfectly conducting surface} \tag{6}$$

where $\hat{n}$ is the normal to the surface of the grooves.

The next step is to match the tangential magnetic fields in regions I and II at the interface $(y = 0)$. This leads to the

$$2e^{-j\alpha_0 x} + \int_{-\infty}^{\infty} R(\alpha) e^{-j\alpha x}\, d\alpha = \sum_{m=0}^{\infty} a_{ml} \cos\left[\frac{m\pi}{c_l}(x - x_l)\right] \cos[\mu_{ml} h_l]. \tag{7}$$

Similarly, the companion equation is derived by matching the tangential electric field $E_x(x,y)$ at the interface and enforcing the boundary condition on the perfectly conducting surfaces. It reads

$$-j\int_{-\infty}^{\infty} \beta R(\alpha) e^{-j\alpha x}\, d\alpha = \sum_{m=0}^{\infty} -\mu_{ml} a_{ml} \cos\left[\frac{m\pi}{c_l}(x - x_l)\right] \times \sin[\mu_{ml} h_l]\, \text{rect}\left(\frac{x - x_l}{c_l}\right) \tag{8}$$

where $\text{rect}(s)$ is the rectangle function defined as

$$\text{rect}(s) = \begin{cases} 1, & \text{for } 0 < s < 1 \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

By projecting (7) and (8) on convenient bases, we can derive an infinite system of equations for the unknown Rayleigh function $R(\alpha)$. Toward this end, we project (7) on the modal eigenfunctions basis of the ith groove, viz., $\{\cos[\frac{n\pi}{c_i}(x - x_i)]\}$ and this enables us to relate the modal amplitudes a_{ni} in terms of $R(\alpha)$ as follows

$$2e^{-j\alpha_0 x_i} I_{ni}(\alpha_0) + \int_{-\infty}^{\infty} R(\alpha) e^{-j\alpha x_i} I_{ni}(\alpha)\, d\alpha = C_{ni} a_{ni} \cos[\mu_{ni} h_i] \tag{10}$$

where

$$I_{ni}(\alpha) = \int_0^{c_i} e^{-j\alpha x} \cos\left[\frac{n\pi}{c_i} x\right] dx \tag{11}$$

and

$$C_{ni} = \left\{ \begin{matrix} c_i & \text{for } n = 0 \\ c_i/2 & \text{for } n \neq 0 \end{matrix} \right\}. \tag{12}$$

Finally, (8) is projected on the basis functions $\{e^{-j\alpha x}\}$, that are orthogonal in the interval $[-\infty, \infty]$. The result is

$$-2\pi j\beta R(\alpha) = \sum_{l=1}^{L} \sum_{m=0}^{\infty} -\mu_{ml} a_{ml} \sin[\mu_{ml} h_l] e^{j\alpha x_l} I_{ml}(-\alpha). \tag{13}$$

Substitution of (10) in (13) yields a Fredholm equation of the second kind for the unknown amplitude distribution function

$R(\alpha)$. This integral equation is discretized and truncated to transform the system into a matrix equation of the form

$$M_{ij}R_j = V_i \tag{14}$$

where $R_j = R(\alpha_j)$. The solution of the above equation is derived by using standard matrix inversion schemes.

The intensity of the scattered field in the direction determined by α_j is defined as

$$I(\alpha_j) = |R(\alpha_j)|^2 \frac{\beta_j}{\beta_0}. \tag{15}$$

III. Numerical Results

We begin this section by interjecting a comment on the numerical implementation of the system of (10) through (13). To transform the original system into (14), the infinite integration interval in the variable α is reduced to the interval $[\alpha_0 - \mathrm{EXT}, \alpha_0 + \mathrm{EXT}]$ where $\mathrm{EXT} = 2.2k$ guarantees the convergence of the results. Once this interval is determined, the integral is discretized. The number of subdivisions is closely connected to the total width of the structure, i.e., to the number of grooves in the surface. However, for the cases illustrated here, it was sufficient to consider 260 subintervals. On the other hand, just 26 modal terms for the representation of the fields inside each cavity was sufficient to ensure the accuracy of the results. As a test of the computations, we also calculated the Poynting flow through a closed-surface enclosing the corrugations and verified that this flow was less than 10^{-16} for all the calculations performed. This guaranteed the fulfillment of the energy balance condition.

In the following examples, we analyze the fields scattered by structures formed by 3, 5, 7, and 15 equally spaced grooves, illuminated by a normally incident plane wave, and plot the intensity of the field scattered in the specular direction [see (15)] as a function of the normalized frequency $(kh = 2\pi h/\lambda)$. The first case considered is a surface with three identical grooves whose geometrical parameters are $c = 0.2h$ and $\Delta = 0.1h$. Fig. 2 displays the behavior of scattered field intensity versus kh for this case. Owing to the symmetry of the normally incident field, the structure admits only two characteristic modes. The first of these exhibits a phase behavior that is uniform across the interface, whereas the second has an alternating phase variation, one in which the phase shifts by π between the adjacent grooves. In the region near $h = \lambda/4$ (i.e., $kh = \pi/2$), both of the eigenmodes of the structure are excited and, consequently, the response curve has two maxima. The maximum with the lower Q is located at $kh = 1.3$ and corresponds to the in-phase mode, whereas the sharp peak is associated with the excitation of the π mode. This mode also has certain interesting properties, as will be evident from the results presented below.

In Fig. 2(b), we plot the far-field pattern corresponding to the high Q resonance. It is evident that the pattern exhibits a superdirective character since the total width W of the structure is small compared to the wavelength ($\simeq 0.2\lambda$). To gain a better understanding of this phenomenon, we examine the behavior of the fields inside the cavities. Fig. 3(a) and (b) shows the amplitude and phase distributions of the resonant field within the interior of the grooves. In these figures, the vertical axis represents the depth of the grooves and the horizontal axis spans the total width of the structure. Fig. 3(b) clearly demonstrates that the relative phase associated with this mode alternates between 0 and π radians along the interface. We also note that the excitation of this mode is accompanied by an enhancement of the field inside the grooves, as shown in Fig. 3(a).

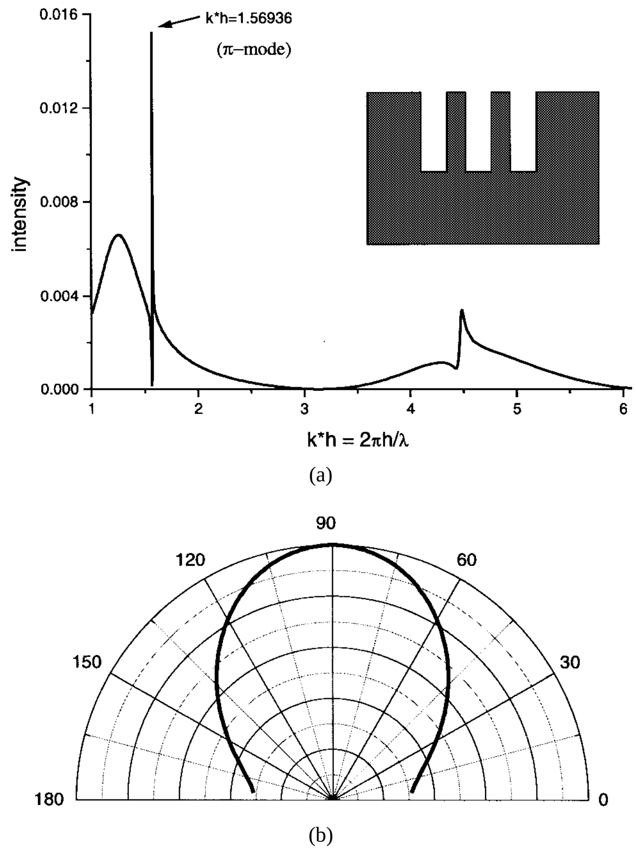
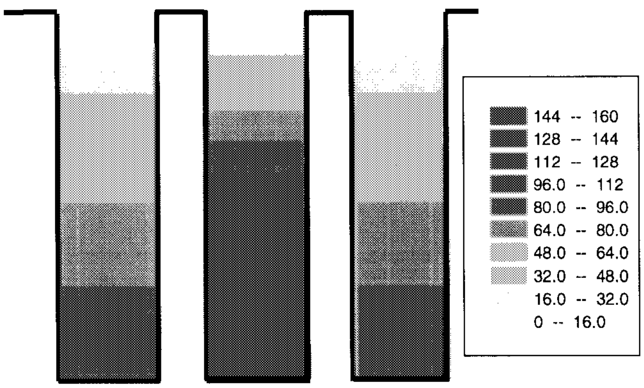
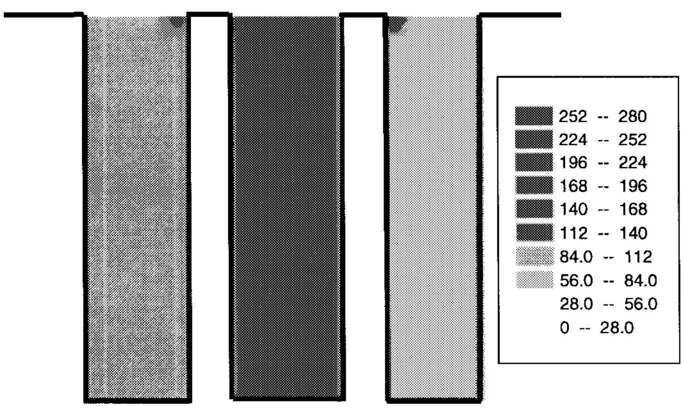


Fig. 2. (a) Frequency scan of the field intensity scattered in the normal direction by an array of three identical grooves with equal spacing. The parameters are: $c = 0.2h$, $\Delta = 0.1h$, and $\theta_0 = 0°$. (b) Far-field pattern of the structure in Fig. 2(a) for the π mode $(kh = 1.569\,36)$.

The next example we consider is formed by five grooves whose geometrical parameters are the same as those for the three-groove case, resulting in a total width of $W = 0.35\lambda$ for the structure. The intensity versus frequency plot is presented in Fig. 4(a) on an extended scale, in order to enable us to capture the fine features of the frequency behavior that are associated with this five-groove structure. We observe that the high Q resonance is now split into two, each of which has a very high Q. Furthermore, the low Q resonance is shifted in frequency relative to that of the three-groove structure. The appearance of a new resonance may be explained by the fact that an increase in the number of grooves is accompanied by an increase in the number of eigenmodes of the system. The π mode $(kh = 1.570\,788)$, which now has a higher Q than it did previously for the three-groove case, also produces

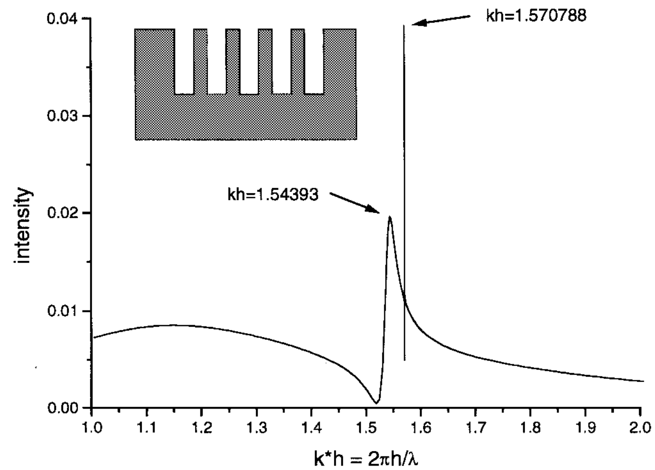


(a)

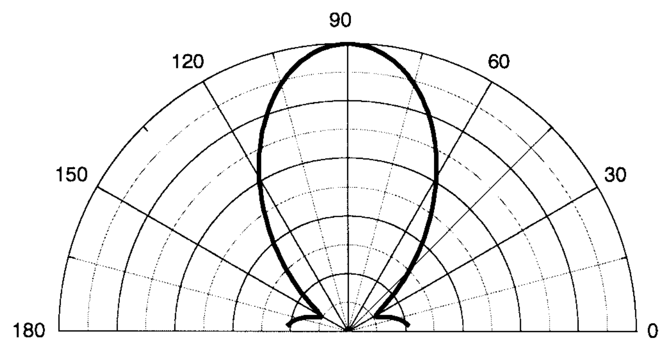


(b)

Fig. 3. Magnetic field distribution inside the cavities of the resonant structure described in Fig. 2 for the high Q resonance frequency $(kh = 1.569\,36)$. (a) Amplitude. (b) Phase.



(a)



(b)

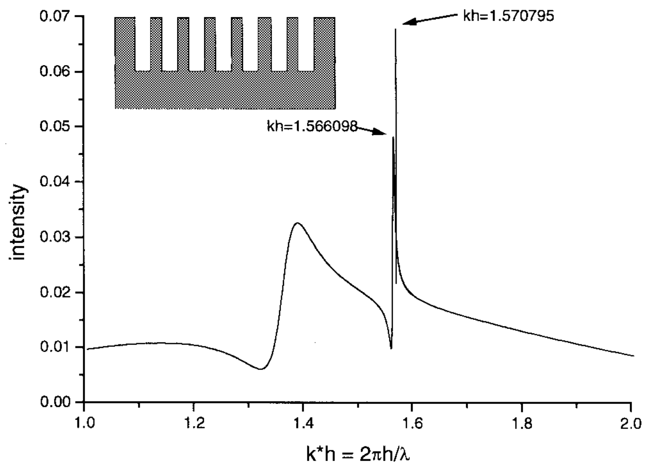
Fig. 4. (a) Frequency scan of the field intensity scattered in the normal direction by an array of five identical grooves with equal spacing. The parameters are: $c = 0.2h$, $\Delta = 0.1h$, and $\theta_0 = 0°$; (b) Far-field pattern of the structure in Fig. 4(a) for the π mode $(kh = 1.570\,788)$.

a narrower far field pattern [see Fig. 4(b)]. This demonstrates that the superdirective property of the structure is enhanced by an increase in the number of grooves, at least for the π mode.

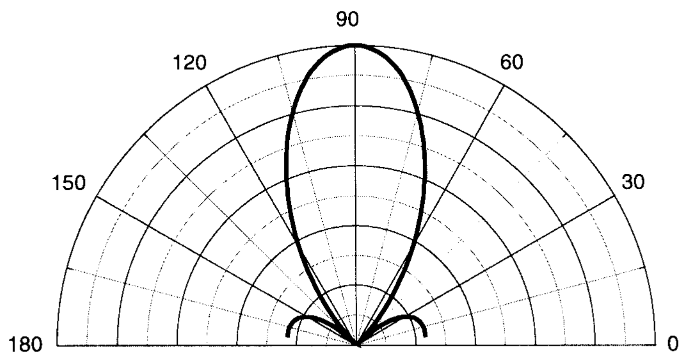
Finally, we increase the number of grooves yet again to seven with $c = \Delta = 0.2h$, which implies a total width $W = 0.65\lambda$. The results for this case, shown in Figs. 5 and 6, again show a trend similar to that we found when the number of grooves was increased from three to five. We observe, once again, that the high Q resonance is split into two very narrow peaks while the lower Q resonance shifts and its quality factor decreases [see Fig. 5(a)]. The far-field pattern corresponding to the π mode is found to be even narrower now than it was in the previous two examples [see Fig. 5(b)]. The distribution of the field inside the grooves is shown in Fig. 6 for the high Q resonance associated with $kh = 1.570\,795$. Its phase distribution, shown in Fig. 6(b), clearly indicates that it corresponds to the π mode. We also see that for this mode the enhancement of the field inside the cavities [see Fig. 6(a)] is considerably larger than in the previous cases with smaller number of grooves and that the same is true for other resonances of this structure. In Fig. 7 we show the far-field pattern associated with different resonant frequencies and, once again, we notice a continuation of the trend that we saw earlier when the number of grooves was increased from three to five.

Next, we address an important question regarding the stability of the superdirective resonances. We inquire, for instance, about the sensitivity of these resonances to the tolerance with which the structure is fabricated and ask to what extent we can vary the depths and widths of the grooves and still maintain the superdirectivity characteristics. This is an important question that we attempt to address by carrying out the numerical experiment described below.

We consider the simple case of a three-groove array and modify the original structure in two different ways, viz.: 1) by changing the depth of the leftmost cavity, i.e., letting $h_1 \neq h_2 = h_3$ and 2) by varying the width of the same groove $(c_1 \neq c_2 = c_3)$. Fig. 8(a) presents a set of curves corresponding to different values of h_1 and an expanded version of the above is plotted in Fig. 8(b) to better display the results in the resonance region where the frequency responses vary very rapidly. The three curves presented in the above figures correspond to depth variations of 1%, 3%, and 5% of the left groove. It is evident that although the resonance peak is still noticeable when the depth deviation is 5%, its amplitude is reduced considerably. Furthermore, Fig. 9 shows that the superdirective characteristic of the far-field pattern totally disappears. This behavior is not entirely unexpected, however, since the resonant excitation of an eigenmode of the multigroove structure is strongly determined by the depths of

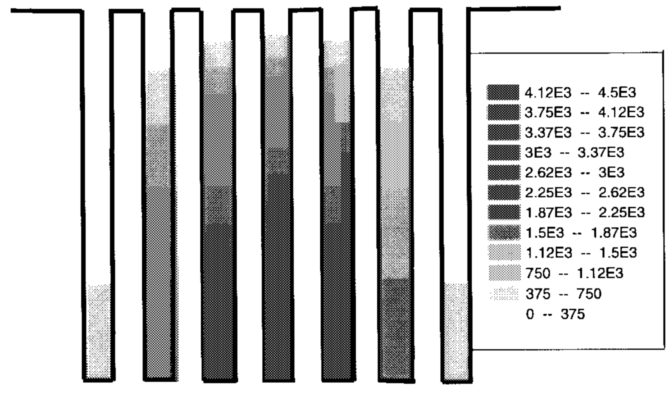


(a)

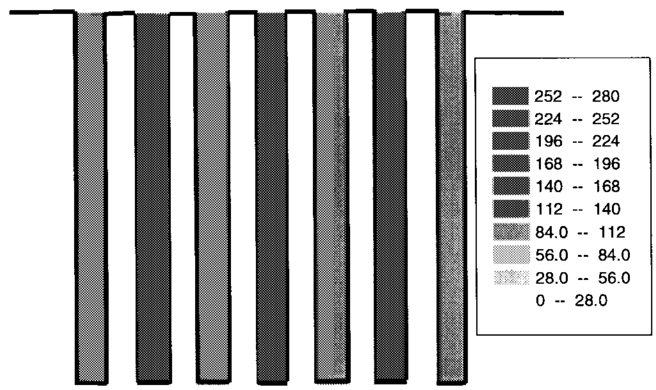


(b)

Fig. 5. (a) Frequency scan of the field intensity scattered in the normal direction by an array ofseven identical grooves with equal spacing. The parameters are: $c = 0.2h$, $\Delta = 0.2h$, and $\theta_0 = 0°$. (b) Far-field pattern of the structure in Fig. 5(a) for the π mode ($kh = 1.570\,795$).



(a)



(b)

Fig. 6. Magnetic field distribution inside the cavities of the resonant structure described in Fig. 5 for the high Q resonance frequency ($kh = 1.570\,795$). (a) Amplitude. (b) Phase.

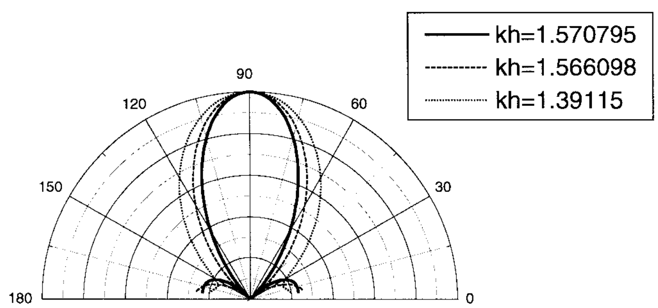


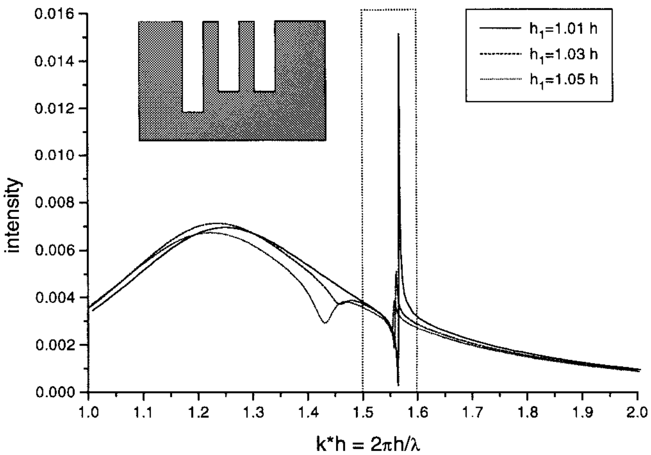
Fig. 7. Far-field patterns radiated by the structure formed by seven grooves shown in Fig. 5 for three different resonant frequencies.

its corrugations. However, in contrast to the depth variation, changes in the width of the left groove by 10%, 50%, or even 100%, alters the frequency characteristic as well as the far-field pattern only very slightly, as may be seen from Fig. 10(a) and (b), which may be compared with Fig. 2(a) and (b). We recall that we are investigating the $\lambda/4$ resonance, which is determined by the depth of the grooves. Their width, on the other hand, does not play an important role in the production of such a resonant behavior.

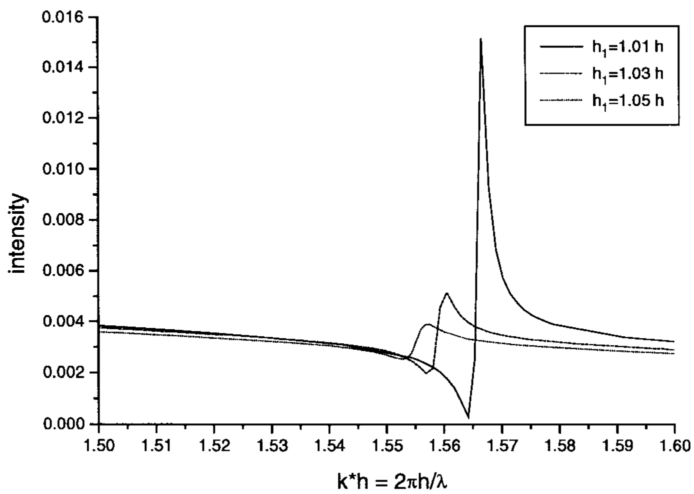
As mentioned earlier, an increase in the number of cavities produces a split in the resonance characteristic of the structure, accompanied by an increase in the Q factors of the peaks. Thus, when the number of grooves is large, it is possible to have a set of very high Q resonances for which the far-field pattern is superdirective. However, although the multigroove structures admit many eigenmode solutions, not all of these are found in the intensity versus frequency curve, since their eigenfrequencies are very similar, which causes their peaks to overlap. For instance, for the case of an array comprising five groups of three cavities (total number of grooves $= 15$, $c = 0.2h$, $\Delta_i = 0.1h$ for $i \neq 4, 7, 10$, and 13; $\Delta_i = 0.7h$ for $i = 4, 7, 10$, and 13; $W = 1.7\lambda$), we obtain the curve shown in Fig. 11(a), where the frequency interval has been reduced even further to facilitate the visualization of the different maxima. Although two extremely narrow peaks appear in this figure, none of the maxima correspond to the π mode. Nonetheless, the radiation patterns associated with the main peaks still exhibit superdirectivity, as may be seen from Fig. 11(b).

IV. MBPE Technique

The examples given in the last section have demonstrated that structures formed by an array of open rectangular cavities exhibit superdirective properties when a high Q resonance mode is excited by the incident plane wave. Furthermore, these resonant frequencies are characterized by a noticeable increase in the energy of the field scattered in the specular direction and are accompanied by an enhancement of the fields excited



(a)



(b)

Fig. 8. (a) Frequency scan of the field intensity scattered in the normal direction by an array of three grooves with equal spacing, equal width $(c = 0.2h, \Delta = 0.1h)$, but different depths $(h_1 \neq h_2 = h_3 = h)$, $\theta_0 = 0°$, for different values of h_1. (b) Enlargement of the marked region of Fig. 8(a).

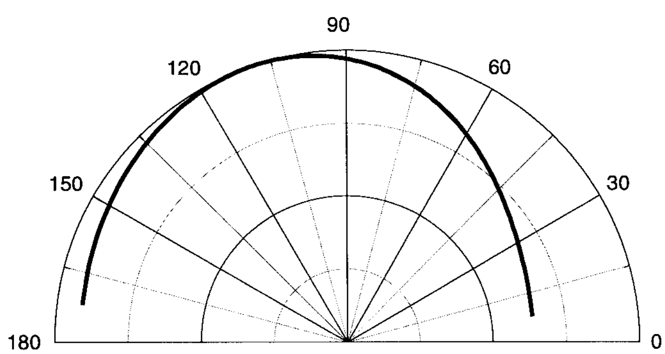
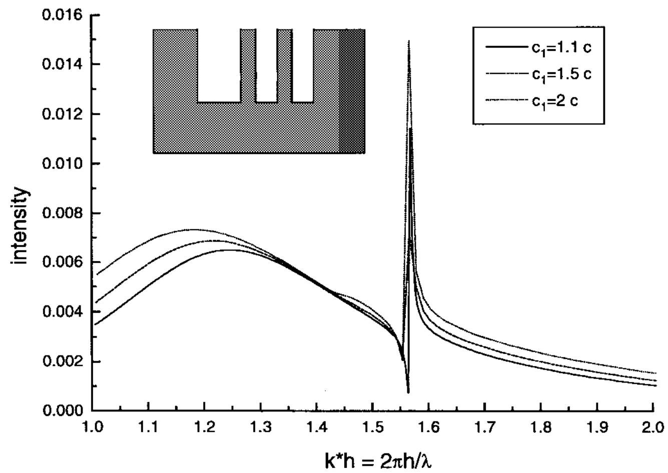
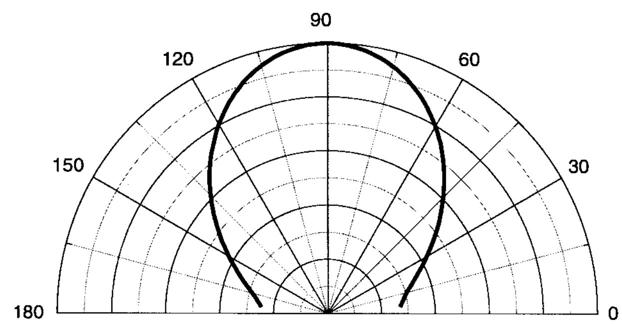


Fig. 9. Far-field pattern radiated by the structure formed by three grooves shown in Fig. 8 for $h_1 = 1.05h$.



(a)



(b)

Fig. 10. (a) Frequency scan of the field intensity scattered in the normal direction by an array of three grooves with equal spacing, equal depth but different widths $(c_1 \neq c_2 = c_3 = c)$, $\Delta = 0.1h$, $\theta_0 = 0°$ for different values of c_1.; (b) Far-field pattern radiated by the structure formed by three grooves shown in Fig. 10(a) for $c_1 = 2c$.

inside the cavities. However, the peaks become extremely narrow with an increase in the number of grooves, making it rather difficult to track them. Thus, it becomes necessary to choose an extremely small step $\Delta\lambda$ while calculating the frequency response of the structure. For instance, we choose $\Delta(kh) = 10^{-8}$ for the seven-groove case to insure that none of the resonant frequencies are skipped. This, in turn, increases the computation time very substantially and prompts us to explore the possibility of using alternate numerical schemes to search for the resonant frequencies.

One such approach is the MBPE technique [13]–[14], which is a numerical algorithm useful for generating the frequency response over a wide-frequency range from a sparse set of sample points, by using a curve-fitting model based on the physics of the problem. A ratio of two polynomials is used as a fitting function and their coefficients are the unknowns to be found using the sample points. The degrees of the polynomials in the numerator and denominator are determined by the number of samples and, typically, four in the numerator and three in the denominator are found to be adequate. Consequently, we only need seven samples to generate the frequency response curve even though it may contain sharp resonances. We should mentioned that sometimes, depending on the structure and the choice of parameters, the matrix for determining the coefficients becomes ill-conditioned, if larger number of samples are used.

Typically, the numerical efficiency of this technique is several orders of magnitude better than the brute force approach, which requires approximately 30 s per frequency point on a Dec Alpha workstation. In the conventional approach we

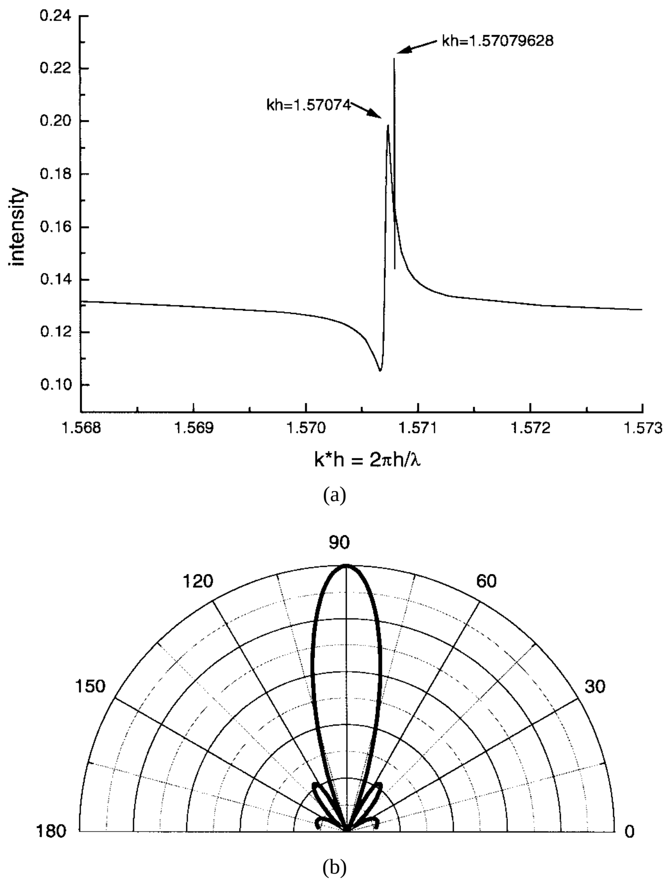


Fig. 11. (a) Frequency scan of the field intensity scattered in the normal direction by an array of fifteen identical grooves distributed in five groups of three grooves. The parameters are: $c = 0.2h$, $\Delta_i = 0.2h$ for $i \neq 4, 7, 10, 13$ and $\Delta_i = 0.7h$ for $i = 4, 7, 10, 13$, $\theta_0 = 0°$. (b) Far-field pattern of the structure in Fig. 11(a), for the high Q resonance frequency ($kh = 1.570\,796\,28$).

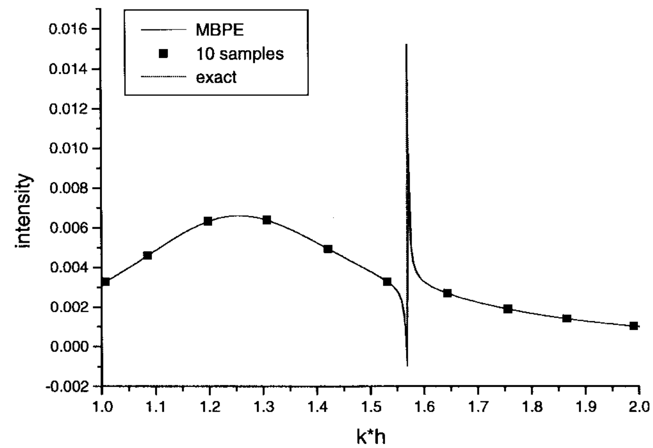


Fig. 12. Comparison of the MBPE (solid line) and the exact (dotted line) results for the scattered field intensity in the normal direction by an array of three grooves shown in Fig. 2.

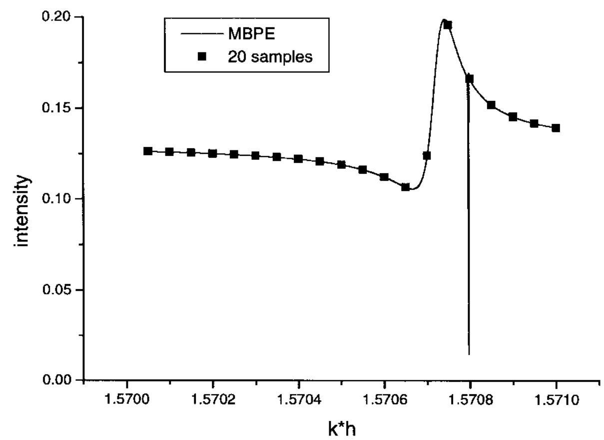


Fig. 13. Comparison of MBPE results (expanded scale) for an array of 15 grooves with those presented in Fig. 11, inthe vicinity of $kh = 1.57$.

start with 100 points, guess the interval where the resonance is located and proceed to use a finer discretization in this interval. Even so, sometimes the resonance point associated with a very high Q system can be missed even when 2000 points are computed in an already-identified interval that can require ~1000 min of computation. This difficulty can be circumvented by using the MBPE technique, which accurately generates the entire curve with only 10 or 20 sample points in about 30 s, and detects the resonance with the needed precision.

To illustrate the application of the MBPE method to the present problem, we present two examples below. The curves in Fig. 12, pertain to a three-groove array with the same parameters as used to generate the results in Fig. 2. The MBPE curve was generated by using four sets of seven samples uniformly distributed, i.e., by working with seven unknowns at a time. We observe that the MBPE method yields a good approximation to the exact curve, even with only a total of ten samples. It is particularly worth noting that the MBPE curve fitting is not just a mere interpolation since it is able to reproduce a resonance peak even without the use of any samples within the interval in which it occurs. Although the MBPE-generated curve deviates slightly from the true response in the vicinity of the resonance peak (out of the resonance the difference between the MBPE and the exact curves does not exceed 0.2%), it nonetheless provides us with a useful tool for finding the location of the resonance frequency. As mentioned earlier, the problem of finding the resonant frequency peak becomes increasingly difficult, with the increase in the number of cavities on the surface. The advantages of using the MBPE algorithm become even more apparent in this case because of substantial time saving realized in the process. For instance, in Fig. 11 we see that there are two sharp and extremely narrow peaks within a very small interval of kh. In fact, during the first pass of the computation, the peak at $kh = 1.570\,796\,28$ was missed even when a very small $\Delta(kh)$ was employed. However, the MBPE method yielded the solid curve in Fig. 12 and immediately hinted the existence of a missing resonant frequency. When the accuracy of the computation was enhanced by using a smaller interval and finer discretization with twenty samples in that interval, we obtained the curve in Fig. 13 in which both the resonance peaks were clearly detected. This leads us to conclude that the MBPE algorithm is a very useful tool for initial computation of the resonant frequencies of the system with good accuracy that can be further refined via finely sampled computation in the vicinity of these frequencies.

V. Conclusion

The scattering characteristics of structures formed by a finite array of rectangular cavities was investigated in this paper by using the modal approach, which was found to be a simple yet useful tool for modeling these structures. It was shown that arrays comprising a finite number of cavities (passive elements) present interesting characteristics, e.g., superdirectivity in the far-field pattern at the resonant frequencies of the system. These resonances are characterized by a phase behavior in the mouths of these cavities that alternates between 0 and π radians and are accompanied by a significant enhancement of the fields inside the grooves. The tolerance effects, viz., the influence of varying the dimensions of the cavities on the resonances of the groove array, has been investigated. It has been found that the resonances are highly sensitive to the variations in the depths of the grooves but not in their widths. The MBPE technique has been employed for efficient computation of the frequency response containing extremely sharp resonances and has been found to be a good tool for locating these narrow resonances.

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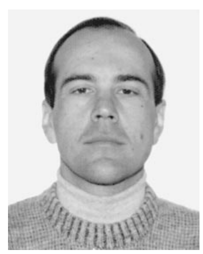

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Raj Mittra (S'54–M'57–SM'69–F'71–LF'96), for a photograph and biography, see p. 486 of the March 1997 issue of this Transactions.