Genetic Algorithms in the Design and Optimization of Antenna Array Patterns

Francisco J. Ares-Pena, *Senior Member, IEEE*, Juan A. Rodriguez-Gonzalez, Emilio Villanueva-Lopez, and S. R. Rengarajan, *Fellow, IEEE*

***Abstract*—This paper demonstrates the application of genetic algorithms (GA's) in array pattern synthesis. GA's have the ability to escape from local minima and maxima and are ideally suited for problems where the number of variables is very high. We present three examples: two for linear arrays and one involving linear and planar arrays.**

***Index Terms*—Antenna arrays, genetic algorithms.**

I. Introduction

In this paper we apply genetic algorithms (GA's) [1] for solving three important problems dealing with the synthesis of antenna array patterns. First, we establish the fast convergence of this method in searching for the optimal solution from a very large solution space for a uniformly null-filled linear array. Then an extension of this procedure is introduced to search for the optimal null-filling topography for the radiation pattern of a linear array. Finally, a hybrid method is presented for array thinning by combining GA's and simulated annealing (SA) [2], which solves the problem of removing unnecessary elements from a planar array.

To get optimal results with the GA, it was found necessary to use (in all the examples studied) a population that was twice the number of variables involved, except for the case of 100 variables. In such a case, the population was 100. The number of chromosomes was a constant in the process and a ranked replacement took place in each iteration if there was an improvement from parents to offspring. One point crossover was always applied and mutation happened every iteration, affecting one gene on every chromosome. The GA package used provided C-source code, which was included in our own source program. It supplied a graphical interface, which was used to fine tune parameters such as population, kind, and condition of replacement, mutation, crossover, and so on. The optimizations themselves were implemented in background mode, i.e., without interaction. The solutions obtained from the best runs are shown in the results.

II. Uniformly Null-Filled Linear Array

In this first example, we started with an $N+1$ element equispaced (0.5λ) linear array. By the substitution $w = e^{j\psi}$, the array factor is written as a polynomial of degree N

$$f(w) = \sum_{n=0}^{N} (I_n/I_N) w^n = \prod_{n=1}^{N} (w - w_n) \qquad (1)$$

where $\psi = kd\cos\theta$.

As is well known, to obtain null filling in the pattern $f(w)$ requires that the roots w_n be complex, i.e., $w_n = \exp(a_n + jb_n)$. By utilizing Orchard's method [3], it is possible to calculate the complex roots $(a_n + jb_n)$ that will yield a pattern with properly filled nulls while maintaining controlled sidelobe levels (SLL's). Reversal of the sign of any a_n leaves the shape of the power pattern unchanged but moves the corresponding root to a reciprocal radial location of the same angle in the complex w plane. Thus, if there are N complex roots, there are 2^N different sets of roots and, hence, that many sets of excitations I_n. Out of these numerous sets of excitations some may be easier to realize physically than the conventional excitation for the pattern without null filling. In the prior literature [4], [5], the problem of searching the optimal pattern has been solved by an exhaustive checking of all the 2^N possible solutions. However, this method is not practical for moderate to large arrays since the computer time required becomes prohibitive as N increases.

An alternative and much faster method using GA's is proposed in this paper. The problem of searching the optimal set of roots may be described in terms of discrete parameters by the process of binary encoding. One bit represents the sign of a_n in each root as "plus" $= 1$ or "minus" $= 0$. For an equispaced linear array of $N+1$ elements, we can define an array of N parameter values or N bits to be optimized. Thus, we have to solve an N-dimensional optimization problem of finding the optimal array of parameters $\{p_1, \cdots, p_N\}$ that minimizes a cost function that represents, for example, a measurement of the variation or ripple in the excitation of the neighboring elements. It is possible to use other cost functions such as dynamic range or even a combination of several performance parameters with different assigned weights depending on their relative importance in the design requirements. The problem of finding this optimal array of bits that minimizes the cost function is solved by GA's [6].

The method has been applied to optimize the $|I_n/I_{n\pm1}|_{\max}$ in sum patterns with a uniform null filling to a level of 5 dB below each sidelobe peak. The initial set of roots $(a_n + jb_n)$ was obtained by means of the Orchard's method [3]. We present results for 20, 40, 60, and 100 element arrays. Table I shows the optimal value obtained (value inside parentheses

Manuscript received June 6, 1997; revised May 27, 1998. This work was supported in part by Xunta de Galicia under contract XUGA 20601A96.

F. J. Ares-Pena, J. A. Rodriguez-Gonzalez, and E. Villanueva-Lopez are with the Departamento de Física Aplicada, Facultad de Física, Universidad de Santiago de Compostela, Santiago de Compostela, 15706 Spain.

S. R. Rengarajan is with the Department of Electrical and Computer Engineering, California State University, Northridge, CA 91330 USA.

Publisher Item Identifier S 0018-926X(99)04428-2.

TABLE I
RESULTS FROM OPTIMIZING $|I_n/I_{n\pm1}|_{\max}$ IN SUM PATTERNS BY UNIFORM NULL FILLING OF THE SIDELOBES

N	*Time*	$\|I_n/I_{n\pm1}\|_{max}$	*Directivity*	*SLL*
20	0.62 s	1.34 (37.55)	17.96 (5.70)	-20
40	6.62 s	1.44 (55.96)	34.94 (4.85)	-24
60	12.85 s	1.46 (57.54)	50.94 (3.17)	-30
100	43.41 s	1.64 (71.00)	83.54 (3.60)	-30

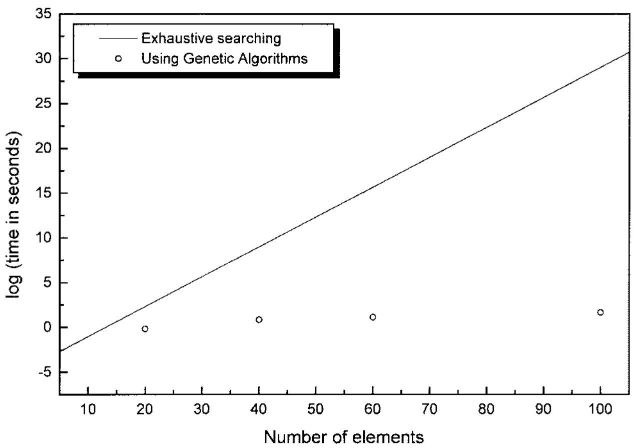


Fig. 1. Comparison between central processing unit (CPU) time for GA and the estimated time for the exhaustive checking of all possible solutions.

is the percentage of improvement from the unfilled pattern) and the CPU time in a Hewtlett-Packard workstation HP 712/80. The directivity of the designed pattern (value inside parentheses is the percentage of loss after null filling) and the SLL are also shown. It is seen from the table that the optimal solution is obtained very rapidly, since only a small fraction of the entire solution space is checked by the GA.

Finally, a comparison between the CPU time for a GA, and the estimated time for the exhaustive checking is shown in Fig. 1. The estimated time for exhaustive checking increases exponentially with the number of elements. Clearly, the required time for the exhaustive checking of all possible solutions even for a 100-element linear array becomes prohibitive!

III. SEARCHING FOR THE OPTIMAL NULL-FILLING TOPOGRAPHY

In the previous section, we synthesized patterns with uniform null filling where each sidelobe minimum was fixed to a certain initial value. We then obtained the optimum pattern from a large solution space generated by considering two possible values corresponding to the real part of each root. This type of optimization allows us to minimize the value of $|I_n/I_{n\pm1}|_{\max}$ or $|I_{\max}/I_{\min}|$ by examining different sets of excitations. We now consider the process of controlling the filling level of each sidelobe null independently. It is possible to design an optimal null-filling topography in the synthesis of array patterns to obtain further improvement in performance.

TABLE II
RESULTS FROM SEARCHING THE OPTIMAL NULL-FILLING TOPOGRAPHY

$\|I_n/I_{n\pm1}\|_{max}$	*Directivity*	*Time*
1.43 (81.23)	5.01	2 m 27.44 s
1.50 (80.31)	4.87	1 m 40.87 s
1.75 (77.03)	5.00	1 m 40.38 s
1.98 (74.02)	4.88	1 m 39.28 s
2.45 (67.85)	4.86	1 m 39.39 s

Let $(a_n^0 + jb_n^0)$ denote the roots of the initial pattern obtained by the Orchard's method [3] as before. Besides changing the real part of roots to get null filling, small perturbations of the imaginary parts were found also to be necessary in order to maintain the sidelobes under strict control. In this case, the chromosome for the GA is given by a binary encoding of the δa_n and δb_n using a floating point representation of 16 bits for each of them. By minimizing a given objective function, small perturbations of the initial roots δa_n and δb_n are calculated in each iteration of the algorithm. The cost function depends on the position of each root, its radial displacement and the design specifications. This function is built by adding three terms. The first term is the summation of the squares of the differences between the obtained and the required SLL's that are controlled individually. The second term is the square of the difference between the obtained directivity and the required one. The last term takes into account the element-to-element magnitude variation or the dynamic range and it is the square of the difference between the obtained and the required value of the desired one. Each term is weighted by a coefficient, thus emphasizing relative importance of each term. The general form of the cost function is given by

$$C = c_1 \cdot \sum_{i=1}^{N} (SLL_{i,\,\mathrm{obt}} - SLL_{i,\,\mathrm{des}})^2 + c_2 \cdot (\mathrm{Dir}_{\mathrm{obt}} - \mathrm{Dir}_{\mathrm{des}})^2 + c_3 \cdot (V_{\mathrm{obt}} - V_{\mathrm{des}})^2 \quad (2)$$

where the subscripts obt and des denote obtained and desired values, respectively, c_1, c_2, and c_3 are the weight factors of each term, Dir is the directivity of the pattern, and V is the variability of the excitations which can be given by $|I_n/I_{n\pm1}|_{\max}$ or $|I_{\max}/I_{\min}|$, depending on the design specifications. A normalized cost function was also used but no improvements were obtained.

In this example, we started with a 16-element equispaced linear array, fixing the value of $|I_n/I_{n\pm1}|_{\max}$ to 1.40, 1.50, 1.75, 2.00, and 2.50, and constraining the percentage of loss in directivity to 5%. We fixed the SLL to -20 dB except for the six inner lobes, which are depressed to -40 dB. Results are shown in Table II. Fig. 2 shows the obtained power pattern corresponding to the first example in Table II as well as the initial pattern with deep nulls.

This technique is also applicable to shaped-beam patterns just by adding a new term to the cost function to control the ripples. We studied an example consisting of a 50-element equispaced linear array with a ripple of ± 0.25 dB and consisting of asymmetrical sidelobes, as shown in the dashed line

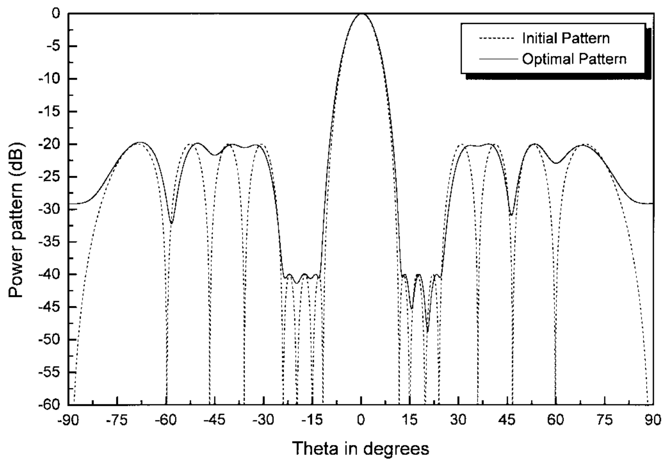


Fig. 2. Power pattern corresponding to the first example in Table II (the optimal null-filling topography) and the initial pattern without filling nulls.

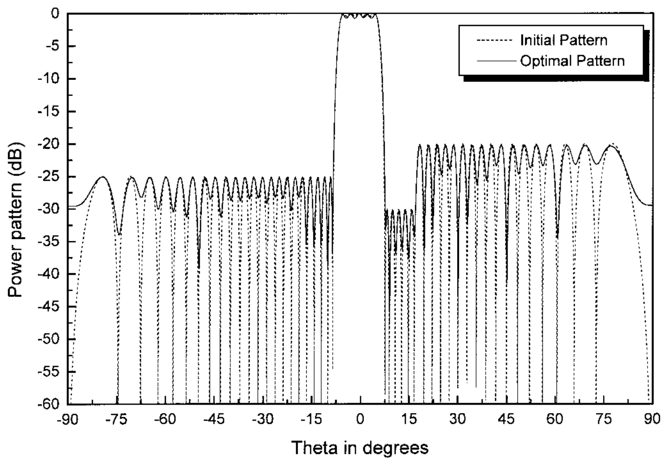


Fig. 3. Power pattern corresponding to a 50-element linear array (the optimal null-filling topography) and the initial pattern.

of Fig. 3. This pattern was obtained by the Orchard–Elliott method [3] as before. By filling the nulls with the use of a GA, the value of $|I_n/I_{n\pm1}|_{\max}$ was decreased from 2.5 to 1.3, thus yielding an improvement of 48%. The final pattern is also shown in the figure.

Recently, Buckley [7] has proposed a method that produces null filling in the sidelobe region in order to constrain excitation coefficients in the synthesis of shaped-beam patterns. Although the element-to-element variation is significantly reduced, his method does not have a strict control over the SLL's or the ripple levels in the shaped region. Orchard's method is better suited for a problem like this since it is possible to have a direct control on the ripple level in the shaped region as well as control on the SLL and null-filling topography.

IV. Design of Thinned Arrays

This last example presents a method to design an optimum aperture distribution of a planar array by minimizing the number of excited elements and the dynamic range of excitation at a minor cost to pattern degradation.

Some of the present techniques that obtain footprint patterns from planar arrays are not efficient. Since the contour of the array must be rectangular such methods utilize many more elements than necessary [8]. The removal of these unnecessary elements would reduce the dynamic range of excitations. This may alleviate the severity of mutual coupling problem, thereby making the design and implementation of the feed network easier. Previous work [9] has attempted to reduce the number of excited elements in the array with some limitations. In that work the criterion employed to select the elements to be removed is not the best possible, since it was based on relative excitations and not on the degradation introduced in the resulting pattern by aperture thinning. Furthermore, the reconstruction of the initial pattern after removing the elements was performed by means of gradient based methods, generally yielding a local minimum. Recently, GA's have been successfully applied to the problem of thinning linear and planar arrays in order to obtain the lowest possible SLL [10]. However, this technique has been applied only to synthesize sum patterns. The problem is more difficult for distributions that produce shaped beams since such distributions exhibit greater dynamic range than those of sum and difference patterns.

The technique proposed in this section uses a GA and SA iteratively in order to solve the problem of array thinning. It begins with an aperture distribution obtained by means of the method described in [11]. After that, this initial distribution is transformed iteratively by the two following steps:

1) *Removal of Unnecessary Radiating Elements:* A GA is employed to remove some elements from the array. The algorithm selects the elements to be removed by comparing the initial pattern with one that results after the elimination of the elements. In this step, a binary coding of the elements of the array is performed. A value of "1" denotes an unchanged element whereas a value of "0" indicates that the element is removed from the array. A group of such binary values would then constitute a chromosome for the GA. A cost function depending on the chromosome under test is built as follows: for each 5° ϕ cut the maximum differences between the desired and obtained patterns are evaluated at the ripple and sidelobe peaks, squared, and added together. A weight coefficient is employed on the ripple term in order to keep it under strict control. The dynamic range ratio is also weighted and added to the result. This definition of the cost function has been found to yield very accurate results. The GA obtains those chromosomes that best minimize the given cost function.
2) *Optimization of the Aperture Distribution:* The second step consists of an optimization of the aperture distribution by means of the simulated annealing technique [2]. In this process the excitations of the remaining elements are modified in order to reduce the degradation of the resulting pattern as well as $|I_{\max}/I_{\min}|$. The cost function employed in the previous step is also used now. To get optimal results, the SA algorithm began with a starting temperature of 100 which was 20% reduced after every iteration.

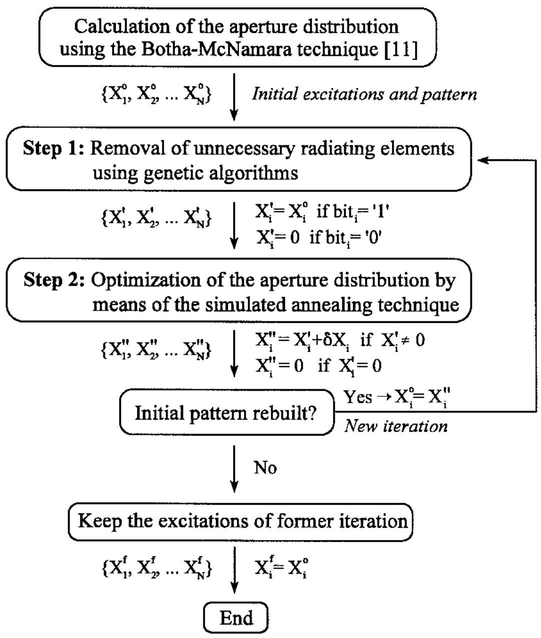


Fig. 4. Flowchart of the array-thinning technique.

TABLE III
RESULTS OF A HYBRID (GA/SA) TECHNIQUE IN ARRAY THINNING

	Step I, GA			*Step II, SA*	
Iteration number	***Number of excited elements***	***Dynamic range ratio*** $\|I_{max}/I_{min}\|$	***Value of the cost function***	***Dynamic range ratio*** $\|I_{max}/I_{min}\|$	***Value of the cost function***
1	59	50.0	456.8	45.8	72.5
2	45	45.8	132.2	32.7	57.5
3	35	32.7	161.4	26.8	50.7
4	33	26.8	104.0	24.5	52.5
5	31	24.5	121.9	19.1	45.4
6	30	19.1	102.2	18.1	44.8

These two steps are iterated until maximum thinning is achieved subject to the satisfaction of the given specification for the pattern and the cost function. The flowchart of the algorithm is shown in Fig. 4.

The technique was applied to an elliptic contour of approximately 12° × 24° (1 : 2). This beam is obtained with an equispaced (0.5λ) rectangular grid planar array of 34 × 12 elements. The initial aperture distribution obtained using [11] yields a ±0.5-dB ripple and a −20 dB SLL in all ϕ cuts. Some of the initial nearly-null excitations were removed so a total of 80 elements per quadrant were excited. The dynamic range for the initial aperture distribution had a value of 1000.

The hybrid technique discussed above reduced the number of excited elements per quadrant to 30 and the dynamic range ratio to 18. This is, a reduction of 98.2% in the dynamic range ratio and a 62.5% in the number of elements. Table III shows the results obtained after each step. A total of six iterations were needed. The pattern was unrecoverable if we applied any further aperture thinning.

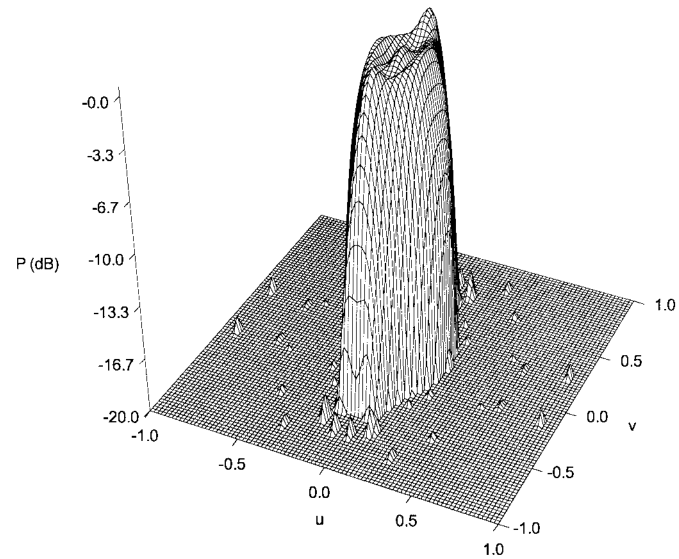


Fig. 5. Power pattern obtained using array-thinning technique.

A three-dimensional plot is shown in Fig. 5, where the power pattern is plotted versus $u = \sin(\theta) \cdot \cos(\phi)$ and $v = \sin(\theta) \cdot \sin(\phi)$, where θ is the elevation angle and ϕ the azimuth angle.

The prior work [9] yielded 46 elements and a dynamic range ratio of 38 for the same problem. The present technique obtains an improvement of about 35% on the number of excited elements in the array and 53% reduction on the dynamic range ratio with respect to [9].

The hybrid technique is valid for footprint patterns with an arbitrary boundary as well as for sum and difference patterns obtained with planar as well as with linear arrays. It is possible to introduce substantial thinning with a small dynamic range for separable distributions that contain a great number of highly under-utilized elements.

V. CONCLUSIONS

GA's have been found to be well suited to apply to several problems of antenna array pattern synthesis. The GA yielded an optimal solution very rapidly by searching a large solution space in the first case. We further demonstrated the utility of GA's in searching for the optimal null-filling topography in pattern synthesis problems. By using a hybrid technique consisting of a GA and SA we have achieved a substantial amount of array thinning with optimal performance characteristics such as a small value of dynamic range.

ACKNOWLEDGMENT

The authors would like to thank Dr. A. Hunter, University of Sunderland, U.K. for writing SUGAL Genetic Algorithm Package, which was used for this work.

REFERENCES

[1] D. E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning.* Reading, MA: Addison-Wesley, 1989.
[2] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, "Optimization by simulated annealing," *Sci.*, vol. 220, no. 4598, pp. 671–679, May 1983.

[3] H. J. Orchard, R. S. Elliott, and G. J. Stern, "Optimizing the synthesis of shaped beam antenna patterns," in *Proc. Inst. Elect. Eng.*, vol. 132, pt. H, pp. 63–68, Feb. 1985.

[4] F. Ares, S. R. Rengarajan, and E. Moreno, "Optimization of aperture distributions for sum patterns," *Electromagn.*, vol. 16, no. 2, pp. 129–143, Mar./Apr. 1996.

[5] F. Ares, S. R. Rengarajan, A. Vieiro, and E. Moreno, "Optimization of aperture distributions for difference patterns," *J. Electromagn. Waves Applicat.*, vol. 10, no. 3, pp. 383–402, 1996.

[6] F. Ares, S. R. Rengarajan, E. Villanueva, E. Skochinski, and E. Moreno, "Application of genetic algorithms and optimizing the aperture distributions of antenna array patterns," *Electron. Lett.*, vol. 32, no. 3, pp. 148–149, Feb. 1996.

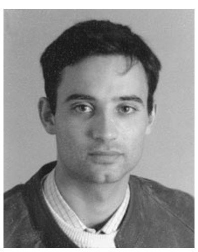
[7] M. J. Buckley, "Synthesis of shaped beam antenna patterns using implicity constrained current elements," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 192–197, Feb. 1996.

[8] F. Ares, R. S. Elliott, and E. Moreno, "Design of planar arrays to obtain efficient footprint patterns with an arbitrary boundary," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 1509–1514, Nov. 1994.

[9] F. Ares, S. R. Rengarajan, A. Vieiro, E. Botha, and E. Moreno, "Improved results for planar arrays in space communication applications," *Microwave Opt. Tech. Lett.*, vol. 12, no. 5, pp. 263–265, Aug. 1996.

[10] R. L. Haupt, "Thinned arrays using genetic algorithms," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 993–999, July 1995.

[11] E. Botha and D. A. McNamara, "A contoured beam synthesis technique for planar antenna arrays with quadrantal and centro-symmetry," *IEEE Trans. Antennas Propagat.*, vol. 41, pp. 1222–1231, Sept. 1993.



Francisco J. Ares-Pena (M'94–SM'96) received the B.S., M.S., and Ph.D. degrees from the University of Santiago de Compostela, Spain, in 1986, 1987, and 1993, respectively, from the same university, all in physics.

He was a Research Scholar in the Department of Electrical Engineering at the University of California, Los Angeles, for two quarters in 1990 and 1991, where he developed the main topic for his Ph.D. thesis. He is currently an Associate Professor in the Department of Applied Physics at the University of Santiago de Compostela, Spain. He has published more than 80 journal and conference papers. His general research interests include numerical method in solving electromagnetic problems and antenna array pattern synthesis.

Dr. Ares is a member of the New York Academy of Sciences. He received the Outstanding Ph.D. Award from the Physics Faculty, University of Santiago de Compostela, in 1994. He is listed in *Marquis Who's Who in the World* and *Marquis Who's Who in Science and Engineering.*

Juan A. Rodriguez-Gonzalez was born in Orense, Spain, in 1972. He received the B.S. and M.S. degrees in physics from the University of Santiago de Compostela, Spain. He is currently working toward the Ph.D. degree (in physics) at the same university.

He also presented several subjects related to electromagnetics at the Open National University Associated Center in Pontevedra, Spain, for two years. His general research interests include numerical methods in solving electromagnetic problems and pattern synthesis. He is also interested in computer programming and software engineering.

Emilio Villanueva-Lopez was born in December 1971 in Santiago de Compostela, Spain. He received the Physics (electronics) and M.S. (antennas) degrees, both in 1995, from the Universidad de Santiago de Compostela.

His interests are related to optimization algorithms applied to microwave antennas.

S. R. Rengarajan (S'77–M'80–SM'85–F'94), for a photograph and biography, see p. 1192 of the August 1996 issue of this TRANSACTIONS.