Anomalous Edge Effects in Finite Arrays

R. C. Hansen, *Life Fellow, IEEE*

***Abstract*—The regular oscillations in scan impedance (normalized by the infinite array values) that occur across a finite array are uniquely altered in two special cases based on computer simulations of finite-by-infinite arrays. First, dipoles with groundplane at broadside in the E-plane, exhibit a slow modulation of the usual oscillations; the period of the modulation varies with dipole radius. For all other cases, the results are insensitive to radius. Second, for a dipole array and lattice spacing and H-plane scan angle that allow a grating lobe to appear at -90° the oscillations disappear, except at the rear edge. These phenomena give some indications of the behavior of the pseudotraveling waves in scan impedance.**

***Index Terms*—Antenna arrays, edge effects, electronic scan.**

I. Introduction

EDGE effects in finite arrays have been studied through the medium of a finite-by-infinite array [1], [2], where the beam is scanned across the finite dimension of the array. Use of colinear or parallel dipoles in the infinite linear arrays allows H-plane or E-plane scan [2]. For a sketch of the geometry see [3]. Since most of the characteristic and essential features of mutual coupling are experienced by scanning an array of thin wire dipoles at or near resonance, more sophisticated elements and moment method analyses are not necessary. Thus, finite-by-infinite arrays of thin wire dipoles with or without ground plane have been used; each dipole has an assumed sinusoidal current distribution.

The vital parameter for a scanned array is scan impedance; the obsolete term "active impedance" is deprecated. The scan impedances of a row of elements across the finite dimension of the array allow all important array parameters to be calculated: directivity, patterns, scan-element pattern, element mismatch. These scan impedances are found from the element voltages divided by the applied element currents; the latter may include an amplitude taper for sidelobe control. The voltage and current vectors are related by an impedance matrix. Each term of the impedance matrix is the sum of mutual impedances from one dipole in one infinite linear array to all the dipoles in another infinite linear array. These mutual impedance sums were calculated in both the spatial and spectral domains [3]. The spatial-domain calculation used Carter mutual impedances [4] with Levin summation acceleration. For the spectral-domain summation for H-plane scan, the sum involved a generalized pattern function and a Hankel function. E-plane scan required a numerical integration, as the integral of the pattern function times the Hankel function cannot be separated [5]. Plots of scan impedance across the array calculated by spatial and spectral methods agreed within a line width, thus validating these array simulators. Details of the calculations are given in [2] and [3].

The finite-by-infinite array simulations showed several interesting and somewhat unexpected results. The scan impedance values oscillated from element to element about the infinite array values with oscillation amplitude increasing from the center toward the array edges. These oscillations occur even at broadside (no scan). The period of the oscillations is regular and increases with scan angle and an excellent fit is provided by $\lambda/2(1-\sin\theta_0)$ [6], [7]. There were two situations that showed unusual behavior and these anomalies are discussed in this paper.

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The author is with R. C. Hansen, Inc., Tarzana, CA 91357 USA.

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II. Modulated Oscillations

Computer simulations of H-plane scan of dipoles and of dipoles/ground plane and of E-plane scan of dipoles gave results insensitive to dipole radius [6], [7]. However, broadside results for E-plane scan impedance of dipoles/ground plane showed an unexpected behavior: the oscillations were modulated and the modulation period varied with the dipole radius [8]. For each radius, the dipole length was adjusted to give a resonant broadside embedded impedance. Figs. 1–4 show scan impedance for half of a 201-element array with values normalized by the infinite array scan impedances. Ground plane spacing was $\lambda/4$. Although all the oscillations have a wavelength period (half-wave element spacing), the modulation period increases with radius/wavelength as seen.

Assuming a heterodyne process, with the "difference frequency" operating, the period P is simply $1/P = 1/P_1 - 1/P_2$ where P_1 is the basic scan impedance period of one wavelength (zero scan) and P_2 is P_1 modified by a factor: $P_2 = P_1(1+\Delta)$. Fig. 5 shows the parameter Δ for the five radii cases simulated. There is some uncertainty in determining period due to attenuation near the array center. This log-linear relationship is due to the change in mutual impedances with change in radius all reflected through the impedance matrix inverse. Why this appears only for E-plane dipoles with ground plane is not known. A reviewer suggests that a virtual space wave heterodynes with a virtual wave guided between the dipole plane and the ground plane; the logarithmic variation of dipole reactance with dipole radius might produce the results of Fig. 5.

The obvious question is, "How does this phenomenon change with scan angle?" Only limited simulations have been done, but at a 30° and greater scan, no modulation is apparent. At 15° scan there is modulation present, but the pattern of it

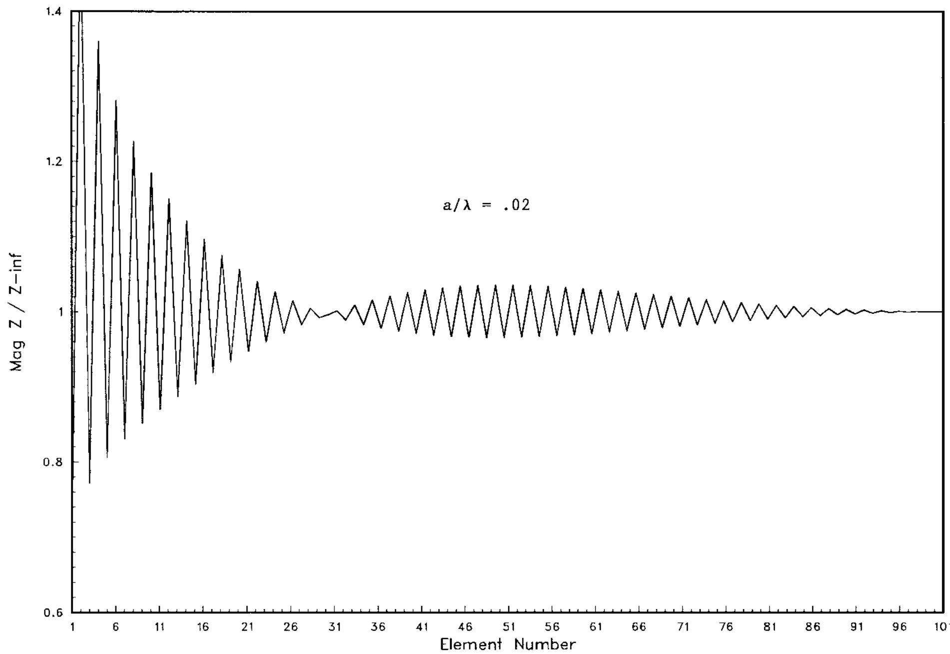

Fig. 1. Two hundred and one linear infinite arrays of dipoles/screen, E-plane scan, and $\Theta = 0°$.

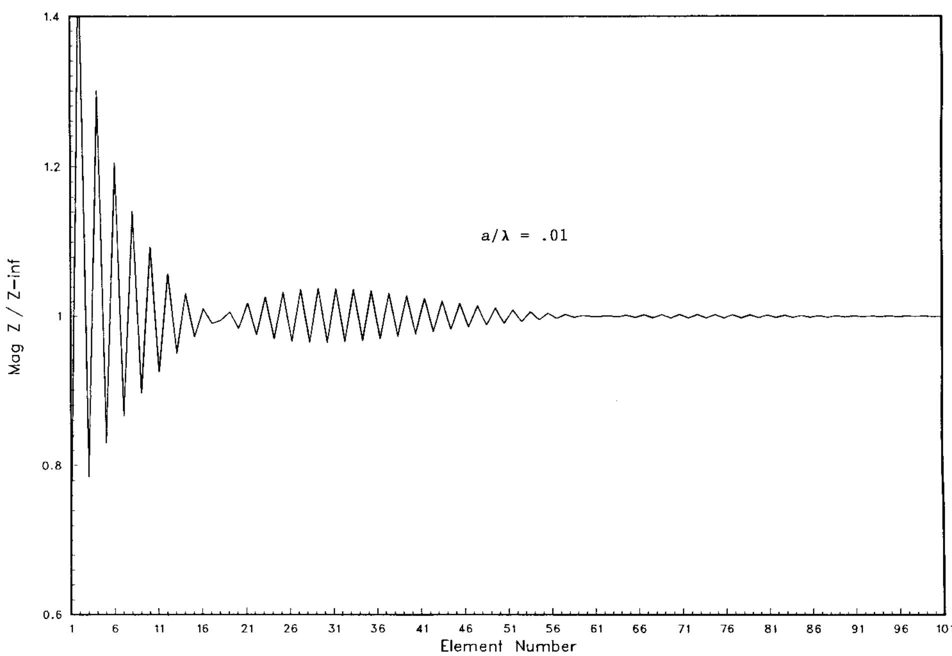

Fig. 2. Two hundred and one linear infinite arrays of dipoles/screen, E-plane scan, and $\Theta = 0°$.

is not clear. Whether the zero scan results can be modeled by empiricism based on electromagnetics is also unknown at this time. This new phenomenon represents a challenge in array modeling and understanding.

III. Scan Impedance at Grating Lobe Incidence

An infinite array of canonical elements such as half-wave or resonant dipoles exhibits a blind angle at grating-lobe incidence in the H-plane as the scan resistance and scan reactance both become infinite. But how does scan impedance behave for a finite array? For the computer simulator, the array lattice of half-wave spacing, resonant dipole length, and dipole radius of 0.005λ were scaled up to provide a grating lobe appearing at $-90°$ for $45°$ scan. Fig. 6 shows the magnitude of scan impedance, now normalized by the center value, again for a 201 element array. The scan impedance is approximately linear, increasing toward the rear of the array. Damped oscillations occur at the rear. These oscillations change as the array size changes. Phase is of also interest; Fig. 7 shows the phase of scan impedance across the array.

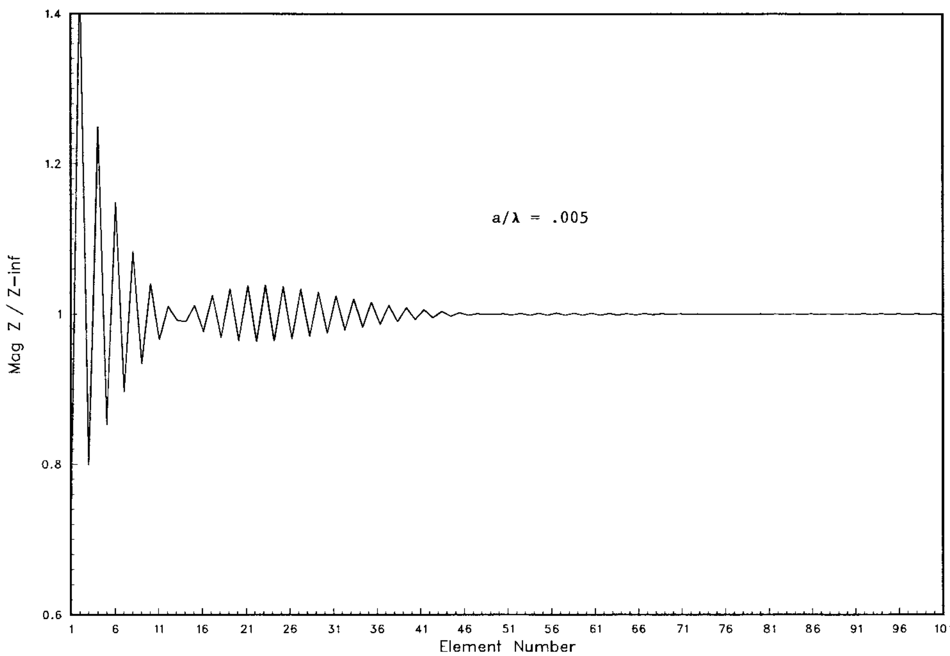

Fig. 3. Two hundred and one linear infinite arrays of resonant dipoles/screen and E-plane scan at 0°.

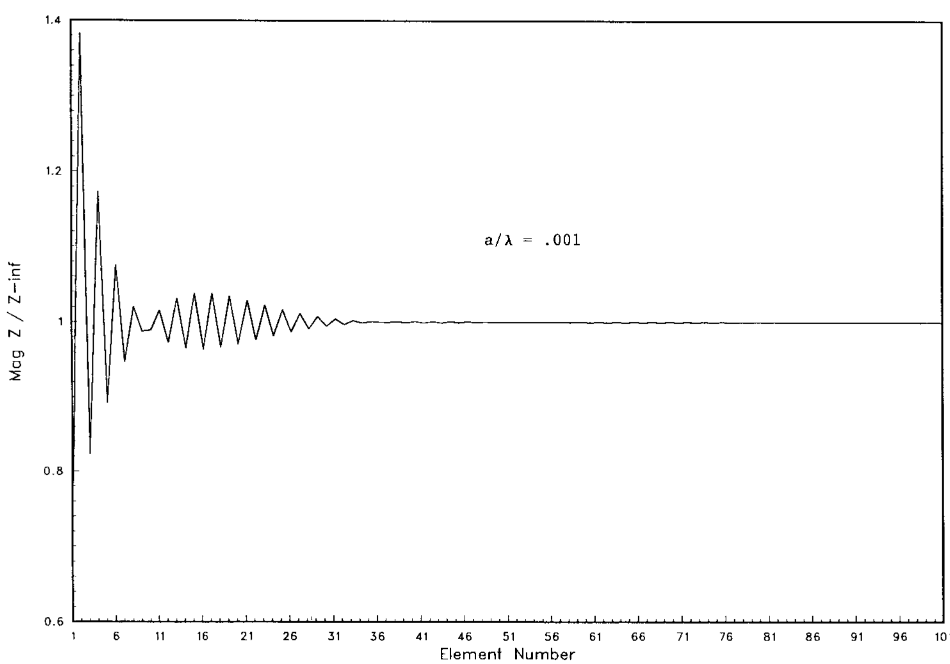

Fig. 4. Two hundred and one linear infinite arrays of resonant dipoles/screen, E-plane scan, and $\Theta = 0°$.

Again, damped oscillations occur at the rear and a dropoff toward zero phase occurs at the front, but over most of the array the phase is approximately 45°. Fig. 8 gives the array center value versus number of elements minus one; as expected, this value becomes large for large arrays. This center value is approximately given by $145\sqrt{N-1}$. There is presently no physical explanation of why the variation is $\sqrt{N}$ instead of N or $\log N$. When the dimensions are adjusted for grating-lobe appearance at 60° scan, the resulting amplitude and phase plots are essentially the same, but the few damped oscillations at the rear have a longer period as expected [9]. Thus, the array (at grating-lobe angle) due to the near absence of oscillations and approximately 45° phase appears approximately as a resistive sheet reminiscent of the current sheet array concept of Wheeler [10]. A further adjustment of dimensions to allow a grating lobe to appear at $-60°$ for a 60° scan gives scan impedances just like those for a half-wave lattice—oscillations about the infinite array value. So the single traveling wave concept is only feasible when the grating lobe first appears at $-90°$.

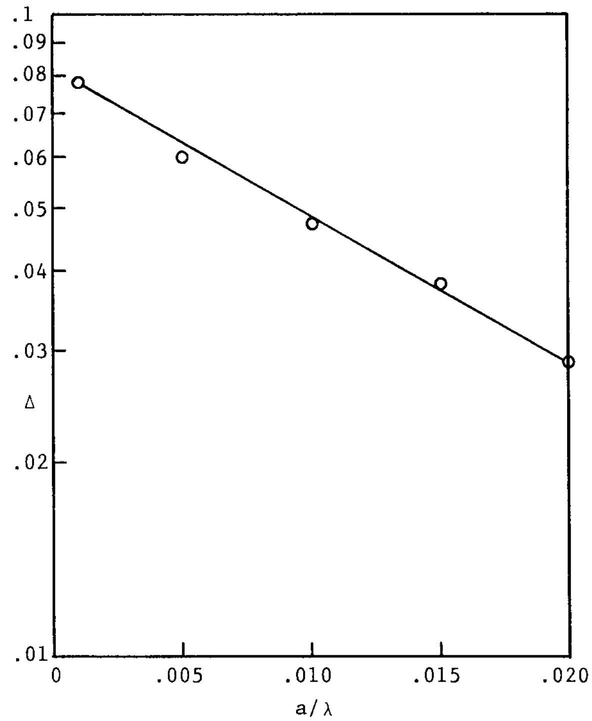

Fig. 5. Modulation factor Δ versus radius.

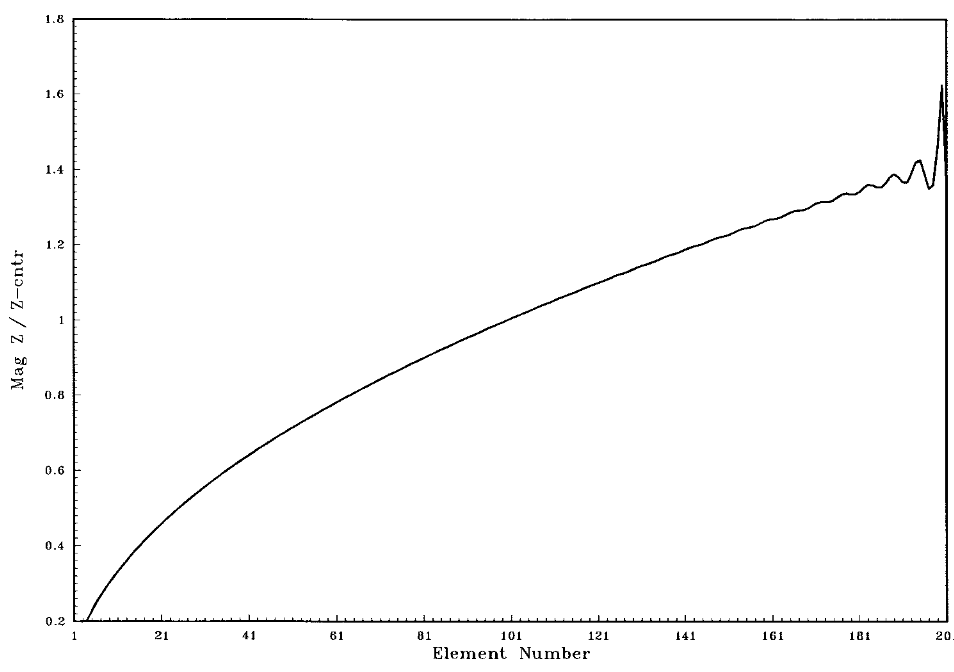

Fig. 6. Two hundred and one linear infinite arrays of dipoles and H-plane scan at 45°.

For E-plane scan with the same grating-lobe spacing, the scan impedance is again that for the half-wave lattice, but the oscillations are smaller.

Because of this resistive sheet analogy, an attempt was made to calculate the edge oscillation values using half-plane diffraction coefficients for a resistive sheet. The Malihiuzhinets function subroutine was generously provided by Volakis [11]. However, none of the combinations of resistive and reactive sheet values tried gave the proper variation of edge value versus scan angle. These edge values would have complemented the Gibbsian models that have been developed [2], [3].

IV. Conclusions

The standing wave of scan impedance across a finite array can be decomposed into two pseudotraveling waves in opposite directions, as indicated by the constant period. These

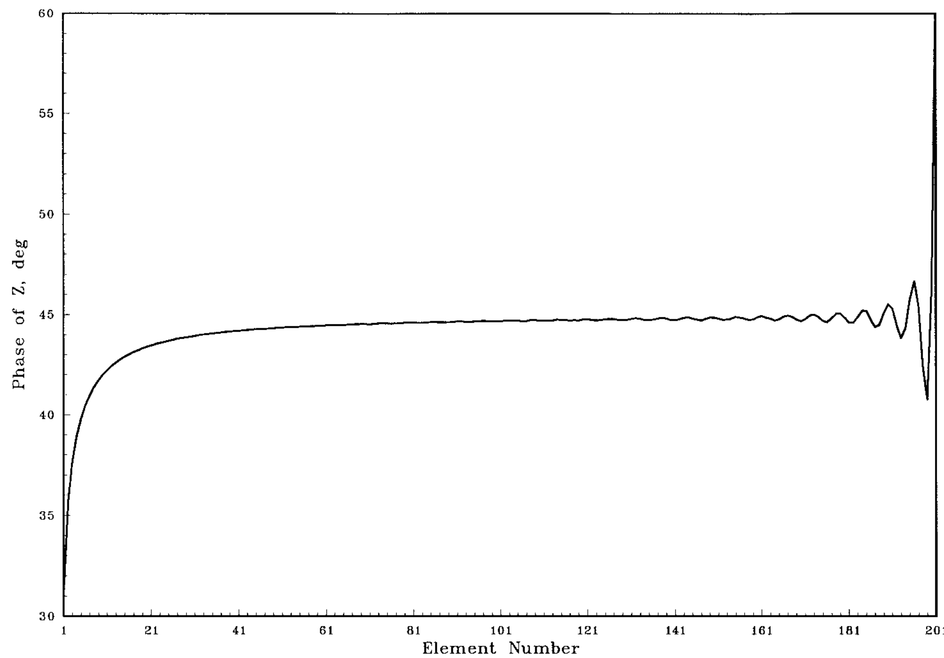

Fig. 7. Two hundred and one linear infinite arrays of dipoles, H-plane scan at 45°.

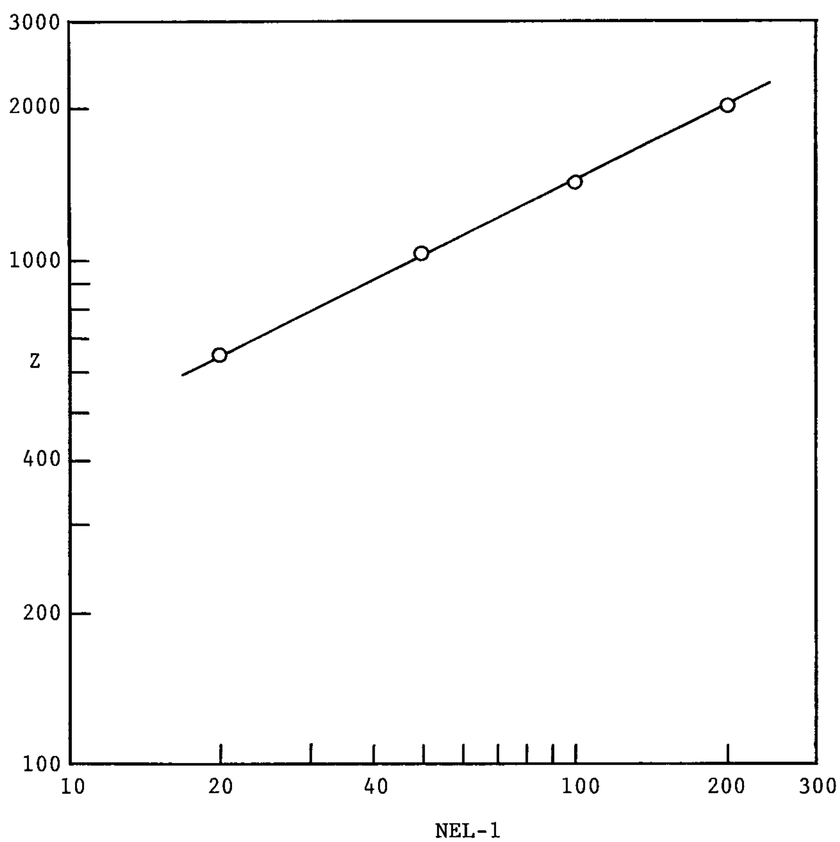

Fig. 8. Array impedance center value.

waves are not electromagnetic waves, of course, and they are esoteric, having an attenuation across the array that is more complicated than a simple exponential. At grating-lobe incidence, one might surmise that only one traveling wave is excited.

The heterodyning (or line splitting, which occurs for broadside E-plane scan of dipoles/screen) may be due to the generation of dual pseudotraveling waves by the mutual coupling. The unique dependence of this case upon dipole radius is still a mystery.

The computer simulations and the apparent traveling waves in scan impedance have engendered a primitive understanding of edge effects in finite arrays, but several important and interesting problems remain to be solved.

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R. C. Hansen (S'47–A'49–M'55–SM'56–F'62–LF'92) received the B.S.E.E. degree from the Missouri School of Mines, Rolla, MO, and the Ph.D. degree from the University of Illinois Urbana-Champaign, in 1955.

Since 1971, he has been a Consulting Engineer for antennas and systems-related problems. From 1967 to 1970 he was with KMS Industries, Ann Arbor, MI. From 1966 to 1967 he was Operations Group Director of the Manned Orbiting Laboratory Systems Engineering Office, Aerospace Corp., Los Angeles, CA, and was responsible for flight crew training, simulator, software system for mission profiles, and mission control center equipment and displays. From 1964 to 1966 he formed and was Director of the Test Mission Analysis Office, responsible for computer programs for the planning and control of classified U. S. Air Force satellites. Prior to that he was Associate Director of Satellite Control, where he was responsible for converting the Air Force satellite control network into a real-time computer-to-computer network. In 1960 he became a Senior Staff Member in the Telecommunications Laboratory, STL, Inc. (now TRW), Redondo Beach, CA, engaged in communication satellite telemetry, tracking, and command. Earlier, he was Section Head of the Microwave Laboratory of Hughes Aircraft Co., Culver City, CA, working on surface wave antennas, slot arrays, near-fields, electronic scanning and steerable arrays, and dynamic antennas. From 1949 to 1955 he worked in the Antenna Laboratory of the University of Illinois, Urbana, on ferrite loops, streamlined airborne antennas, and direction finding and homing systems. He has written over 100 papers on electromagnetics, has been an associate editor of *Microwave Journal* (1960–1995), *Radio Science* (1967–1969), and *Microwave Engineer's Handbook* (1971), editor of *Microwave Scanning Antennas, Vol. I* (1964), *Microwave Scanning Antennas, Vols. II and III* (1966), *Significant Phased Array Papers* (1973), *Geometric Theory of Diffraction* (1981), and *Moment Methods in Antennas and Scattering* (1990). He is the author of *Phased Array Antennas* (New York: Wiley, 1998). He was editor of the APS Newsletter (1961–1963)

Dr. Hansen has been involved in many professional activities including chair of U.S. Commission B URSI (1967–69), chair of the 1958 WESCON Technical Program Committee, President of IEEE Antenna and Propagation Society (1963-1964 and 1980), Chair of Standards Subcommittee 2.5 (which revised *Methods of Testing Antennas*), member APS AdCom (1959–1974), member IEEE Publication Board (1972 and 1974), and Director IEEE (1975). He is a registered Professional Engineer in California and England, and is a member of the American Physical Society, Tau Beta Pi, Sigma-Xi, Eta Kappa Nu, and Phi Kappa Phi. He was awarded an honorary Doctor of Engineering degree by the University of Missouri-Rolla in 1975. The University of Illinois Electrical Engineering Department gave him a Distinguished Alumnus Award in 1981. The College of Engineering awarded a Distinguished Alumnus Service Medal to him in 1986. The IEEE AESS Barry Carlton Best Paper prize was awarded him in 1991 and he received the IEEE APS Distinguished Achievement Award in 1994. He was elected to the National Academy of Engineering in 1992.