Design of Unequally Spaced Arrays for Performance Improvement

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***Abstract*— Classical antenna array synthesis techniques such as Fourier, Dolph–Chebyshev and Taylor synthesis efficiently obtain array current distributions for equally spaced arrays that generate a desired far-field radiation pattern function or keep important parameters like beamwidth and sidelobe level within prescribed performance bounds. However, the concept of optimization of the field pattern (e.g., by decreasing sidelobes or beamwidth) of an given equally spaced array realization by *altering its element spacings* still represents a challenging problem having considerable practical advantages. These include reduction in size, weight, and number of elements of the array. This paper describes a new approach to synthesis of unequally spaced arrays utilizing a simple inversion algorithm to obtain the element spacings from prescribed far-zone electric field and current distribution, or current distributions from prescribed far-zone electric field and element spacings.**

***Index Terms*—Antenna arrays.**

I. Introduction

OVER the past 60 years, the theory of uniformly spaced antenna arrays has been studied in depth and is certainly well documented. For example, given a desired radiation pattern (e.g., pencil-beam, sectoral, cosec^2, etc.) and the number of elements, it is possible to employ such traditional synthesis procedures as Dolph–Chebyshev, Taylor, Fourier inversion or numerical optimization to obtain the required array current distribution for a uniformly spaced array.

The analysis of unequally spaced antenna arrays originated with the work of Unz [1], who developed a matrix formulation to obtain the current distribution necessary to generate a prescribed radiation pattern from an unequally spaced linear array (with prespecified geometry). Subsequent to the initial concept of Unz, recent design techniques focus on two categories of nonuniform arrays: *arrays with randomly spaced elements and thinned arrays*, which are derived by selectively zeroing some elements of an initial equally spaced array.

In the first category, Harrington [2] developed a method to reduce sidelobe levels of uniformly excited N-element linear arrays by employing nonuniform spacing. Furthermore, he demonstrated that the close-in sidelobes can be reduced in height to approximately $2/N$ times the main-lobe field intensity level. Andreasan [3] exploited the use of emerging digital computation techniques to develop empirical results on unequally spaced arrays. Two important conclusions resulted from his work: 1) the 3-dB beamwidth of the mainlobe depends primarily on the length of the array and 2) the sidelobe level depends primarily on the number of elements in the array and to a minimal extent on the average spacing of the array when the latter exceeds about two wavelengths. Utilizing the Poisson sum expansion of the basic array factor, Ishimaru's classical analysis [4] of unequally spaced arrays addressed the following aspects: 1) sidelobe reduction in comparison with a linear array with uniform excitation; 2) secondary beam suppression of the linear array by use of the Anger function; and 3) azimuthal frequency scanning by means of an unequally spaced circular array. A purely analytical technique for unequally spaced linear array synthesis was developed by Miller and Goodman [5], who applied Prony's method to estimate the parameters (array current distribution and element spacing) of a sum of exponentials. The advantages of the method are its simplicity (the computations required are the solution of two sets of linear equations and the roots of one polynomial) and its noniterative nature. The method does require high-accuracy computer routines for obtaining the polynomial roots, however, and the linear equations must be well conditioned.

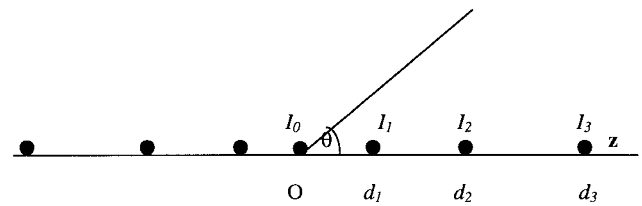

Fig. 1. Geometry of nonuniformly spaced linear symmetric array.

Another appealing design method for large unequally spaced arrays (array lengths of 1000λ or greater) employs a purely statistical approach utilizing probability distributions to decide the optimal space taper of an array with a prescribed current distribution. Lo and Lee [6] studied the probabilistic properties of a planar antenna array when its elements are placed randomly over an aperture.

The *thinned array* class of design methods is typified by the application of dynamic programming employed by Skolnik [7]. This approach stresses the power of computer-aided optimization tools to design unequally spaced arrays by treating the goal of sidelobe reduction as an objective function and the limits in the placement of adjacent elements as optimization constraints. In this approach, the density of elements located within a given aperture is made proportional to the amplitude distribution of the conventional equally spaced array. The latter method was extended by Mailloux

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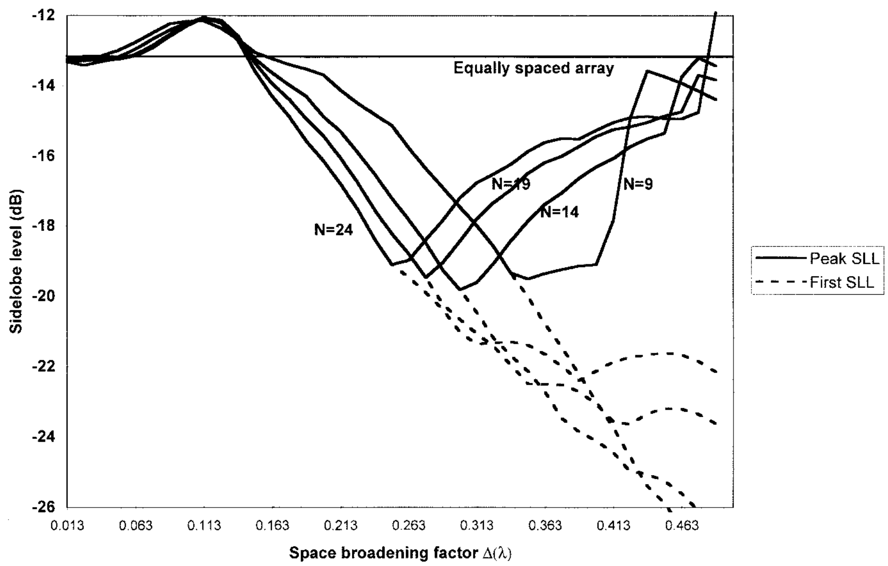
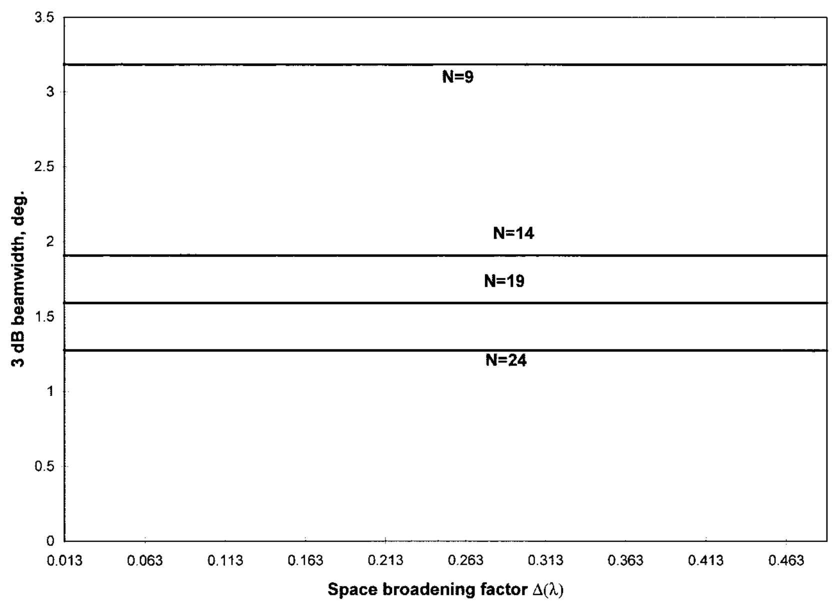


(a)



(b)

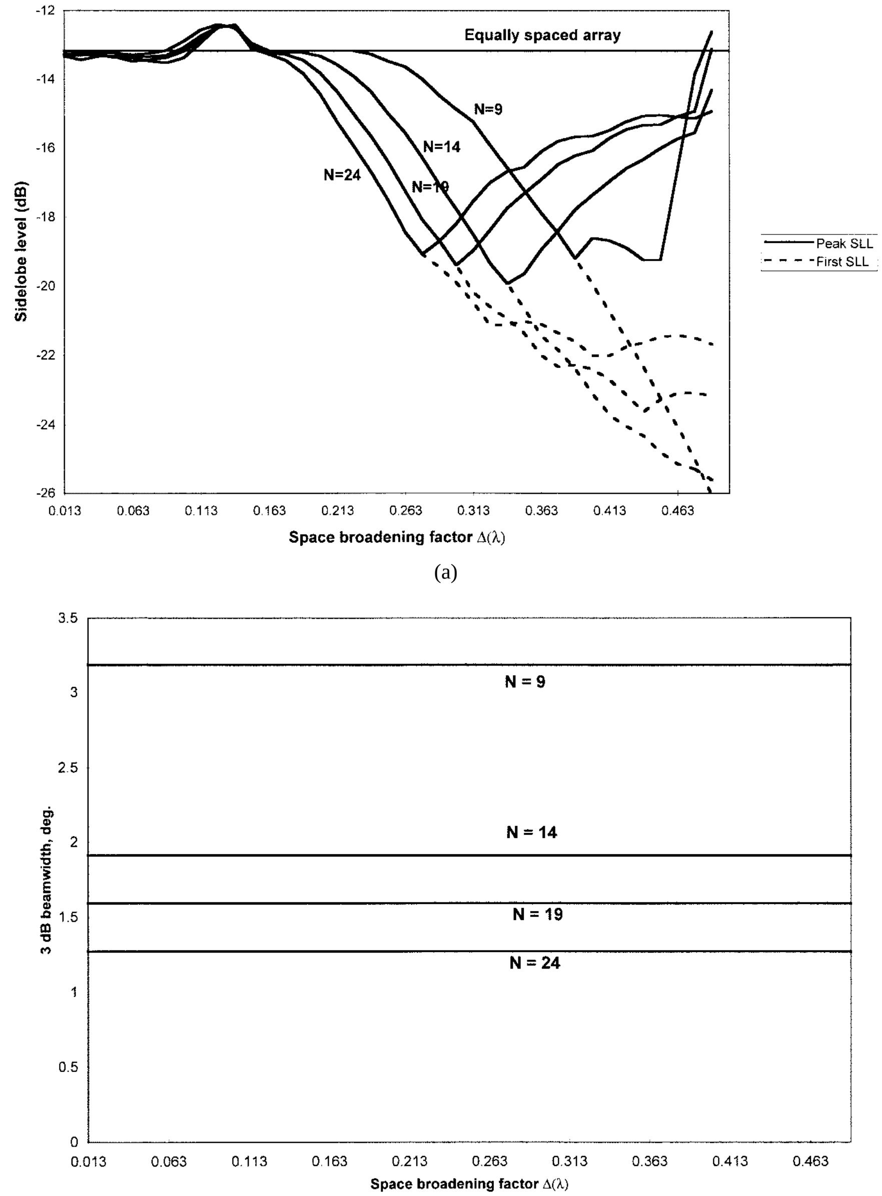
Fig. 2. (a) Sidelobe level of pencil-beam pattern with abrupt skirt. (b) Beamwidth of pencil beam pattern with abrupt skirt.

and Cohen [8], who utilized statistical thinning of arrays with quantized element weights to improve sidelobe performance in large circular arrays. Their results demonstrated the possibility of obtaining considerable sidelobe reduction by a combination of probabilistic thinning and discrete amplitude quantization. Another recent approach to thinned linear and planar array design is the application of genetic algorithms to design optimal spacings [9].

This paper presents a new method for unequally spaced array synthesis, which yields appropriate element spacing values for a prescribed array factor in a simple, recursive manner. This technique starts with a prescribed array pattern and synthesizes unequally spaced arrays under the constraint that adjacent element spacings are limited by the *space broadening factor* Δ. The inversion algorithm necessary in this approach has been employed earlier by the authors in far-field analysis of spherical [10] and nonuniformily spaced linear arrays [11–14].

II. Synthesis Technique and Applications

A general representation of a $2N+1$ element linear array is illustrated in Fig. 1 where the d_i and I_i represent the respective element spacings and currents. A fundamental question for antenna pattern synthesis concerns the potential of achieving radiation pattern improvement (e.g., lower peak sidelobe levels) utilizing nonunifom element spacings in comparison with a uniformly spaced array (e.g., $\lambda/2$). Basic constraints in this consideration would be that the number of elements and the current distributions would be the same for both uniform and nonuniform arrays and that the nonuniform element spacings would lie between 0.5λ and 1.0λ.



(a)

(b)

Fig. 3. (a) Sidelobe level of pencil-beam pattern with linear skirt. (b) Beamwith of pencil-beam pattern with linear skirt.

An affirmative answer to the above assertion is based on the fact that since different sets of nonuniform element spacings generate different radiation patterns, one such pattern could provide improvement on the equally spaced array pattern (e.g., by providing lower peak sidelobe level (PSLL), narrower beamwidth or closer mean-squared fit to the prescribed pattern response). *Although this possibility exists, general nonuniform spaced array design is more challenging than uniformly spaced design based on several considerations.*

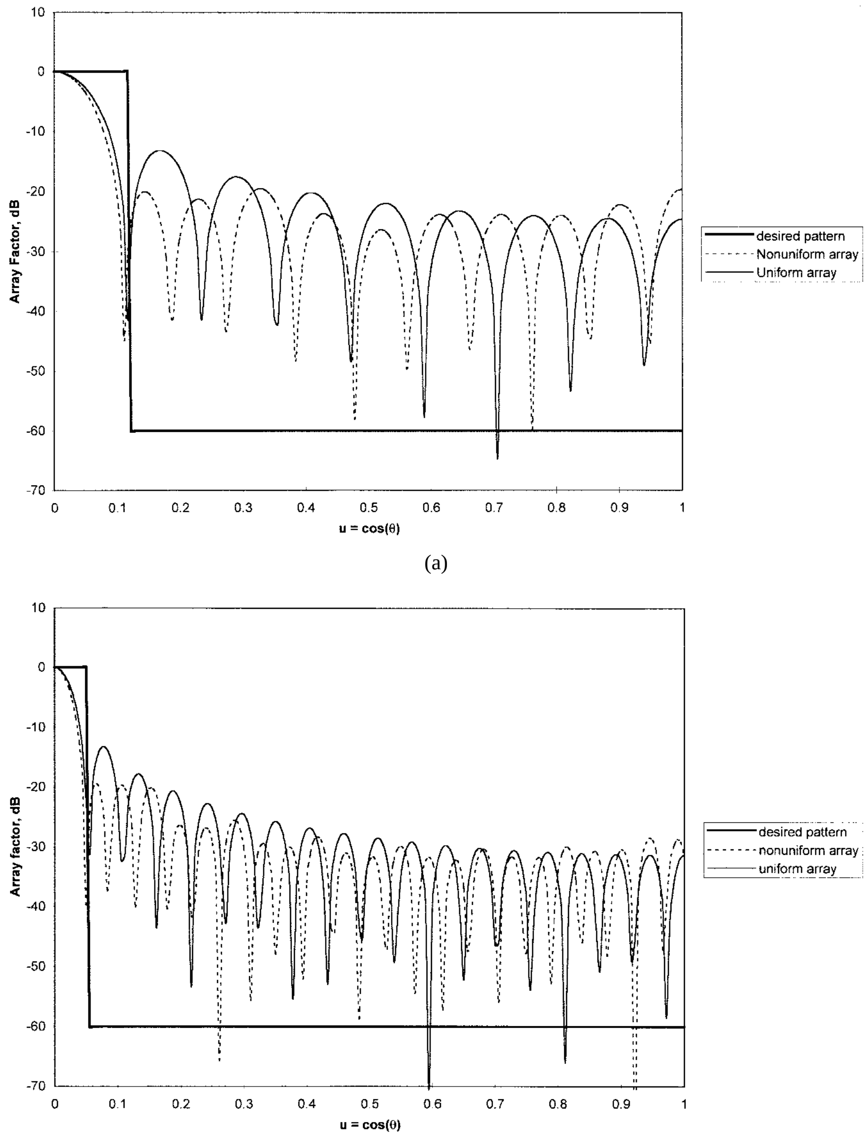
1) Since the element spacings occur as exponential or trigonometric functions, element spacing synthesis is a *nonlinear* problem whereas the array current synthesis is a *linear* problem.
2) Constraints have to be placed on the solutions for the element spacings; viz., they *cannot be complex numbers* and the adjacent element positions should be $\geq 0.5\lambda$ to reduce the array element count.

These considerations suggest that any synthesis technique for unequally spaced arrays should contain several aspects:

1) a nonlinear inversion algorithm to obtain the element spacings by the solution of a nonlinear set of equations;
2) a mechanism to place *a priori* constraints on the element spacings such as—they must be real and positive and greater than 0.5λ to reduce the array element count.

In order to delineate the synthesis algorithm, the desired array pattern is specified as $E_d(u)$ over the interval $-1 \leq u \leq 1$, where $u = \cos(\theta), 0 \leq \theta \leq \pi$ radians. Since the array factor is symmetric, i.e., $E_d(-u) = E_d(u)$, the synthesis problem is addressed only over the interval $0 \leq u \leq 1$, or $0 \leq \theta \leq \pi/2$ radians.

The new technique is presented below in terms of four fundamental steps. Step 1 formulates the synthesis problem where the prescribed array factor is sampled at M uniformly



(a)

(b)

Fig. 4. (a) Comparison of pencil-beam patterns (desired pattern with abrupt skirt) $N = 9$. (b) Comparison of pencil-beam patterns (desired pattern with abrupt skirt) $N = 19$.

spaced points over the observation range. The second step contains the key development in the synthesis technique, viz., the Legendre transformation of the array factor. In Step 3, the limiting property of the Legendre polynomials is effected and this leads to the generation of a triangular system of equations. The actual design equations for element spacing values are presented in Step 4. The following paragraphs present these steps in detail.

Step I. Definition of the Synthesis Problem

The actual array pattern $E(u)$ of a $2N+1$ element *nonperiodic* symmetric array of point sources (Fig. 1) is given by [15]

$$E(u_m) = \sum_{n=0}^{N} I_n \cos(m\beta_n), \qquad m = 0, M-1 \qquad (1)$$

where $\beta_n = kd_n\Delta u$ and the sampling interval $\Delta u = 1/(M-1)$.

The objective is to obtain feasible solutions for the array currents I_n and/or element positions d_n by matching the desired and actual array patterns at M points in the interval $0 \le u \le 1$ such that

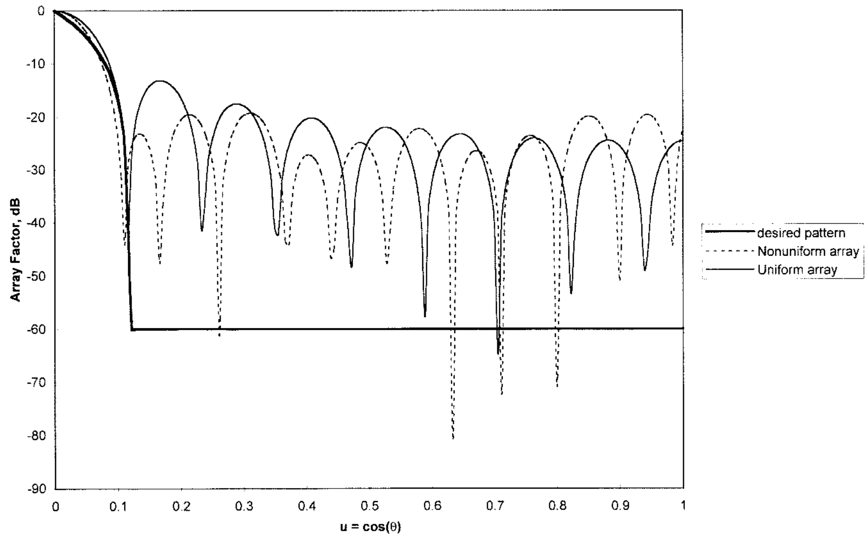
$$E(u_m) = E_d(u_m) \qquad (2)$$

where $u_m = m\Delta u, m = 0, 1 \cdots M-1$.

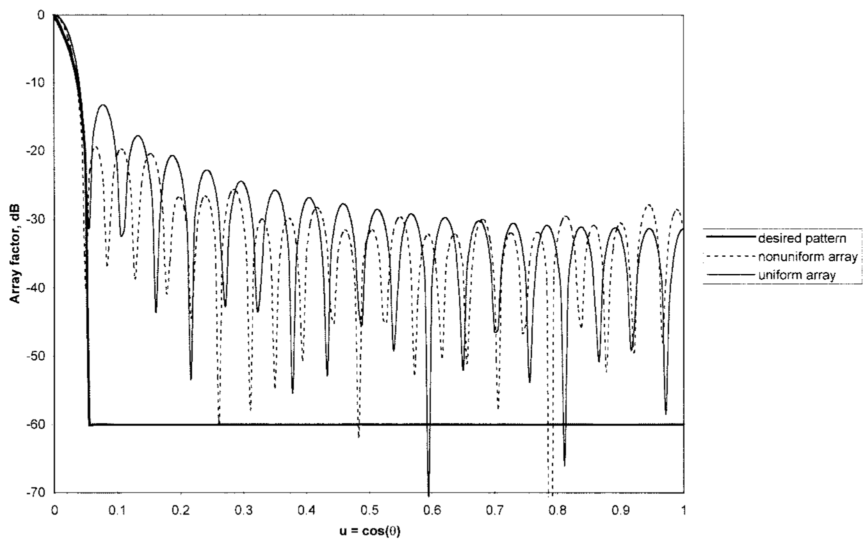
Step II. Legendre Transformation of the Desired Array Pattern $E_d(u)$

The following steps define the procedure for transforming the array function $E_d(u_m)$ into its Legendre transform $F(\alpha_p)$. To proceed we form the expression

$$F(\alpha_p) = \sum_{m=0}^{M-1} \varepsilon_m E_d(u_m) P_{m-1/2}(\cos\alpha_p)$$
$$p = 0, 1, 2, 3 \cdots N \qquad (3)$$



(c)



(d)

Fig. 4. (*Continued.*) (c) Comparison of pencil-beam patterns (desired pattern with linear skirt) $N = 9$. (d) Comparison of pencil-beam patterns (desired pattern with linear skirt) $N = 19$.

where $\varepsilon_m = 1, m = 0; = 2, m > 0$; and $P_{m-1/2}$ is the Legendre function of half integer order. The range and values of $\alpha_p; p = 0, 1, 2, 3 \cdots N$ will be selected in accordance with the element positions as will be described shortly. Substituting (1) and (2) for $E_d(u_m)$ into (3) yields the final transformed equation

$$F(\alpha_p) = \sum_{n=0}^{N} I_n \sum_{m=0}^{M-1} \varepsilon_m P_{m-1/2}(\cos \alpha_p) \cos(m\beta_n). \quad (4)$$

Step III. Generation of the Recursive Equations

The Legendre transformation of the desired array pattern $E_d(u)$ is motivated by consideration of the following limiting relation for the Legendre polynomial of fractional order [16]:

$$\sum_{m=0}^{\infty} \varepsilon_m P_{m-1/2}(\cos \alpha) \cos(m\beta) = \begin{cases} [2/(\cos \beta - \cos \alpha)]^{1/2}, & 0 \le \beta < \alpha \\ 0, & \alpha < \beta < \pi. \end{cases} \quad (5)$$

Applying the property in (5) to (4) permits the transformed array function to be expressed as

$$F(\alpha_p) = \sum_{n=0}^{N} I_n f(\alpha_p, \beta_n) \quad (6)$$

where

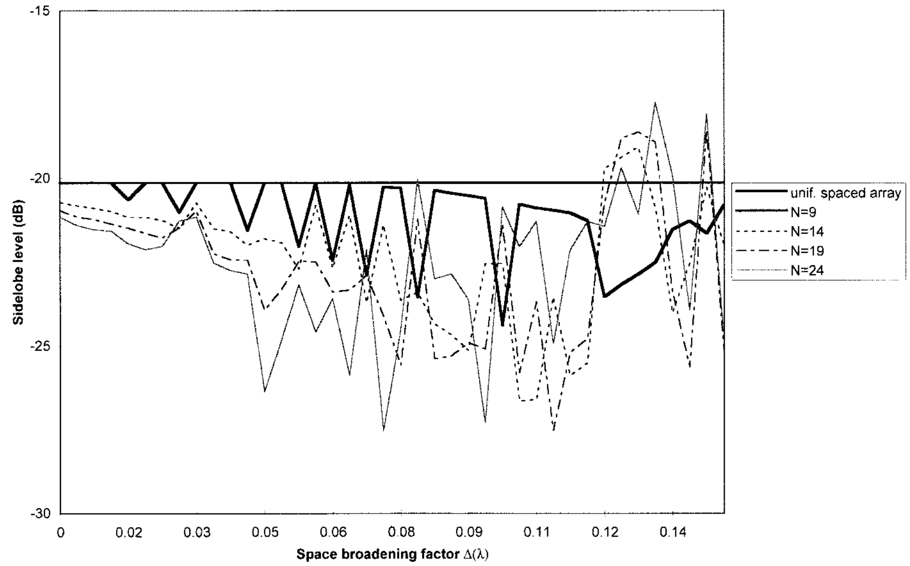
$$f(\alpha_p, \beta_n) = \begin{cases} [2/(\cos \beta_n - \cos \alpha_p)]^{1/2}, & 0 \le \beta_n < \alpha_p \\ 0, & \alpha_p < \beta_n < \pi. \end{cases} \quad (7)$$

From (2) and (7), it follows that

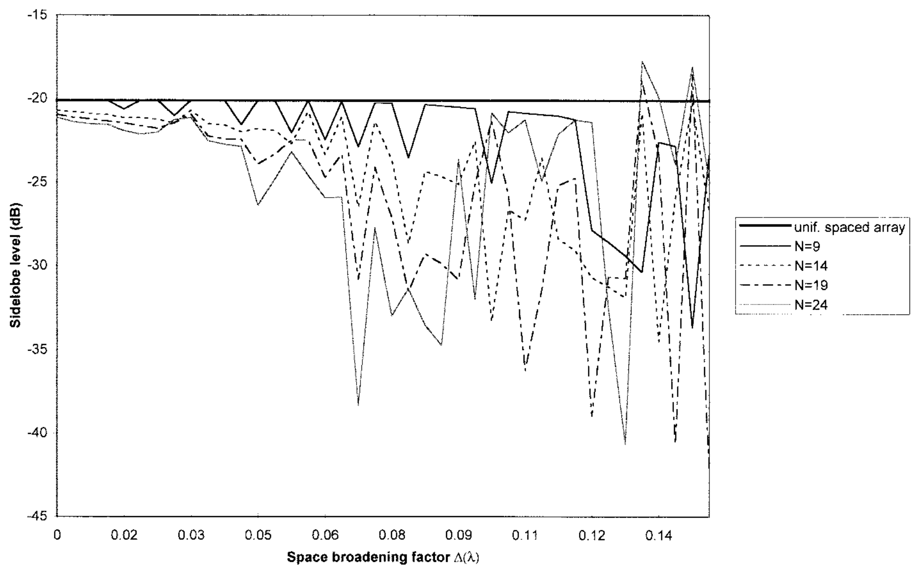
$$\beta_{n\,\max} = \beta_N < \pi \quad (8)$$

where $\beta_N = kd_N \Delta u$ and the sampling interval $\Delta u \approx 1/(M)$ since M is large. Equation (8) yields the limiting condition on M

$$M \ge \frac{kd_N}{\pi}. \quad (9)$$



(a)(i)



(a)(ii)

Fig. 5. (a)(i) Peak sidelobe level of flat-top beam with abrupt skirt. (a)(ii) First sidelobe level of flat-top beam with abrupt skirt.

As an illustration, if the adjacent element spacings of the array are limited to $\leq \lambda$, then $d_N \leq N\lambda$ and (9) reduces to

$$M \geq 2N. \tag{10}$$

However, in the actual implementation of the synthesis algorithm as described in Section III, higher number of sampling points were necessary for the convergence of the algorithm.

The selection of the α grid is the important constituent in the reconstruction of the array currents and positions in recursive form. This grid is defined by the following relation:

$$\alpha_p = \beta_p + c \tag{11}$$

where the constant $c; c > 0$ is limited by the condition that the α and β values intersperse one another as follows:

$$\begin{array}{lcccccccccc} \beta \text{ Space:} & \beta_0 & & \beta_1 & & \beta_2 & & \beta_3 & \cdots & \beta_N & \\ \alpha \text{ Space:} & & \alpha_0 & & \alpha_1 & & \alpha_2 & & \alpha_3 & \cdots & \alpha_N \end{array}$$

Now, utilizing (6) and (7) we obtain the following *triangular system of equations*:

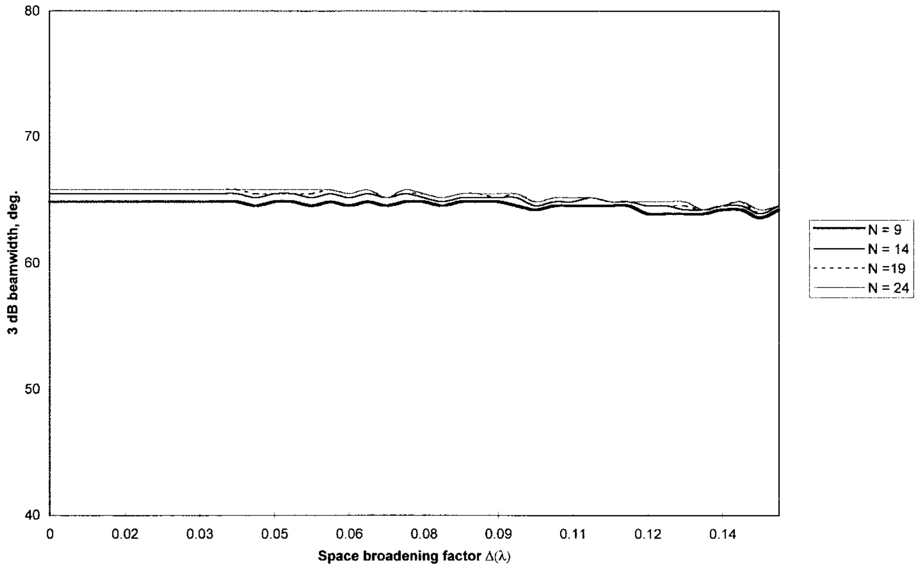
$$F(\alpha_p) = \sum_{n=0}^{p} I_n f(\alpha_p, \beta_n) \tag{12}$$

This system is invertible as follows:

$$\begin{aligned} I_0 f(\alpha_0, \beta_0) &= F(\alpha_0) \\ I_p f(\alpha_p, \beta_p) &= F(\alpha_p) - \sum_{n=0}^{p-1} I_n f(\alpha_p, \beta_n) \\ p &= 1, 2, 3 \cdots N \end{aligned} \tag{13}$$

Step IV. Application of Inversion Algorithm in Step III to Synthesize Currents and Positions

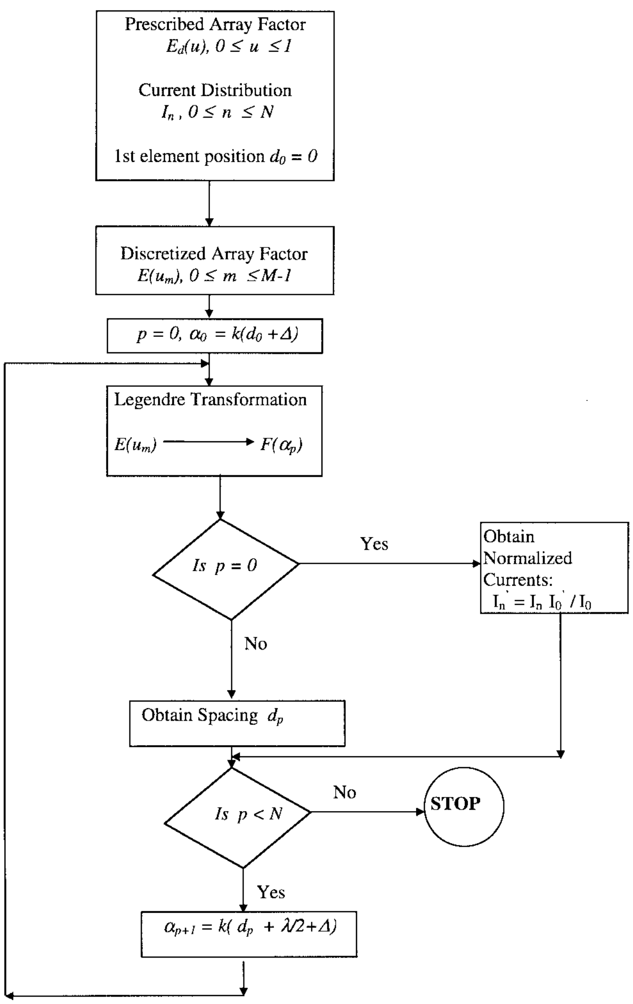
The algorithm described in (1)–(13) is utilized to yield the synthesized array spacings in the following manner.



(b)

Fig. 5. (*Continued.*) (b) Beamwidth of flat-top beam with abrupt skirt.

TABLE I
FLOW CHART OF SYNTHESIS ALGORITHM



1) The first array element is positioned at $d_0 = 0$ or $\beta_0 = 0$.
2) The spacing between adjacent elements of the array is constrained between the limits $0.5\lambda \le d_p - d_{p-1} < 0.5\lambda + \Delta$ where Δ, the *space broadening factor* is maintained less than 0.5λ.

In this way, it is ensured that the *lower limit* of adjacent element spacing is at least 0.5λ in order to *reduce mutual coupling effects* and in the upper limit, the adjacent element spacing is less than λ, *in order to prevent grating lobes*. Hence, the values of $\alpha_p, p = 0, 1, 2 \cdots N$ are selected as follows:

$$\alpha_0 = k(d_0 + \lambda/2)\Delta u.$$

The normalized value of the central current I_0 is obtained from (13) as

$$I_0 = F(\alpha_0)/f(\alpha_0, \beta_0) \tag{14}$$

Equation (14) is the first design equation of the array. In order to synthesize the second-element current/position, α_1 is selected as

$$\alpha_1 = k(d_0 + \lambda/2 + \Delta)\Delta u$$

and from (13)

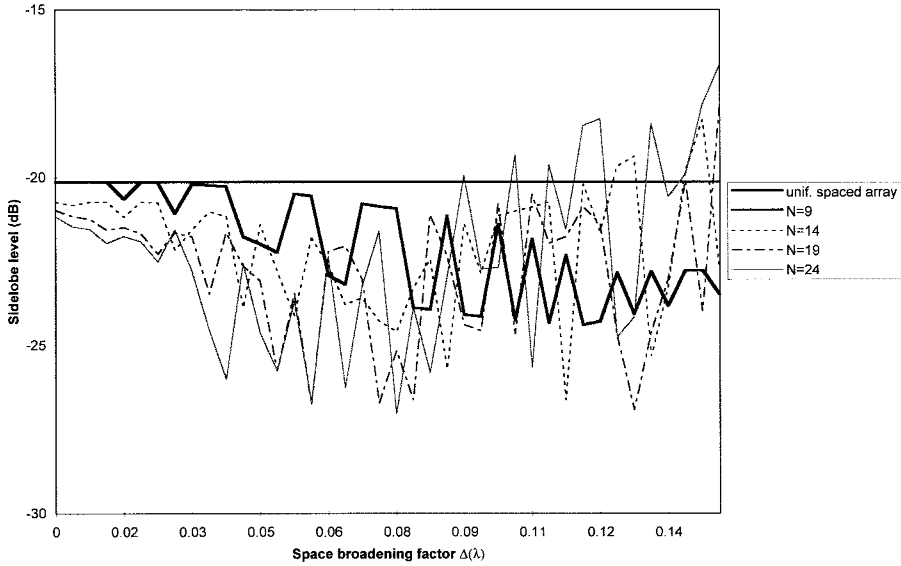
$$I_1 f(\alpha_1, \beta_1) = F(\alpha_1) - I_0 f(\alpha_1, \beta_0) \tag{15}$$

Equation (15) is the second design equation of the array since the left-hand side of the equation contains both the array current I_1 and the element position term $\beta_1 = kd_1\Delta u$. Proceeding in this manner, the successive values of α_p are selected as

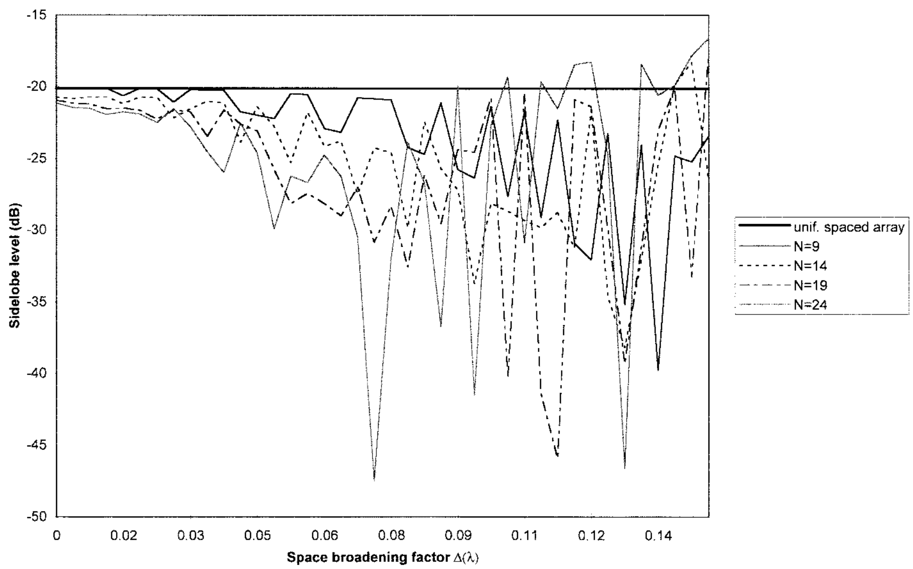
$$\alpha_p = k(d_{p-1} + \lambda/2 + \Delta)\Delta u$$

and the *pth design equation of the array* is obtained as

$$I_p f(\alpha_p, \beta_p) = F(\alpha_p) - \sum_{n=0}^{p-1} I_n f(\alpha_p, \beta_n). \tag{16}$$



(a)(i)



(a)(ii)

Fig. 6. (a)(i) Peak sidelobe level of flat-top beam with linear skirt. (a)(ii) First sidelobe level of flat-top beam with linear skirt.

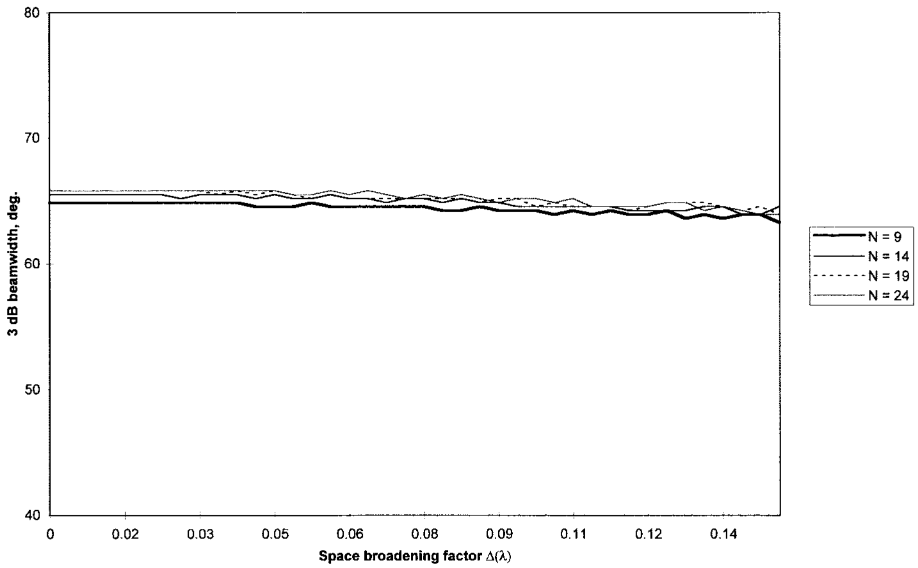
The proposed synthesis technique can be represented by the flowchart form as shown in Table I. The organization of the flowchart follows the analysis given above as a four-step process originating with the desired array field pattern and culminating with the generation of array element positions.

The above development [(14)–(16)] provides a means to solve the following distinct synthesis problems.

1) Synthesis of Array Currents of a Nonuniformly Spaced Array with Prescribed Element Positions: This problem is formulated as follows. Given a prescribed symmetrical pattern $E_d(u)$ over the interval $-1 \leq u \leq 1$ and prespecified nonuniform element positions of a $2N+1$ element symmetric array, obtain the appropriate set of currents $I_p, p = 0, 1, \cdots N$. The element currents can be obtained utilizing (14) and (16)

$$I_0 = \frac{F(\alpha_0)}{f(\alpha_0, \beta_0)}$$
$$I_p = \frac{F(\alpha_p) - \sum_{n=0}^{p-1} I_n f(\alpha_p, \beta_n)}{f(\alpha_p, \beta_p)}; \qquad p = 1, 2, \cdots N. \quad (17)$$

2) Synthesis of Array Element Positions Only with a Prespecified Array Current Distribution: This problem is the one of more interest. Given a desired symmetrical pattern $E_d(u)$ expressed in the interval $-1 \leq u \leq 1$ and a prespecified array current distribution for a $2N+1$ element array, obtain the appropriate nonuniform set of element positions $d_p, p = 0, 1, \cdots N$. The pth design formula for the array is given from



(b)

Fig. 6. (*Continued.*) (b) Beamwidth of flat-top beam with linear skirt.

(16) as

$$f(\alpha_p,\beta_p) = \frac{F(\alpha_p) - \sum_{n=0}^{p-1} I_n f(\alpha_p,\beta_n)}{I_p} \tag{18}$$

and from (5) and (18), we obtain

$$\beta_p = \cos^{-1}\left(\frac{2I_p^2}{\left[F(\alpha_p) - \sum_{n=0}^{p-1} I_n f(\alpha_p,\beta_n)\right]^2} + \cos(\alpha_p)\right) \tag{19}$$

and $d_p = \beta_p/(k\Delta u)$.

To exclude infeasible solutions, the following conditions are placed on these values of β_p and d_p:

1) if the value of the argument within the large bracket in (19) is greater than one, then $d_p = d_{p-1} + 0.5\lambda$ is taken as default value;
2) if $\beta_p \geq \alpha_p$, then $d_p = d_{p-1} + 0.5\lambda$ is taken as default value;
3) if $d_p - d_{p-1} < 0.5\lambda$, then $d_p = d_{p-1} + 0.5\lambda$ is taken as default value.

The value of β_p is again calculated using the default spacing value d_p and the algorithm is continued.

III. Applications of the Synthesis Technique

The efficacy of the above technique for pattern synthesis and improved array performance is illustrated below by synthesis of specific pattern shapes with prescribed current distributions. These are, respectively, a pencil-beam pattern with uniform current distribution and a flat-top beam pattern with a prescribed current distribution. In all these cases, the desired pattern is sampled uniformly at M points for $0 < u \leq 1.0$.

The desired array pattern function can generally be expressed as

$$E_d(u) = \begin{cases} E_1(u); & 0 \leq u \leq u_m \\ E_2(u); & u_m < u \leq 1.0 \end{cases} \tag{20}$$

where the boundary u_m delineates the mainlobe and sidelobe regions.

1) Synthesis of Pencil-Beam Pattern Utilizing Uniform Current Distribution: As alluded to above, in this first application the array current distribution is uniform and the initial array geometry consists of $2N + 1\lambda/2$ point sources. Based on previous computations, the selection of the prescribed pattern [(20)] is understood to significantly affect the solutions obtained for the element spacings. Two cases are presented below for illustrative purposes:

a) Prescribed pattern with abrupt skirt:

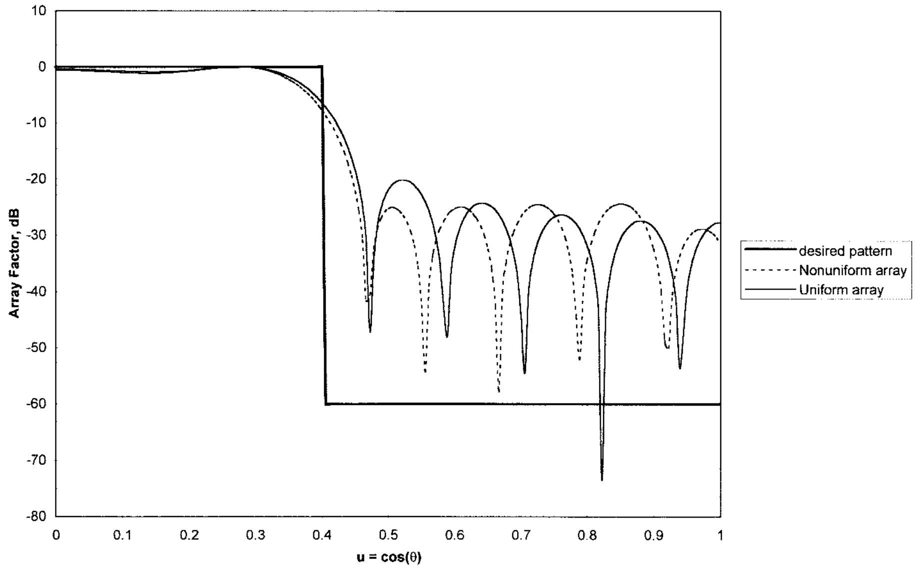
$$E(u) = \begin{cases} E_{\max}; & 0 \leq u \leq u_m \\ E_{\min}; & u_m < u \leq 1.0 \end{cases} \tag{21}$$

b) Prescribed pattern with linear skirt:

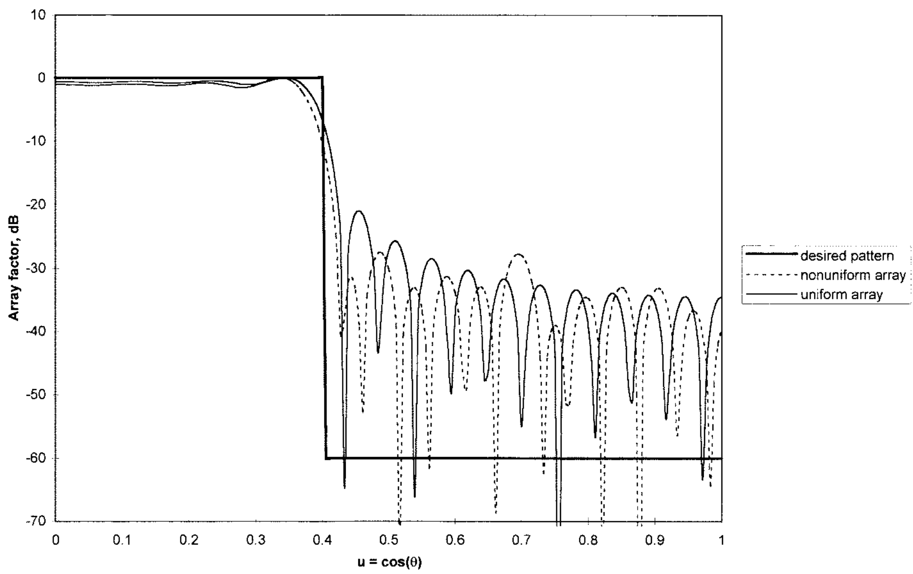
$$E(u) = \begin{cases} 1 - u\dfrac{E_{\max} - E_{\min}}{u_m}; & 0 \leq u \leq u_m \\ E_{\min}; & u_m < u \leq 1.0 \end{cases}. \tag{22}$$

In both cases, the values for $E_{\max}$ and $E_{\min}$ are $E_{\max} = 1.0$, $E_{\min} = 0.001$, the dividing boundary u_m is assumed as the position of the first null (at $u \sim (2/2N + 1)$) of the *initial* equally spaced array.

The synthesis algorithm (Section II) has been applied in both cases a) and b) to obtain the appropriate adjacent element spacings. For case a) Fig. 2(a) describes the results obtained for the variation of *first sidelobe level* (FSLL) and peak sidelobe level (PSLL) as a function of the space broadening factor Δ for different values of the array number ($N = 9, 14, 19,$ and 24). The 3-dB beamwidth [Fig. 2(b)] of the nonuniform array remains unchanged for case (a) as a function of the space



(a)



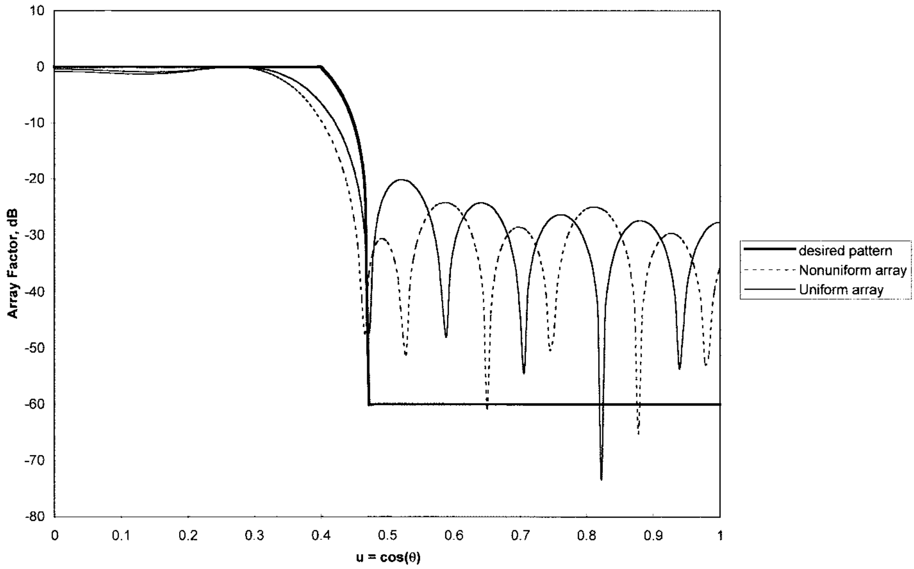
(b)

Fig. 7. (a) Comparison of flat-top beam patterns (desired pattern with abrupt skirt) $N = 9$. (b) Comparison of flat-top beam patterns (desired pattern with abrupt skirt) $N = 19$.

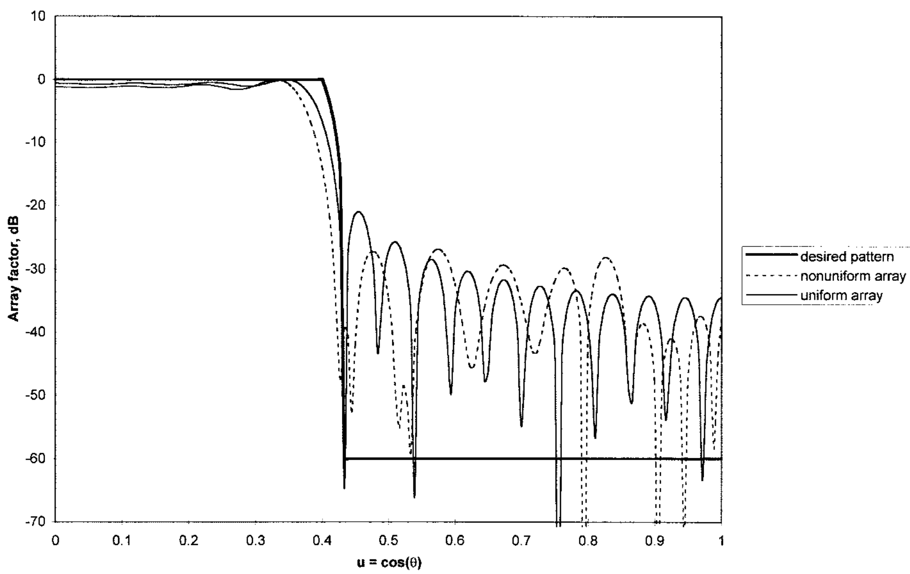
broadening factor: 3.185°, 1.91°, 1.592°, and 1.273° for $N = 9, 14, 19,$ and $24,$ respectively. Similarly Fig. 3(a) illustrates the sidelobe characteristics of the synthesized nonuniform arrays for case b) utilizing the prescribed function in (22). The 3-dB beamwidth [Fig. 3(b)] of the nonuniform array remains unchanged as in case a), with variation of the space broadening factor: 3.185°, 1.91°, 1.592°, and 1.273° for $N = 9, 14, 19,$ and $24,$ respectively. A perusal of the results yields the following conclusions.

1) The value of the FSLL decreases continually as the value of Δ (in wavelengths) ranges from 0.1 to 0.5λ; however, the corresponding value of the PSLL decreases along with the FSLL up to a point and then begins increasing again.
2) In case a), as the value of the array number N increases, the PSLL reaches its lowest value for a smaller value of the space broadening factor Δ. Case b) results are similar to those of case a).
3) The 3-dB beamwidth essentially remains unchanged as a function of Δ for both cases a) and b).

Synthesized field patterns are depicted in Fig. 4(a)–(d) for the two cases of target functions defined in (21) and (22). In each figure, a comparison is made among the specified target pattern $E_d(u)$, the pattern of the $2N+1$ element equally spaced array, and the pattern of the synthesized unequally spaced array. From Fig. 4(a) and (b) [case a)], it is observed that by employing the nonuniform array synthesis technique, the PSLL is reduced by ~6.5 dB over that of a uniform array for both $N = 9$ and $N = 19$. Fig. 4(c) and (d) compares the uniform and nonuniform array patterns for $N = 9$ and $N = 19$, respectively, corresponding to case b). In these figures, it is observed that in comparison with the equally spaced array,



(c)



(d)

Fig. 7. (*Continued.*) (c) Comparison of flat-top beam patterns (desired pattern with linear skirt) $N = 9$. (d) Comparison of flat-top beam patterns (desired patterns with linear skirt) $N = 19$.

the reduction of the PSLL is ~6.5 dB. The synthesized array spacings d_i are presented in Tables II(a) and (b) for cases a) and b), respectively.

2) Synthesis of Flat-Top Beam Pattern: The next application synthesizes an unequally spaced array to generate a flat-top beam pattern. As in the case of the pencil-beam array, the initial array geometry consists of $2N+1$ points sources having $\lambda/2$ spacings; however, in present case, the current distribution is nonuniform as described below in (25). Also, two forms of the prescribed array factor will be detailed as cases a) and b):

a) Prescribed with abrupt skirt:

$$E(u) = \begin{cases} E_{\max}; & 0 \le u \le u_m \\ E_{\min}; & u_m < u \le 1.0. \end{cases} \tag{23}$$

b) Prescribed pattern with linear skirt:

$$E(u) = \begin{cases} E_{\max}; & 0 \le u \le u_m \\ E_{\max} - (u - u_m)\dfrac{(E_{\max} - E_{\min})}{(u_{m1} - u_m)}; & u_m \le u \le u_{m1} \\ E_{\min}; & u_{m1} < u \le 1.0. \end{cases} \tag{24}$$

The current distribution required to generate the prescribed array factor can be derived from Fourier synthesis of an equally spaced array [15] and is given as

$$I_n = \sin(n\pi u_m)/(n\pi u_m), \qquad -N \le n \le N. \tag{25}$$

In both cases a) and b) $E_{\max} = 1.0$, $E_{\min} = 0.001$, the boundary $u_m = 0.4$, and the second boundary u_{m1} is the

TABLE II
(a) SYNTHESIZED ELEMENT SPACINGS (λ) PENCIL-BEAM PATTERN WITH ABRUPT SKIRT. (b) SYNTHESIZED ELEMENT SPACINGS (λ) PENCIL-BEAM PATTERN WITH LINEAR SKIRT

N = 9	N =19
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.59	0.5
0.717	0.5
0.769	0.5
0.796	0.5
	0.576
	0.633
	0.668
	0.694
	0.713
	0.726
	0.737
	0.744
	0.75
	0.754

(a)

N = 9	N = 19
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.683	0.5
0.781	0.5
0.833	0.5
0.866	0.5
	0.589
	0.633
	0.664
	0.687
	0.707
	0.722
	0.735
	0.746
	0.754
	0.761

(b)

TABLE III
(a) SYNTHESIZED ELEMENT SPACINGS (λ) FLAT-TOP BEAM WITH ABRUPT SKIRT. (b) SYNTHESIZED ELEMENT SPACINGS (λ) FLAT-TOP BEAM (WITH LINEAR SKIRT)

N=9	N=19
0.5	0.5
0.5	0.5
0.519	0.532
0.5	0.5
0.597	0.5
0.5	0.5
0.5	0.5
0.562	0.565
	0.52
	0.5
	0.5
	0.5
	0.581
	0.527
	0.609
	0.5
	0.516
	0.592

(a)

N=9	N=19
0.5	0.5
0.5	0.5
0.578	0.575
0.501	0.515
0.616	0.5
0.5	0.5
0.553	0.5
0.597	0.603
	0.507
	0.5
	0.5
	0.561
	0.61
	0.5
	0.627
	0.5
	0.604
	0.604

(b)

position of the first null in the pattern of the corresponding equally spaced array.

Fig. 5(a)(i) and (a)(ii) describes the variation of PSLL and FSLL, respectively, of the synthesized nonuniform array for case a) as a function of the space-broadening factor Δ for $N = 9, 14, 19$, and 24. Fig. 5(b) depicts the variation of the 3-dB beamwidth with the factor Δ. Similarly, Fig. 6(a) and (b) illustrates the variation of the sidelobe levels and beamwidth, respectively, for the case b). Analysis reveals that the pattern synthesis of the flat-top beam is *highly sensitive* to the space-broadening factor and as a consequence, hence, the range of Δ variation was reduced as compared with the previous pencil beam examples. The following conclusions are based on the results in Figs. 5(a) and (b) and 6(a) and (b).

1) The values of FSLL and PSLL decrease in an oscillatory manner as the value of Δ increases from 0–0.14λ. This implies that the array pattern is very sensitive to small changes in Δ.
2) The 3-dB beamwidth essentially remains unchanged as a function of Δ.

Computed pattern responses are presented in Fig. 7(a) and (b) for case a) and Fig. 7(c) and (d) for case b). Fig. 7(a) compares the patterns corresponding to case a) for $N = 9$, where it is observed that the PSLL is improved over that of the uniform array by ~5 dB. For $N = 19$, Fig. 7(b) reveals an approximately 7-dB reduction in PSLL. Similarly, Fig. 7(c) and (d) illustrates the results for case b) for $N = 9$ and $N = 19$, respectively. The PSSL is again reduced by ~5 dB and ~7 dB for $N = 9$ and $N = 19$, respectively. The synthesized array spacings are given in Tables III(a) and (b) for case a) and case b), respectively.

The above numerical results demonstrate that utilizing the nonuniform synthesis method presented in this paper, peak sidelobe levels can be reduced by ~6.5 dB in comparison with uniformly excited pencil-beam arrays and up to ~7 dB for flat-top beam arrays while essentially maintaining the same beamwidth. The design of unequally spaced arrays by genetic algorithms [9] yields ~ -20-dB peak sidelobe level for thinned linear and planar arrays optimized over both scan angle and bandwidth of operation. Another comparison can be made between the current method and recent optimization methods [17], where the peak sidelobe level achieved is ~ -18.5 dB (~ -20 dB for current case with a 14-element array). A primary advantage of the method presented in this paper is that it is noniterative in nature and, hence, less prone to errors. Optimization and genetic algorithms, on the other hand, are inherently iterative techniques, with the associated potential for speed, convergence, and accuracy problems.

The extension of the method to low sidelobe symmetric planar arrays is straightforward and can be considered as the two-dimensional counterpart of the algorithm described in Section II. In the planar case, a two-dimensional Legendre transformation can be effected on a desired array pattern $E_d(u, v)$, where $u = \sin(\theta)\cos(\phi)$ and $v = \sin(\theta)\sin(\phi)$ define the complete volumetric space around the array. Further work is in progress to develop an efficient synthesis algorithm for planar arrays.

IV. Conclusion

This paper describes a new analytical approach to the synthesis of unequally spaced antenna arrays. The synthesis technique is unique and practical in that it enables a designer to determine pattern the appropriate element spacings for a prescribed pattern (and a given array current distribution) by means of a Legendre transformation of the array factor, while maintaining constraints on the element-spacing values. Numerical results have been presented for pencil and flat-top beam arrays. The effects of unequal spacing, as reflected by the sidelobe levels and 3-dB beamwidth, are studied in detail. The results show that considerable improvement in array performance can be obtained in comparison with uniformly spaced arrays having the same number of elements and identical current distribution.

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