Optimum Design of Feed Structures for High G/T Passive and Active Antenna Arrays

Simsek Demir and Canan Toker

***Abstract*—In this work, noise analysis of parallel feed structures is presented. Signal and noise behavior of the feed structures are signified by the newly introduced concepts of "coherent" and "incoherent" impedance match of power-combining structures. It is also shown that a feed structure can be redesigned for low-noise operation without affecting the radiation characteristics. Optimum design of parallel feed structures for low-noise operation is explained. Also an optimum use of active elements in such structures is investigated to have a low overall noise temperature of the antenna array with minimum number of active elements. In the analysis, a new method is introduced where a "noise-equivalent line length" (NELL) is defined. This definition, which unifies the contribution of noise from different array elements, is used in the design of a parallel feed structure and as an active circuit replacement criteria in passive arrays.**

***Index Terms*—Antenna arrays, feeds.**

I. Introduction

NOISE performance is an important figure of merit for receive antennas. Noise calculations for receive arrays date back to the 1960's where antenna arrays were investigated for better signal-to-noise ratios [1]–[5]. In recent years, there is an intensive research on active antenna structures for both transmit and receive operations [6]–[13]. Some special studies for noise performance evaluation and measurement were also carried out [14]–[15]. However, in these analysis, noise contribution of feed lines was not specifically taken into account. Noise analysis and noise performance evaluation of an antenna array is not complete without assessment of noise contribution from its feed structure, particularly for large arrays.

Large antenna arrays are commonly based on printed circuit technology, which has wide application areas [16]–[18]. There is extensive theoretical and practical work about these structures and the facts are well understood but still, yet, intensive research is going on. Without loss of generality, the antenna array examples, explained in this paper, will be of printed type. The main advantages of such antennas are easy integration with active devices, low cost, low profile, flexible structure, which is suitable for conformal surfaces, and large variety of antenna element types. On the other hand, due to the dielectric and conductor losses as well as the spurious radiation from the feed network, efficiency of such a structure is low [19] and, consequently, noise performance is moderate. Feed network design is an important part of a printed antenna array design. There are some standard forms of such structures [20]–[22]. Also, some specific studies on these structures such as radiation loss of feed structures, specific feed types, and application of Thevenin theorem to feed network design are present in the literature [23]–[25]. However, noise analysis and general guide lines for low-noise design of feed structures do not appear in the literature.

Receive antenna arrays are power combining structures and their design for certain radiation characteristics is related to the case where all elements receive coherent signal. Feed structure designs in the literature are mainly concerned with impedance matching for coherent excitation. However, impedance matching for coherent excitation does not imply the incoherent impedance matching, which is the case where the signals combined in the structure are incoherent. The response of a combiner (such as a feed network) to excitations from incoherent sources such as thermal noise of the lines is totally different and such a concept was not considered previously. A method called "active element approach" similiarly uses excitation of a single element [26], but this is intended for mutual coupling analysis among antenna elements in the array. In our work, incoherent matching of the line segments in a feed structure is introduced for the first time. Moreover, for receive antenna arrays it is difficult to measure the noise temperature of the array [14]–[15]. Therefore, reliable theoretical prediction is of ultimate importance for noise characterization of these structures.

Antenna array feed structures are mainly of two types: 1) parallel (parallel feed and corporate feed) and 2) series structures. A parallel feed structure is, indeed, a system of power combiners; the received power by individual antenna elements in the array are sequentially combined in an order prior to the output of the array. As a general definition, feed structures are used to combine the received power of antenna elements of an antenna array with predetermined combination coefficients, as in the case of a tapered feed structure, for a low sidelobe level array. Signal combination is based on coherent operation. There is a strong correlation in the noise received by antenna elements from space; therefore, space noise can be treated as a signal coming from outside the antenna and signal-to-noise ratio calculations can be carried out accordingly [1]–[5].

Antenna arrays are, in general, composed of identical elements. Therefore, unequal power combiners together with impedance transformers are used for feed tapering, where both structures are realized using line segments. Feed struc-

Manuscript received May 29, 1997; revised June 3, 1998. This work is supported in part by the European COST 245 Action and by the Turkish Scientific and Technical Research Council, Tubitak.

The authors are with the Electrical and Electronics Engineering Department, Middle East Technical University, Ankara, 06531 Turkey.

Publisher Item Identifier S 0018-926X(99)04441-5.

ture is designed by adjusting the lengths and characteristic impedances of these line segments.

Sources of the noise at the output of a receive array are the space noise, antenna element noise, and the feed structure noise. In active arrays, noise contribution of active circuits within the structure should also be considered. Our analysis is concerned with the thermal noise generated by the feed lines and noise contribution of the active elements with special emphasis on noise performance of the feed structure.

In practice, line segments in a feed structure are lossy. A lossy line generates thermal noise which is uncorrelated with other noise sources in the system. Therefore, unlike space noise, noise generated inside the antenna structure is incoherent. Noise generated by a line segment is delivered to the receive port through the feed structure and superposed at the output with other noise signals to yield the overall noise power. Response of an antenna array to the internal noise signals is different than its response to received signal. We will focus on coherent and incoherent operation of the array structure where coherent operation corresponds to receiving a signal and incoherent operation stands for response to internal noise.

In Section II, a new concept which is termed as "noise-equivalent line length" (NELL) is defined whereby noise contributions from different parts of a feed structure can be identified by a single parameter. This parameter, i.e., "NELL" is in dimensions of physical length and, hence, provides a suitable means of comparison of noise contributions from different sections of the feed structure. In Section III, analysis of double section quarter-wave-line impedance transformers is presented and NELL of such a transformer is given. The characteristic impedances of the quarter-wave-line segments which yield optimum noise performance are also given. In Section IV, noise analysis of unequal power combiners is given. Following this analysis, "weighted NELL" and its application in calculation of NELL of combiners are introduced. In Section V, a situation where active circuits are placed in the feed structure is considered and noise performance of such a feed structure is formulated. An optimum placement criteria of active circuits, corresponding to the minimum use of active elements for low-noise operation is derived. Examples are given in Section VI demonstrating the use of formulations given in previous sections to evaluate the overall noise performance of a 4 $\times$ 8 corporate fed patch antenna array and 2 $\times n$ parallel arm feed structure. An example which demonstrates the improvement in noise performance for various placement points of active circuits in a 16 $\times$ 16 array is given.

II. Noise-Equivalent Line Length

As stated above, the main disadvantage of the feed lines is the loss associated with them which is a thermal noise source. Characteristic impedance of the line Z_{line}, attenuation per unit length of the line α, length of the line l, and the ambient temperature T determine the generated noise power [27].

The attenuation factor of the line introduces an imaginary part to the Z_{line} but it is assumed that line is low loss and, therefore, Z_{line} remains practically real. The source and load reflection coefficients are defined with respect to the characteristic impedance of the line

$$\Gamma_g = \frac{Z_g - Z_{\text{line}}}{Z_g + Z_{\text{line}}} \tag{1}$$

$$\Gamma_L = \frac{Z_L - Z_{\text{line}}}{Z_L + Z_{\text{line}}} \tag{2}$$

The loss factor L is defined as

$$L = e^{2\alpha l}. \tag{3}$$

Noise power P_n of a lossy transmission line delivered to the load is given in [27, eq. (5.60)] as

$$P_n = kT\Delta f \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_L \Gamma_g e^{-2\gamma l}|^2 L}(L - |\Gamma_g|^2/L - 1 + |\Gamma_g|^2) \tag{4}$$

where $\gamma = \alpha + j\beta$ is the propagation constant and here L is defined as in (3).

The available noise power is essential for evaluating the noise performance. In the following lines, available noise power of a low-loss line is formulated. The output impedance of a transmission line with a source reflection coefficient of Γ_g is

$$Z_{\text{out}} = Z_{\text{line}} \frac{1 + \Gamma_g e^{-2\gamma l}}{1 - \Gamma_g e^{-2\gamma l}}. \tag{5}$$

The conjugately matched load impedance Z_L is Z^*_{out}, thus

$$\begin{aligned}\Gamma_L &= \frac{Z^*_{\text{out}} - Z_{\text{line}}}{Z^*_{\text{out}} + Z_{\text{line}}} \\ &= \left(\frac{1 + \Gamma^*_g e^{-2\gamma^* l}}{1 - \Gamma^*_g e^{-2\gamma^* l}} - 1\right) \bigg/ \left(\frac{1 + \Gamma^*_g e^{-2\gamma^* l}}{1 - \Gamma^*_g e^{-2\gamma^* l}} + 1\right) \\ &= \Gamma^*_g e^{-2\gamma^* l}.\end{aligned} \tag{6}$$

Substituting this definition of Γ_L in (4), available noise power of a low-loss transmission line is obtained as

$$P_{n,\text{available}} = kT\Delta f \frac{L - |\Gamma_g|^2/L - 1 + |\Gamma_g|^2}{(1 - |\Gamma_g/L|^2)L}. \tag{7}$$

Available noise power of a low-loss transmission line is a function of source mismatch and loss factor. This power increases with increasing magnitude of source reflection coefficient which is clearly seen in the following arrangement of (7):

$$P_{n,\text{available}} = kT\Delta f \frac{L-1}{L}\left(1 + \frac{1+L}{(L/|\Gamma_g|)^2 - 1}\right). \tag{8}$$

As $|\Gamma_g|$ increases, the denominator of the second term in the summation decreases, hence, the summation itself increases; that is, available noise power increases with increasing source mismatch. The upper limit of this available power is found by substituting the upper limit of $|\Gamma_g|$, i.e., $|\Gamma_g| = 1$, into the above equation

$$P_{n,\text{available,max}} = kT\Delta f. \tag{9}$$

This result means that in the limit of mismatch, a low-loss transmission line introduces an available noise power equal to the available noise power of a resistor. Therefore, incoherent impedance matching of the feed structure is of ultimate importance from point of view of noise generation.

The above noise analysis is used toward the definition of a NELL which simplifies the noise performance evaluation of the feed structures considerably. In doing so, the noise contribution from each element of a feed structure to the output noise power is expressed in terms of a single parameter, which explicitly puts the strength of the contribution relative to the theoretical minimum. Simple addition of these equivalent line lengths yields the overall noise performance of the feed structure. Moreover, this uncomplicated parameter is used as a replacement criteria of active circuits in an antenna array for the optimum noise performance as explained in Section V.

Let L be the loss factor of the line under investigation which has a source reflection coefficient Γ_g and a loss coefficient α. Its available noise power P_{line} is given by (7). Let L_{neqv} be the loss factor of a line that has a line length l_{neqv} and has the same loss coefficient α. Furthermore, let this line be impedance matched at its source side so that its available noise power is given by

$$P_{\text{eqv}} = kT\Delta f \frac{L_{neqv} - 1}{L_{neqv}}. \tag{10}$$

Now, we can make the two lines "equivalent" as far as their available noise powers are concerned. By equating P_{line} to P_{eqv}, the loss factor of the equivalent line turns out to be

$$L_{neqv} = \frac{L^2 - |\Gamma_g|^2}{(L^2 - |\Gamma_g|^2) - (L-1)(L + |\Gamma_g|^2)}. \tag{11}$$

Consequently, NELL l_{neqv} of the *line* turns out to be

$$l_{neqv} = \frac{1}{2\alpha} \ln\left(\frac{L^2 - |\Gamma_g|^2}{(L^2 - |\Gamma_g|^2) - (L-1)(L + |\Gamma_g|^2)}\right). \tag{12}$$

Notice that in this equation the equivalent line is also assumed to have the same attenuation coefficient α. Equations (11) and (12) in the form above will be used toward the definition of a weighted equivalent line length, however, they can also be arranged as

$$\begin{aligned} L_{n,\text{eqv}} &= \frac{L^2 - |\Gamma_g|^2}{L(1 - |\Gamma_g|^2)} \\ l_{n,\text{eqv}} &= \frac{1}{2\alpha} \ln\left(\frac{L^2 - |\Gamma_g|^2}{L(1 - |\Gamma_g|^2)}\right). \end{aligned} \tag{13}$$

Using the equivalent line length, output noise temperature can be obtained through (10). Notice that noise-equivalent length of a mismatched line is longer than its physical length, where minimum noise-equivalent length which corresponds to matched case is equal to its physical length. Without comparing the actual noise temperatures, one can deduce the effect of mismatch on noise generation by comparing the noise equivalent and physical lengths. In Fig. 1, NELL of a line normalized with its physical length is given as a function of the source-reflection coefficient.

The potential of the NELL is its ease of use in noise calculation of line type structures. Let us consider two transmission lines connected in cascade. Let $l_{\text{eqv},1}$ and $l_{\text{eqv},2}$ be the NELL's of these two transmission lines and $L_{\text{eqv},1}$ and $L_{\text{eqv},2}$ be the corresponding noise-equivalent loss factors. Available noise powers generated by these two lines will be given by

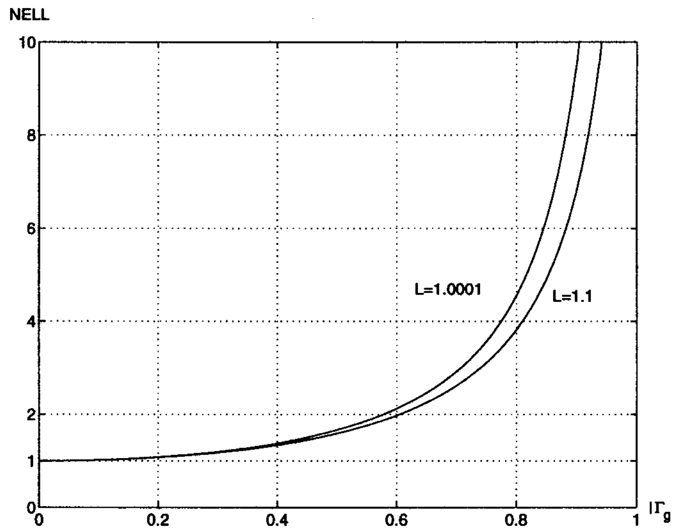


Fig. 1. Variation of NELL with the source-reflection coefficient, normalized with its physical line length.

$$\begin{aligned} P_1 &= kT\Delta f \frac{L_{\text{eqv},1} - 1}{L_{\text{eqv},1}} \\ P_2 &= kT\Delta f \frac{L_{\text{eqv},2} - 1}{L_{\text{eqv},2}}. \end{aligned} \tag{14}$$

The noise power generated by the first line will be attenuated through the second line and the total available noise power of the cascaded lines will be obtained by

$$\begin{aligned} P_{\text{total}} &= \frac{P_1}{L_{\text{eqv},2}} + P_2 \\ &= kT\Delta f \left(\frac{L_{\text{eqv},1} - 1}{L_{\text{eqv},1} \cdot L_{\text{eqv},2}} + \frac{L_{\text{eqv},2} - 1}{L_{\text{eqv},2}}\right) \\ &= kT\Delta f \frac{L_{\text{total}} - 1}{L_{\text{total}}}. \end{aligned} \tag{15}$$

The noise-equivalent loss for the cascaded lines L_{total} is obtained as follows:

$$\begin{aligned} \frac{L_{\text{total}} - 1}{L_{\text{total}}} &= \frac{L_{\text{eqv},1} - 1}{L_{\text{eqv},1} \cdot L_{\text{eqv},2}} + \frac{L_{\text{eqv},2} - 1}{L_{\text{eqv},2}} \\ &= \frac{L_{\text{eqv},1} - 1}{L_{\text{eqv},1} \cdot L_{\text{eqv},2}} + \frac{L_{\text{eqv},1}(L_{\text{eqv},2} - 1)}{L_{\text{eqv},1} \cdot L_{\text{eqv},2}} \\ &= \frac{L_{\text{eqv},1} \cdot L_{\text{eqv},2} - 1}{L_{\text{eqv},1} \cdot L_{\text{eqv},2}}. \end{aligned} \tag{16}$$

That is

$$L_{\text{total}} = L_{\text{eqv},1} \cdot L_{\text{eqv},2} \tag{17}$$

and also

$$l_{\text{total}} = l_{\text{eqv},1} + l_{\text{eqv},2}. \tag{18}$$

In conclusion, the overall NELL of cascaded line sections is given by addition of NELL's of the individual line sections.

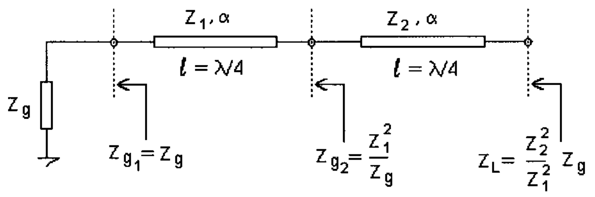


Fig. 2. Schematic representation of a double section quarter-wave-line impedance transformer.

III. Noise Analysis of Quarter-Wave-Line Impedance Transformers

The power combination weights in a combiner are proportional to the impedances of the combining arms at the junction point of the arms. Therefore, impedance transformation should be introduced to realize a tapered feed structure. If no impedance transforming structures are used, the array will be a uniform fed array. Impedance transformation might be introduced at the branches connecting antenna elements to the parallel arm or, in general, on the line segments of the parallel arm, between connection points of two successive elements. Quarter-wave-line segments are most suitable for this impedance transformation.

Quarter-wave-line impedance transformers are generally used as double section transformers. For these structures, ratios of characteristic impedances of consecutive line sections are important but the characteristic impedances of the individual transmission line sections are not important for the transformation operation. On the other hand, as explained above, characteristic impedance of a line section is effective in the noise contribution of that line section. In the following section, first, noise contribution of a double section quarter-wave-line transformer is given and then, the method to determine the characteristic impedance of the lines for optimum noise performance is explained.

In Fig. 2, a double section quarter-wave-line transformer is shown. Z_1 and Z_2 stand for the characteristic impedances of the two line segments. Thus, the impedance transformation ratio q^2 is $(Z_2/Z_1)^2$. Attenuation factor α is not a strong function of the line width, i.e., of the characteristic impedance; therefore, for the sake of simplicity and without loss of generality, the two consecutive line sections are assumed to have the same attenuation factor.

For the first-line section source, impedance is $Z_{g_1} = Z_g$ this line section generates an available noise power P_{n_1} and delivers it to the load impedance. The impedance transformer structure is designed to be a matched structure, therefore, the delivered power is equal to the available power. The available noise power of the first line section is

$$P_{n_1} = kT\Delta f \left(1 - \frac{L((Z_g - Z_1)^2 - (Z_g + Z_1)^2)}{(Z_g - Z_1)^2 - L^2(Z_g + Z_1)^2}\right). \quad (19)$$

For the second line section source, impedance is $Z_{g_2} = \frac{Z_1^2}{Z_g}$ and this line section generates an available noise power P_{n_2} and delivers it to the load impedance

$$P_{n_2} = kT\Delta f \left(1 - \frac{L((Z_g - qZ_1)^2 - (Z_g + qZ_1)^2)}{(Z_g - qZ_1)^2 - L^2(Z_g + qZ_1)^2}\right). \quad (20)$$

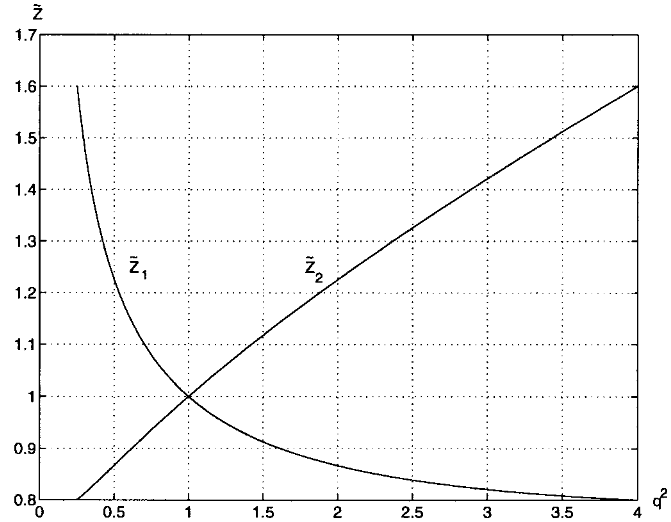


Fig. 3. Characteristic impedance of the line segments of a double section quarter-wave-line impedance transformer normalized with source impedance as a function of impedance transformation ratio q^2.

The total noise power delivered to the load by the lossy quarter wave transmission lines is

$$P_{\text{Total}} = \frac{P_{n_1}}{L_2} + P_{n_2}. \quad (21)$$

This total noise power is a function of Z_1. In order to minimize this noise power an optimum Z_1 can be chosen and then Z_2 can be adjusted to satisfy the required impedance transformation; that is, Z_1 is chosen such that

$$\frac{\partial P_{\text{Total}}}{\partial Z_1} = 0. \quad (22)$$

The characteristic impedance Z_1, which satisfies the above equation can be represented by the following approximation for low-loss line segments:

$$Z_1 \approx \left(\frac{(q-1)^2 + ((q-1)^4 + 4q^2)^{1/2}}{2q^2}\right)^{1/2} \times Z_g. \quad (23)$$

For a source impedance of 50 Ω, variation of optimum Z_1 as defined in the above equation with respect to impedance transformation ratio is shown in Fig. 3. NELL of double section quarter-wave-line impedance transformers as a function of Z_1 for various values of q^2 is given in Fig. 4, where the equivalent length is normalized with the physical length. The minima of the curves on these graphs correspond to the optimum noise performance of the transformer for a given impedance transformation ratio q^2.

Substituting (23) into (19) and (20) and using (12), NELL is obtained as

$$l_{neqv,\text{tr}} = \frac{1}{2\alpha} \ln\left(\frac{1}{1 - P_{\text{Total}}/kT\Delta f}\right). \quad (24)$$

The variation of normalized NELL with impedance transformation ratio q^2 for the optimum value of Z_1 as determined by (23), is given in Fig. 5.

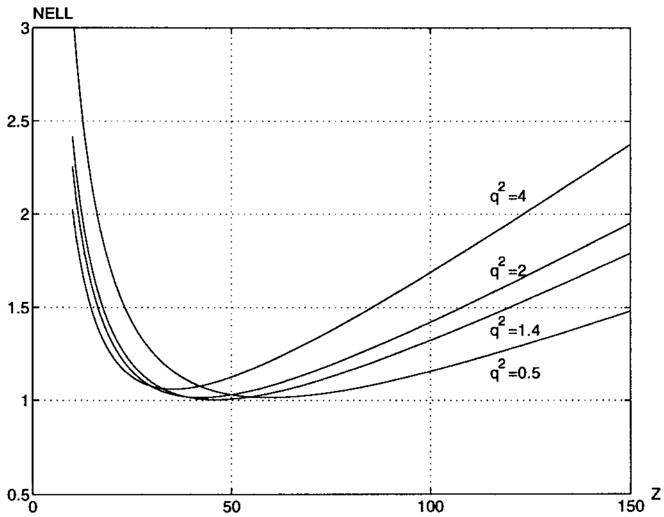


Fig. 4. NELL of double section impedance transformers normalized with their physical line lengths as a function of the characteristic impedance of the first line segment.

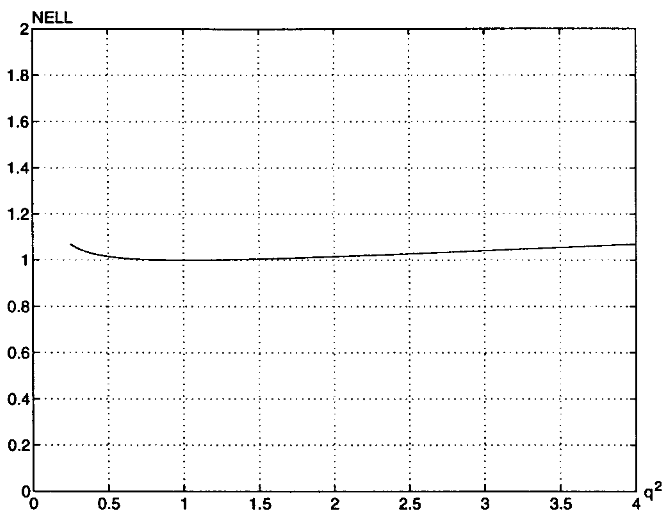


Fig. 5. NELL of a double section quarter-wave-line impedance transformer normalized with its physical line length as a function of impedance transformation ratio q^2, where optimum characteristic impedance line segments are used.

IV. Unequal Power Combiner and Noise-Equivalent Line Length

In this section, analysis of unequal power combiners with special emphasis on noise response is introduced. The schematics of an unequal power combiner is shown in Fig. 6. A detailed power combination analysis of such a structure is given in [28]. The power combination ratio is determined by the impedance of the combining arms seen by the combiner at the junction point. In order to have a power combination ratio of $1 : p^2$, impedances seen at the junction points should be adjusted to

$$\begin{aligned} Z_{\mathrm{arm}_2} &= (1+p^2)Z_{\mathrm{load}} \\ Z_{\mathrm{arm}_3} &= \frac{1+p^2}{p^2}Z_{\mathrm{load}} \end{aligned} \tag{25}$$

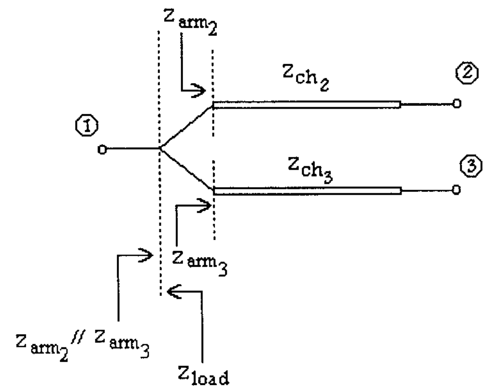


Fig. 6. Schematic representation of an unequal transmission line power combiner.

where combined power is delivered to a load impedance of Z_{load}.

With these impedances the system is matched for coherent excitation from the arms and power combination ratio is $1 : p^2$, i.e., one arm's contribution is $1/(1+p^2)$ and the other arm's contribution is $p^2/(1+p^2)$ to the output power, where p is a positive real number. For $p = 1$ this structure corresponds to an equal power combiner. In even mode excitation, i.e., coherent excitation, no power is reflected back from the input ports, i.e., input ports are matched. Sum of the available power of two sources is equal to the output power. In odd-mode excitation, all the power is reflected back from the input ports and no power is coupled to the output port.

Excitation from a single port is constructed by superposing the input sources of the even and odd mode excitations with a proper ratio. The power combiner is matched for coherent excitation, i.e., excitation from both ports, but it is not matched for excitation from a single port, i.e., incoherent excitation. The power levels at the ports for such an excitation are as follows:

$$\begin{aligned} P_{\text{in from port 2}} &= \frac{1+2p^2}{(1+p^2)^2}P_{\text{available}} \\ P_{\text{reflected from port 2}} &= \frac{p^4}{(1+p^2)^2}P_{\text{available}} \\ P_{\text{coupled to port 3}} &= \frac{p^2}{(1+p^2)^2}P_{\text{available}} \\ P_{\text{delivered to port 1}} &= \frac{1+p^2}{(1+p^2)^2}P_{\text{available}} \end{aligned} . \tag{26}$$

Using p and $1/p$ alternatively in the above equations, the results for the two different arms of the same unequal power combiner can be obtained.

Consequences:

1) Notice that equal excitation of an unequal combiner does not correspond to even mode excitation. When the two arms of the combiner are excited with equal amplitude coherent sources, some power is reflected back from the input ports and the power delivered to the output port is a proportional combination of the two sources, which is seen from simultaneous interpretation of even- and odd-mode excitations. This case corresponds to receiving signal by an-

tenna elements. The antenna elements are identical and the incident signal wave is uniform. Therefore, the induced open-circuit voltages on the antenna elements are equal in magnitude. In a tapered fed antenna array, some part of the incident power reflects back from the unequal combiner inputs, whereas in a uniform fed array all the received power is transferred to the output port. This means that with the same number of antenna elements, i.e., with the same aperture, a uniform fed array receives more power. This explains the reduced aperture efficiency in tapered feeds.

2) When only one arm of the combiner is excited, delivered power to the output port is equal to the power combination ratio of that arm times the available power of the source, as seen from (26). This case corresponds to the response of the combiner to incoherent sources such as noise.

Noise power at the output port of the combiner due to noise signals at the input ports can be expressed as

$$\begin{aligned} P_o &= P_1 + P_2 \\ &= w_1 P_{\text{available},1} + w_2 P_{\text{available},2} \end{aligned} \tag{27}$$

where $w_1 = \frac{1+p^2}{(1+p^2)^2}$ and $w_2 = \frac{p^2}{1+p^2}$. Notice that $w_1 + w_2 = 1$. In other words, output power is a weighted sum of the input noise powers.

Let us consider a line connected to one of the input ports of a combiner. Depending on its source impedance incoherent match conditions, it will have a NELL. Moreover, due to the weighting of the arms it will have another NELL seen from the output port of the combiner. In this way, the equivalent line whose available noise power is equivalent to the noise power contribution of the line to the output of the combiner is defined as the weighted noise-equivalent line length (WELL). The equivalent noise power is obtained by

$$P_{\text{out,line}} = w \cdot P_{\text{line}} \tag{28}$$

where w is the weight of the combining arm.

As stated previously, a feed structure is basically a power combiner structure whose inputs are the antenna elements. As in a two arm combiner, each input arm, i.e., each antenna element has a specific weight, which is determined by its excitation coefficient. There may be several levels of subarrays in an antenna array structure. Each level of subarrays is also a combiner structure. Therefore, overall contribution weight of an antenna element is determined by the combined effects of the combiners in different levels. Similiarly, weight of an individual line segment in the feed structure is determined by the combiners proceeding the line segment up to the output port.

Using the definition of NELL, the following expression is obtained for the weighted noise-equivalent line length of a line section:

$${}^{w}l_{\text{neqv}} = \frac{1}{2\alpha} \ln\left(\frac{L^2 - |\Gamma_g|^2}{(L^2 - |\Gamma_g|^2) - w(L-1)(L + |\Gamma_g|^2)} \right) \tag{29}$$

where w is determined as described above.

By this definition weighted equivalent length of each line segment in a structure can be calculated. These weighted equivalent lengths explicitly give the strength of each noise source in the structure, which enables the designer to see the relatively noisy parts of the feed structure. Summation of weighted equivalent line lengths gives the overall NELL.

In view of (27), if the power inputs to the input arms are equal to each other, i.e., $P_{\text{available},1} = P_{\text{available},2} = P_{\text{available}}$, then output power is equal to the input power from one of the input ports

$$P_o = P_{\text{available}}. \tag{30}$$

This means that noise-equivalent line of the combiner is the line on one of the input arms when the two input arms of the combiner are identical regardless of the combination weights

$$l_{\text{combiner}} = l_{\text{input arm}}. \tag{31}$$

When the incoming noise powers are not equal to each other, the NELL of the combiner can be approximated using (12) and (27) as follows:

$$l_{\text{combiner}} \approx w_1 \cdot l_{\text{arm},1} + w_2 \cdot l_{\text{arm},2}. \tag{32}$$

This approximation is valid within about 10% error for noise temperatures less than 200 °K for any combination ratio. Note that highest noise temperature for a passive structure is the ambient temperature, i.e., 290 °K, which corresponds to a resistor and for low-loss transmission lines; this figure is usually very much less than 200 °K.

Output impedance of a parallel fed antenna array decreases rapidly with the increasing number of parallel connected antenna elements, which makes it difficult to attain impedance match for coherent and incoherent operation. This situation is prevented by implementing impedance transformers at the connection points of subarrays. In a corporate feed the number of these connection points is maximum because the number of antenna elements in the subarrays is the minimum, i.e., two. Therefore, coherent and incoherent matching can be achieved more easily in a corporate feed structure.

Weighted noise-equivalent line length formulation is applied to a $N = 2^n$ element uniform corporate fed microstrip rectangular patch antenna array which is incoherent impedance matched. Following is the approximate NELL for this structure based on (16) and (32):

$$\begin{aligned} l_{\text{total},n\,\text{even}} &\approx \sum_{k=1}^{n/2} \lambda \cdot 2^{k-1} - \lambda/4 \\ &\approx \lambda \cdot 2^{n/2} - \frac{5}{4}\lambda \\ l_{\text{total},n\,\text{odd}} &\approx l_{\text{total},n-1} + \frac{1}{2}\lambda \cdot 2^{(n-1)/2}. \end{aligned} \tag{33}$$

Because of spurious radiation losses and conductor losses [25], realizable microstrip transmission line widths are limited. Consequently, characteristic impedances required for incoherent impedance match in parallel arm structures are not always practical and incoherent mismatch is usually unavoidable. Such a structure may have coherent impedance match but because of incoherent impedance mismatch, its NELL will be much longer than the minimum equivalent length. A $2 \times p$ element uniform fed parallel arm structure has the following approximate NELL, including incoherent mismatch:

$$l_{\text{total}} \approx \sum_{k=1}^{p/2-1} \frac{1}{2\alpha} \ln\left(\frac{L^2 - |\Gamma_{g,k}|^2}{L^2 - |\Gamma_{g,k}|^2 - w(L-1)(L+|\Gamma_{g,k}|^2)}\right) \tag{34}$$

where

$$w = \frac{2k}{p}$$
$$\Gamma_{g,k} = \frac{Z_{\text{ant}}/2k - Z_{\text{line},k}}{Z_{\text{ant}}/2k + Z_{\text{line},k}}.$$

A two-level parallel arm structure has the shortest physical line length, but because of the incoherent mismatch, the NELL might be several times longer than the physical line length. For a corporate feed structure, the total physical line length is a maximum, but because of easy integration of impedance transformers, incoherent impedance can be attained and NELL can be kept at the minimum. The NELL is a measure to determine the topology of the array for low-noise operation from this point of view.

V. Active Circuit Placement for Low-Noise Operation

Lengths of the line segments in a feed structure are determined with phasing requirements and characteristic impedances should be chosen for the incoherent impedance match. However, in some cases, incoherent impedance match might be unattainable and noise performance might be moderate. For a large antenna array, active circuit integration might be a solution to achieve low-noise operation. In that case, NELL will be a design guide to determine the insertion level of the active devices in the feed structure. This level depends on the noise figure and gain of the active circuit and the loss factor of the line segments.

Formulations (33) and (34) of NELL's for different feed structures that accept the number of antenna elements as a parameter, enable the designer to decide on the number of elements in a subarray. In the following lines, it will be shown that a level in the structure may exist such that insertion of the active device at this level will improve the noise performance. Fig. 7 shows, schematically, the levels in a corporate feed structure where active elements are placed at one of them. Placing active devices at level one is the best choice for lowest noise performance at the expense of large number of active circuits, i.e., 2^n active circuits. However, there will be a level in the structure where noise improvement is just achieved with decreased number of active elements.

Usually, low-noise amplifiers (LNA) are used at the output, i.e., at level $n+1$, of the passive receive arrays to decrease the noise contribution of the proceeding stages in the receiving system. However, a passive antenna and a LNA connected to level $n+1$ has a lower G/T than the passive antenna itself as shown below.

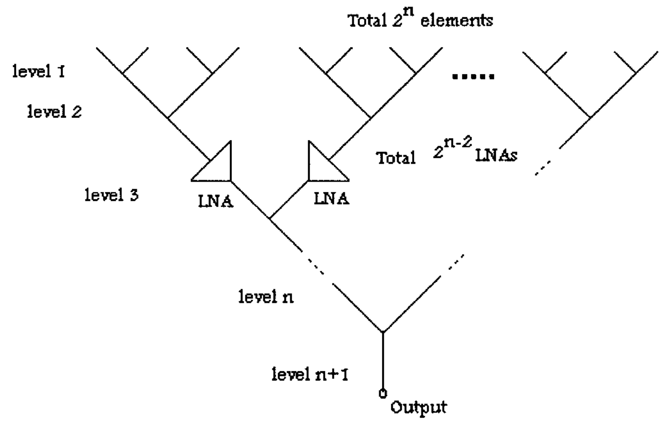


Fig. 7. Schematic representation of a $N = 2n$ element corporate feed structure.

Signal power at the output of the combination is

$$P_{\text{signal, sys}} = G_{\text{amp}} P_{\text{signal, ant}} \tag{35}$$

and noise is

$$\begin{aligned} P_{\text{noise, sys}} &= G_{\text{amp}} P_{\text{noise, ant}} + P_{\text{amp. noise}} \\ T_{\text{sys}} &= G_{\text{amp}} T_{\text{ant}} + T_{\text{amp}} \end{aligned} \tag{36}$$

which yields a $(G/T)_{\text{passive ant}}$ greater than the $(G/T)_{\text{active sys}}$

$$\begin{aligned} (G/T)_{\text{ant}} &= \frac{P_{\text{signal ant}}/P_{\text{in, ant}}}{T_{\text{ant}}} \\ &= \frac{G_{\text{amp}} P_{\text{signal ant}}/P_{\text{in, ant}}}{G_{\text{amp}} T_{\text{ant}}} \\ &\geq \frac{G_{\text{amp}} P_{\text{signal ant}}/P_{\text{in, ant}}}{G_{\text{amp}}(T_{\text{ant}} + T_{\text{amp}}/G_{\text{amp}})} \\ &\geq (G/T)_{\text{sys}}. \end{aligned} \tag{37}$$

On the other hand, placing such amplifiers at level k, $k \neq n+1$, in the array to form an active array with distributed amplification, in any case, improves the G/T performance compared to such a system of antenna and a LNA connected to level $n+1$. In order to see this fact let us consider the two cases given in Fig. 8. In this figure, the connection point A of two successive levels k and $k+1$ is shown with noise-equivalent length l_{neqv1} corresponding to levels one to k of the structure shown in Fig. 7 and noise-equivalent length l_{neqv2} corresponding to levels $k+1$ to n. This series representation of NELL's is possible because of the results given in Section IV [see (18)]. A feed structure with noise temperature T_f and efficiency η_f can be partitioned into two consecutive sections with noise temperatures and efficiencies T_1, η_1, and T_2, η_2, respectively, where $T_f = T_1 \times \eta_2 + T_2$ and $\eta_f = \eta_1 \times \eta_2$ and to improve the performance an LNA can be placed between these two partitions, as shown in Fig. 8(b). The noise power generated at the output by the configuration in Fig. 8(a) (where a single amplifier is used) is given by

$$\begin{aligned} P_{\text{out,single}} &= k\Delta f((\eta T_B + T_{\text{LNA}}) + T_f)G \\ &= kT_B \Delta f G\eta + kT_f \Delta f G + kT_{\text{LNA}} \Delta f G. \end{aligned} \tag{38}$$

The second case, as shown in Fig. 8(b), corresponds to the distributed amplification and has the following noise power at

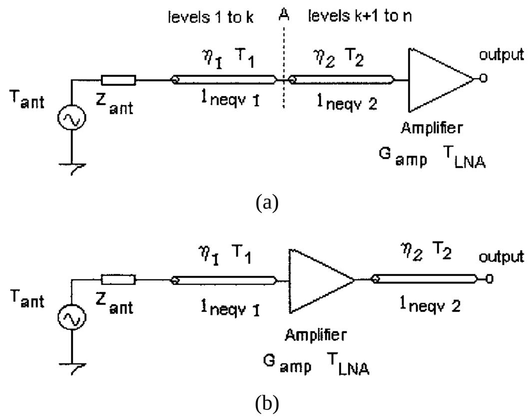


Fig. 8. Active circuit integration with antenna arrays. (a) LNA connected to level $n+1$. (b) LNA's connected to level $k+1$.

the output:

$$\begin{aligned} P_{\text{out,dist}} &= k\Delta f(G\eta_2(\eta_1 T_B + T_1 + T_{\text{LNA}}) + T_2) \\ &= kT_B \Delta f G\eta + kT_{\text{LNA}} \Delta f G \eta_2 \\ &\quad + kT_1 \Delta f G \eta_2 + kT_f \Delta f. \end{aligned} \tag{39}$$

The difference of the two noise-power expressions is

$$\begin{aligned} &P_{\text{out,single}} - P_{\text{out,dist}} \\ &\quad = k\Delta f G\left(T_f - \eta_2 T_1 - \frac{T_2}{G} + T_{\text{LNA}}(1-\eta_2)\right). \end{aligned} \tag{40}$$

Since $\eta_2 < 1$ and $T_f > T_1\eta_2 + T_2/G$, the single amplifier case creates strictly more noise power at the output. From the signal output point of view, the two systems are equivalent. Therefore, noise performance is the determining factor when comparing the two systems for G/T performance, which shows that distributed amplification, in any case, has better G/T performance.

In addition, depending on η_2 and T_2, distributed amplification may yield even a better G/T compared to $(G/T)_{\text{passive}}$. When comparing noise powers of these two systems, active system's noise power should be normalized since the signal power of active system is G times more than that of the passive system. Distributed amplification yields a better G/T than the passive case when the following equation yields a result greater than zero:

$$\begin{aligned} P_{\text{out,passive}} - \frac{P_{\text{out,dist}}}{G} &= T_f - \eta_2(T_{\text{LNA}} + T_1) - \frac{T_2}{G} \\ &= T_2\left(1 - \frac{1}{G}\right) - \eta_2 T_{\text{LNA}}. \end{aligned} \tag{41}$$

To satisfy this condition, high G and low T_{LNA} values are required. This equation also implies that active circuit integration is necessary for low-efficiency systems. Using this calculation maximum length of noise-equivalent line of the second partition of the feed structure can be determined, where an active circuit with a certain G and T_{LNA} should be inserted for better noise performance. This point will be the optimum point where the number of required LNA's is a minimum and the noise performance of the passive array is improved.

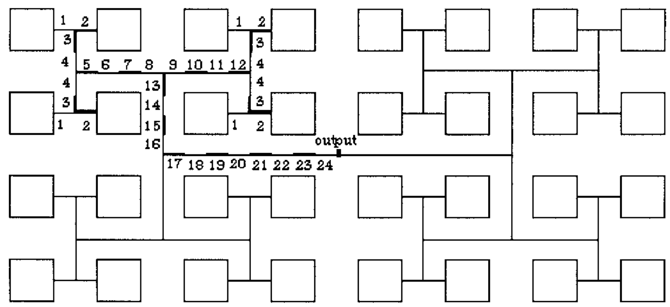


Fig. 9. Schematic representation of a 4×8 corporate feed antenna array. The characteristic impedance of the numbered line segments are given in Table II.

TABLE I
CHARACTERISTIC IMPEDANCE OF THE LINE SEGMENTS IN THE CORPORATE FEED EXAMPLE FOR ONLY COHERENT IMPEDANCE MATCHED AND COHERENT AND INCOHERENT IMPEDANCE MATCHED

#	$Z_{coh.}$	Z_{both}	#	$Z_{coh.}$	Z_{both}	#	$Z_{coh.}$	Z_{both}	#	$Z_{coh.}$	Z_{both}
1	200	200	7	100	164	13	100	105	19	100	100
2	189	189	8	105	195	14	100	122	20	136	100
3	100	100	9	100	168	15	100	139	21	100	100
4	100	200	10	100	149	16	100	161	22	136	100
5	100	116	11	100	132	17	100	100	23	100	100
6	105	138	12	100	117	18	136	100	24	136	100

VI. DESIGN EXAMPLES

A. Passive Array

In order to demonstrate the importance of incoherent match in feed structures, two microstrip rectangular patch antenna arrays are designed for two different cases: 1) only coherent impedance match and 2) both coherent and incoherent impedance match. In both designs, microstrip lines which have moderate radiation and conductor loss are used with characteristic impedances in between 100 Ω and 200 Ω [25]. The patch antenna input impedance is taken as 300 Ω.

The first array is a corporate fed antenna array with a 0.9^n feed taper in one direction. The schematics of this 8×4 antenna array is given in Fig. 9. The output impedance of the array is designed to be 50 Ω. The feed structure which is symmetric is investigated as a combination of $\lambda/4$ line segments. The numbering of these line segments is shown on the figure. In Z_{coh} columns of Table I, characteristic impedances of the line segments for only coherent match are given. Data in the Z_{both} columns of the same table is obtained by considering both coherent and incoherent match via (12), (23), (32), and (29). The impedance transformers in the coherent match case are used for feed tapering, and in the latter, used for both feed tapering and incoherent impedance match. As can be seen in Table I, the characteristic impedances Z_{both} obtained for optimum noise performance are quite different than those values obtained without considering incoherent impedance match. To show the improvement, the NELL's of both feed structures are calculated and normalized with the

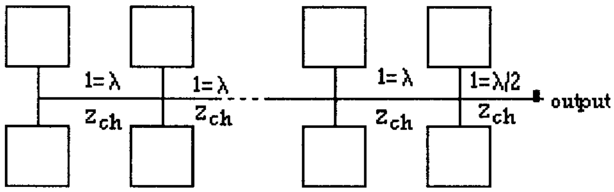


Fig. 10. A $2 \times n$ parallel arm feed structure.

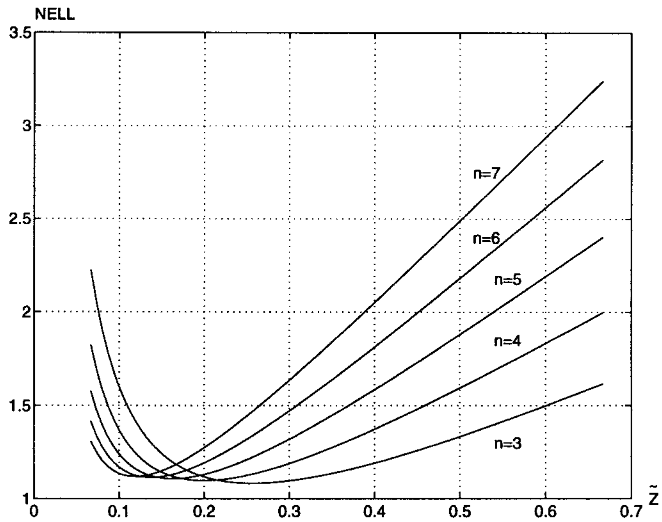


Fig. 11. Variation of normalized NELL of a $2 \times n$ parallel arm antenna array as a function of characteristic impedance of the parallel arm for various values of n.

completely incoherent impedance matched situation

$$l_{neqv,\text{case}\,1} = 2.13 l_{\text{matched}}$$
$$l_{neqv,\text{case}\,2} \approx 1.02 l_{\text{matched}}. \tag{42}$$

As it is explicit from (42), the improvement ratio is $2.13/1.02$ which is greater than two.

The second array is a parallel arm structure, which is a uniform fed array of $2 \times n$ elements as shown in Fig. 10. It is assumed that the line segments in the structure are all of equal characteristic impedance. The output impedance of the array is not matched to 50 Ω. NELL's of the feed structures are calculated and plotted in Fig. 11 for different values of n. The NELL's shown in the plot are normalized with the incoherent impedance matched equivalent length. As seen from this figure, for each value of n a value for Z_{ch} exists for optimum noise performance.

B. Active Array

A 16 × 16 corporate-fed microstrip antenna array on a RT-DUROID 5880 substrate is designed. Quarter-wave-line impedance transformers are placed according to the results of Section III, to the feed structure to decrease the noise generation by the lossy feed lines. The rectangular patch antennas are resonant at 10 GHz having a 276 Ω resistive input impedance. The uniform fed antennas are equispaced by a distance d, which is determined by radiation pattern considerations. The total NELL of this 16 × 16 patch array is calculated to be about 14.8λ which corresponds to $T = 101\,°$K for $T_{\text{amb}} = 290\,°$K.

TABLE II
NUMBER OF AMPLIFIERS TO BE CONNECTED TO 16 × 16 PATCH ANTENNA ARRAY AND THE RESULTANT OUTPUT NOISE TEMPERATURE FOR DIFFERENT LEVELS AS CONNECTION POINTS OF AMPLIFIERS

Level	1	2	3	4	5	6	7	8	output
# of amp.	256	128	64	32	16	8	4	2	1
T_{level} (dB-K)	30.7	30.9	31.1	31.7	32.1	32.9	33.6	34.7	35.6

Using microstrip matching elements and an ultra low-noise pHEMT NEC 32 484A, a low-noise amplifier is designed. The noise figure of the amplifier is 0.6 dB and the gain is 14 dB. It is assumed and shown that appropriate matching circuits whose total physical length will be about $\lambda/2$ can be designed independent of wherever the amplifier is inserted in the feed structure [29]. In the case, where this amplifier is connected to the output of the 16 × 16 passive array, noise temperature at the output of the amplifier turns out to be $T = 35.6$ dB K. Using weighted equivalent line length method and the results of Section V, the noise temperatures corresponding to different amplifier placement levels are calculated and presented in Table II. The improvement of noise performance with the added number of amplifiers is clearly seen from this table.

Moreover, without considering the directive gain, G/T of the passive antenna is

$$(G/T)_{\text{passive}} = \eta_{\text{array}} - 10\log(101)$$
$$\approx \eta_{\text{array}} - 20. \tag{43}$$

For the active array, without considering the directive gain which is the same as for the passive array, the G/T of the active antenna is

$$(G/T)_{\text{active}} = \eta_{\text{array}} + G_{\text{amp}} - T_{\text{level}}$$
$$= \eta_{\text{array}} + 14 - T_{\text{level}} \tag{44}$$

where T_{level} is as given in Table II.

Using Table II, above equations yield that active antenna has a better G/T than the passive antenna for connection level seven, which is also implied by (41). It can be said that for this structure and the amplifiers concerned, the level where the improvement is just achieved is level seven, where four active elements are required. Further improvements can be achieved using more active elements at the different levels as can be seen from Table II.

VII. CONCLUSION

In this work, noise analysis of low-loss transmission lines is given and a new concept named as "NELL" is introduced. Quarter-wave-line impedance transformers are investigated and it is shown that there is an optimum choice of characteristic impedance for the lowest NELL of a double section transformer. After establishing the importance of coherent and incoherent impedance matching concepts, noise sources in a parallel feed antenna array are identified and effects of these sources are expressed in terms of their NELL's. This new concept provides a suitable design guide for low-noise parallel feed structures. Coherent impedance matching, which is important for efficient radiation characteristics, does not imply incoherent impedance matching, which is important for noise generation of the feed structure. It is shown that a feed structure can be redesigned to be incoherent impedance matched without affecting the radiation characteristics. It is also shown that through this optimization procedure, the noise temperature of a typical corporate feed structure can be decreased by a factor of two. The noise performance of different parallel feed structures are investigated and it is shown that incoherent impedance matched corporate feed has a good noise performance, whereas the noise performance of a parallel arm feed structure rapidly degrades with increasing number of antenna elements.

The NELL concept is also utilized as an active circuit placement criteria in antenna arrays, which showed that G/T performance of a passive antenna array may be improved by inserting active circuits to certain levels in the feed structure. With the development of very low-noise figure transistors, it seems that active devices inserted in proper places in a feed structure causes this improvement. On the other hand, earlier transistors having higher noise figures could not lead to such improvements.

REFERENCES

[1] Y. T. Lo, S. W. Lee, and Q. H. Lee, "Optimization of directivity and signal-to-noise ratio of an arbitrary antenna array," *Proc. IEEE*, vol. 54, pp. 1033–1045, Aug. 1966.

[2] D. K. Cheng and F. I. Tseng, "Signal-to-noise ratio maximization for receiving arrays," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 792–794, Nov. 1966.

[3] D. K. Cheng, "Optimization techniques for antenna arrays," *Proc. IEEE*, vol. 59, pp. 1664–1674, Dec. 1971.

[4] L. P. Winkler and M. Schwartz, "A fast numerical method for determining the optimum SNR of an array subject to a Q factor constraint," *IEEE Trans. Antennas Propagat.*, vol. AP-20, pp. 503–505, July 1972.

[5] W. Wasylkiwskyj and W. K. Kahn, "Efficiency as a measure of size of a phased array antenna," *IEEE Trans. Antennas Propagat.*, vol. AP-21, pp. 879–884, Nov. 1973.

[6] R. A. York, R. D. Martinez, and R. C. Compton, "Active patch antenna element for array applications," *Electron. Lett.*, vol. 26, no. 7, pp. 494–495, Mar. 1990.

[7] R. Gillard, H. Legay, J. M. Floch, and J. Citerne, "Rigorous modeling of receiving active microstrip antenna," *Electron. Lett.*, vol. 27, no. 25, pp. 2357–2358, Dec. 1991.

[8] H. An, B. Nauwelaers, and A. van de Capelle, "Broadband active microstrip array elements," *Electron. Lett.*, vol. 27, no. 25, pp. 2378–2379, Dec. 1991.

[9] B. Robert, T. Razban, and A. Papiernik, "Compact amplifier integration in square patch antenna," *Electron. Lett.*, vol. 28, no. 19, pp. 1808–1810, Sept. 1992.

[10] P. S. Hall and P. M. Haskins, "Microstrip active patch array with beam scanning," *Electron. Lett.*, vol. 28, no. 22, pp. 2056–2057, Oct. 1992.

[11] P. S. Hall, "Analysis of radiation from active microstrip antennas," *Electron. Lett.*, vol. 29, no. 1, pp. 127–129, Jan. 1993.

[12] D. Sanchez-Hernandez and I. Robertson, "60 GHz band active microstrip patch antenna for future mobile system applications," *Electron. Lett.*, vol. 30, no. 9, pp. 677–678, Apr. 1994.

[13] H. An, B. Nauwelaers, and A. R. van de Capelle, "Broad-band active microstrip antenna design with the simplified real frequency technique," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 1612–1619, Dec. 1994.

[14] H. An, B. Nauwelaers, and A. van de Capelle, "Noise figure measurement of receiving active microstrip antennas," *Electron. Lett.*, vol. 29, no. 18, pp. 1594–1596, Sept. 1993.

[15] W. Grabherr and W. Menzel, "Broadband, low noise active receiving microstrip antenna," in *Proc. 24th Eur. Microwave Conf.*, Cannes, France, Sept. 1994, pp. 1785–1790.

[16] R. J. Mailloux, J. McIlvenna, and N. P. Kernweis, "Microstrip array technology," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 25–37, Jan. 1981.

[17] M. A. Weiss, "Microstrip antennas for millimeter waves," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 171–176, Jan. 1981.

[18] P. S. Hall, "Printed antennas: Responding to new applications," in *Proc. 26th Eur. Microwave Conf.*, Prague, Czech Republic, Sept. 1996, pp. 464–470.

[19] Y. T. Lo and S. W. Lee, *Antenna Handbook: Theory, Applications and Design*. New York: Van Nostrand Reinhold, 1988.

[20] F. Lalezari and C. D. Massey, "mm wave microstrip antennas," *Microwave J.*, vol. 30, no. 4, pp. 87–96, Apr. 1987.

[21] R. J. Mailloux, "Antenna array technology," *Proc. IEEE*, vol. 80, pp. 163–172, Jan. 1992.

[22] P. Bhartia, K. V. S. Rao, and R. S. Tomar, *Millimeter Wave Microstrip and Printed Circuit Antennas*. Boston, MA: Artech House, 1991.

[23] E. H. Newman and J. E. Tehan, "Analysis of a microstrip array and feed network," *IEEE Trans. Antennas Propagat.*, vol. AP-33, pp. 397–403, Apr. 1985.

[24] M. L. Oberhart and Y. T. Lo, "New simple feed network for an array module of four microstrip elements," *Electron. Lett.*, vol. 23, no. 9, pp. 436–437, Apr. 1987.

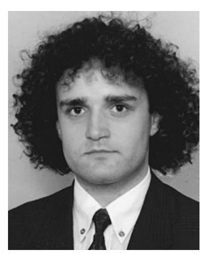
[25] E. Levine, G. Malamud, S. Shtrikman, D. Treves, "A study of microstrip array antennas with the feed network," *IEEE Trans. Antennas Propagat.*, vol. 37, no. 4, pp. 426–434, Apr. 1989.

[26] D. M. Pozar, "Finite phased arrays of rectangular microstrip patches," *IEEE Trans. Antennas Propagat.*, vol. AP-34, pp. 658–665, May 1986.

[27] R. E. Collin, *Antennas and Radiowave Propagation*. New York: McGraw-Hill, 1985.

[28] L. I. Parad and R. L. Moynihan, "Split-tee power divider," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 91–95, Jan. 1965.

[29] S. Demir, C. Toker, and A. Hizal, "Design of an active microstrip array using a microwave circuit simulator," in *Proc. IEEE MTT-S Top. Symp. Tech. Wireless Applicat.*, Vancouver, Canada, Feb. 1997, pp. 103–106.



Simsek Demir was born in Amasya, Turkey, on July 2, 1969. He received the B.Sc., M.Sc., and Ph.D. degrees from the Middle East Technical University, Ankara, Turkey, in 1991, 1993, and 1998, respectively.

He has been with the Electrical and Electronics Engineering Department of the Middle East Technical University since 1991 and is currently an Instructor there. He was a Visiting Scholar at tje University of Massachusetts at Amherst in 1995 under a NATO A2 Fellowship. His research interests are monolithic microwave integrated circuits (MMIC) design, nonlinear analysis of microwave circuits, nonlinear modeling of MESFET's, noise analysis, and low-noise design of active and passive antenna array feed structures.

Dr. Demir received the Student Researcher Award from the Raj Mittra Travel Grant Committee in 1998.

Canan Toker was born in Safranbolu, Turkey, on October 1, 1939. He received the B.Sc. and Ph.D. degrees from the University College of North Wales, Bangor, U.K., in 1962 and 1965, respectively.

He joined the Department of Electrical and Electronic Engineering, Middle East Technical University, Ankara, Turkey, in 1965, where he is currently a Professor. He was the Chairman of the Department of Electrical and Electronic Engineering, Gaziantep Campus, Middle East Technical University, between 1974 and 1978 and the Chairman of the Department of Electrical and Electronic Engineering, Middle East Technical University, Ankara, between 1982 and 1985. He was the Deputy Director of the Turkish Scientific and Technical Research Council between 1990 and 1992. His research interests include varactor diodes and parametric amplifiers, tunnel diodes, nonuniform transmission lines, directional couplers, and periodic structures. In recent years he has been involved with the monolithic microwave integrated circuits (MMIC), particularly with the modeling of MESFET's.

Dr. Toker received the Science Promotion Award of the Turkish Scientific and Technical Research Council in 1974.