Diffuse Like and Cross-Polarized Fields Scattered from Irregular Layered Structures—Full-Wave Analysis

Ezekiel Bahar, *Fellow, IEEE*, and Yuzhi Zhang

***Abstract*— The full-wave solution for scattering from two-dimensional (2-D) irregular layered structures is expressed as a sum of radiation fields, the lateral waves, and the surface waves. Only the radiation far fields are considered in this work. The lateral waves and surface waves are ignored since excitations of plane waves are considered and the observation points are in the far fields. The scattering coefficients appearing in the full-wave generalized telegraphists' equations for irregular layered structures are proportional to the derivatives of the surface heights at each interface. Using a first-order iterative procedure to solve the generalized telegraphists' equations, the diffusely scattered fields from irregular layered structures are expressed as a sum of first-order fields scattered at each rough interface. In this paper, the like and cross-polarized diffuse scattered fields are derived for three medium irregular structures with 2-D rough interfaces. The thickness of the coating material or thin film between the two interfaces is arbitrary, however, in this work it is assumed to be constant. Thus, in this case, both interfaces are rough and there are five different scattering processes identified in the full-wave results. A physical interpretation is given to the five different scattering mechanisms that contribute to the diffusely scattered fields. This work can be used to provide realistic analytical models of propagation across irregular stratified media such as ice or snow covered terrain, remote sensing of coated rough surfaces, or the detection of buried objects in the presence of signal clutter from the rough interfaces.**

***Index Terms*— Electromagnetic scattering, nonhomogeneous media.**

I. Introduction

THE diffusely scattered electromagnetic fields for the two irregular structures have been investigated previously. In one, the upper interface is flat and the lower interface is rough [1]–[4]. In the other, the upper interface is rough and the lower interface is flat [5].

In this work, the model of the irregular structure considered consists of two random rough (upper and lower) interfaces. The thickness of the coating material (film) between the two random rough interfaces is assumed to be constant, however, when this is not the case, procedures presented to the earlier work can be followed. This model is referred to as the "coated rough-surface model." The physical mechanism for scattering from coated rough surfaces is schematically illustrated in Fig. 1. The upper interface for $y = h_{01s}(x_s, z_s)$ between medium zero and medium one is

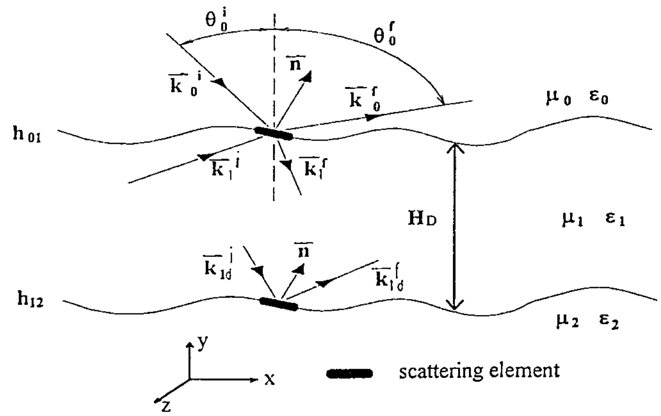

Fig. 1. Illustration of uniform coating model.

$$h_{01s}(x_s, z_s) = \begin{cases} h_{s1}(x_s, z_s), & |x_s| < l, |z_s| < L \\ h_{01}, & |x_s| \geq l, |z_s| \geq L \end{cases} \tag{1a}$$

where the mean value of h_{01s} is $\langle h_{01s} \rangle = h_{01}$. The lower interface $y = h_{12s}(x_s, z_s)$ between medium one and medium two has exactly the same dependence on x_s and z_s as the upper interface except that $\langle h_{12s} \rangle = h_{12}$.

$$h_{12s}(x_s, z_s) = \begin{cases} h_{s2}(x_s, z_s), & |x_s| \leq l, |z_s| \leq L \\ h_{12}, & |x_s| > l, |z_s| > L \end{cases} \tag{1b}$$

The thickness of the coating layer H_D is constant, thus

$$H_D = h_{01s}(x_s, z_s) - h_{12s}(x_s, z_s) = h_{01} - h_{12}. \tag{1c}$$

The unit vector normal to the large scale rough interface between medium zero and one and between medium one and two is

$$\begin{aligned} \vec{n} &= \frac{-h_x \vec{a}_x + \vec{a}_y - h_z \vec{a}_z}{\sqrt{1 + h_x^2 + h_z^2}} \\ &= \sin\gamma \cos\delta \vec{a}_x + \cos\gamma \vec{a}_y + \sin\gamma \sin\delta \vec{a}_z \end{aligned} \tag{2a}$$

where

$$h_x = \frac{dh_{s1}}{dx_s} = \frac{dh_{s2}}{dx_s}, \quad h_z = \frac{dh_{s1}}{dz_s} = \frac{dh_{s2}}{dz_s} \tag{2b}$$

$$\gamma = \arccos\left[\frac{1}{\sqrt{1 + h_x^2 + h_z^2}}\right] \quad \delta = \arctan\left[\frac{h_z}{h_x}\right]. \tag{2c}$$

Manuscript received July 3, 1997; revised July 24, 1998.

The authors are with the Department of Electrical Engineering, Center for Electro-Optics, University of Nebraska-Lincoln, Lincoln, NE 68588 USA

Publisher Item Identifier S 0018-926X(99)02487-4.

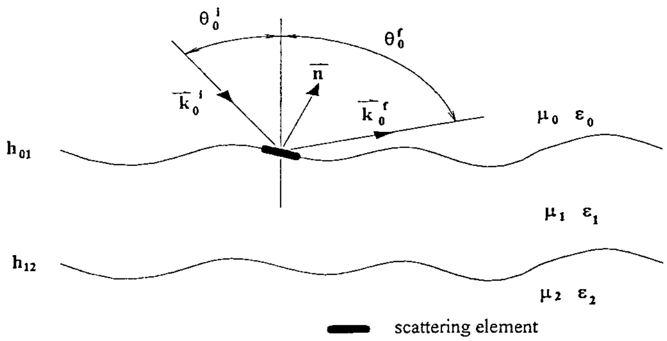

Fig. 2. Scattering upon reflection in medium zero above the upper interface.

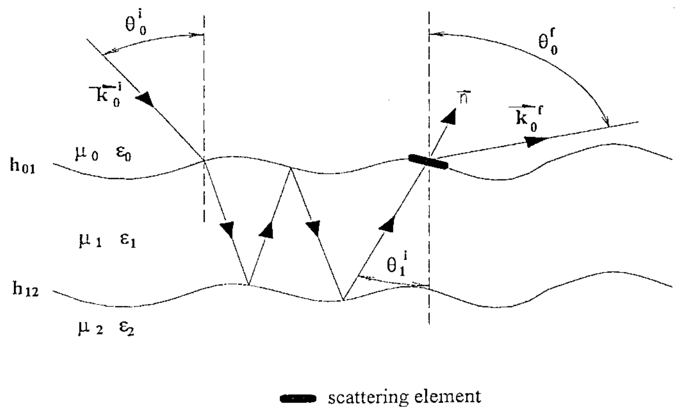

Fig. 3. Scattering upon transmission (across upper interface) from medium one to medium zero.

II. Diffuse Scattered Fields from Coated Rough Surfaces

It is shown in Appendix A that on solving the generalized telegraphists' equations, the diffuse first order scattered fields can be expressed as the sum

$$E_S^{PQ} = E_{SU}^{PQ}(\vec{r}) + E_{SD}^{PQ}(\vec{r}) \tag{3}$$

where $E_{SU}^{PQ}(\vec{r})$ is associated with scattering from the upper interface, and $E_{SD}^{PQ}(\vec{r})$ is associated with scattering from the lower interface. For $e^{i\omega t}$, time-harmonic plane wave excitations the incident electric field of magnitude E_0^{iP} is

$$\vec{E}_P^i = E_0^{iP} e^{-i\vec{k}_0^i\cdot\vec{r}}\vec{a}_p. \tag{4}$$

In (4) $\vec{a}_P$ is parallel $(P = V)$ or perpendicular $(P = H)$ to the reference plane of incidence (normal to $\vec{n}_0^i \times \vec{a}_y$) where $\vec{n}_0^i$ is the unit vector in the direction of the wave vector $\vec{k}_0^i$ for the incident waves). For plane wave excitations and for observation points above the upper interface $y \geq h_{01s}(x_s, z_s)$, the scattered fields due to the rough upper and lower interfaces are given by ([2], [5]–[7])

$$\begin{aligned} &E_{SU}^{PQ}(\vec{r}) \\ &= \frac{E_0^{Qi}}{(2\pi i)^2}\iint\iint \frac{e^{i(\vec{k}_0-\vec{k}_0^i)\cdot\vec{r}_{s1}-i\vec{k}_0\cdot\vec{r}}}{s_0^i c_\phi^i - s_0 c_\phi} \\ &\quad\times\Bigg\{F_{00U}^{PQ}(\vec{k},\vec{k}_0^i) + \frac{T_{01}^{Qi}R_{21}^{Qi}F_{01U}^{PQ}(\vec{k},\vec{k}_1^i)e^{-i2v_1^iH_D}}{1-R_{01}^{Qi}R_{21}^{Qi}e^{-i2v_1^iH_D}} \\ &\quad+\frac{\frac{c_0}{c_1}F_{10U}^{PQ}(\vec{k},\vec{k}_0^i)R_{21}^PT_{01}^Pe^{-i2v_1^iH_D}}{1-R_{01}^PR_{21}^Pe^{-i2v_1H_D}} \\ &\quad-\frac{n_r\frac{c_0}{c_1}T_{10}^{Qi}R_{21}^{Qi}\cdot F_{11U}^{PQ}(\vec{k},\vec{k}_1^i)R_{21}^PT_{01}^Pe^{-i2v_1^iH_D-i2v_1H_D}}{\left[1-R_{01}^{Qi}R_{21}^{Qi}e^{-i2v_1^iH_D}\right]\left[1-R_{01}^PR_{21}^Pe^{-i2v_1H_D}\right]}\Bigg\} \\ &\quad\times\frac{dh_{s1}}{dx_s}\,dx_s\,dz_s\frac{k_0}{u}\,dv_0\,dw \end{aligned} \tag{5a}$$

and (5b), shown at the bottom of the page, where F_{mnU}^{PQ} $(m, n = 0, 1)$ and F_{11D}^{PQ} are scattering coefficients associated with the upper and lower interfaces. They are given explicitely in Appendix B. The integration is over the rough surface variables x_s and z_s as well as the wave-number variables v_0 and w of the scattered wave vector $\vec{k}_0$. The superscripts of E_s^{PQ} denote $P(P = H, V)$ polarized scattered fields due to $Q(Q = H, V)$ polarized incident fields. The fields expressed by (5) are at the observation point $y \geq h_{01s}$

$$\vec{r} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z. \tag{6a}$$

The position vectors to the upper and lower rough interfaces are

$$\vec{r}_{s1} = x_s\vec{a}_x + h_{s1}(x_s, z_s)\vec{a}_y + z_s\vec{a}_z \tag{6b}$$

$$\vec{r}_{s2} = x_s\vec{a}_x + h_{s2}(x_s, z_s)\vec{a}_y + z_s\vec{a}_z. \tag{6c}$$

The incident and scattered wave vectors are

$$\begin{aligned} \vec{k}_j^i = k_j\vec{n}_j^i &= k_j\left[\sin\theta_j^i\cos\phi^i\vec{a}_x \mp \cos\theta_j^i\vec{a}_y + \sin\theta_j^i\sin\phi^i\vec{a}_z\right] \\ &= k_j\left[s_j^ic_\phi^i\vec{a}_x \mp c_j^i\vec{a}_y + s_j^is_\phi^i\vec{a}_z\right], \quad \mp\text{for } j = 0, 1 \end{aligned} \tag{6d}$$

$$\vec{k}_j = k_j\vec{n}_j = k_j[s_jc_\phi\vec{a}_x + c_j\vec{a}_y + s_js_\phi\vec{a}_z], \quad j = 0, 1, 2 \tag{6e}$$

$$\begin{aligned} \vec{k}_{1D}^i = k_1\vec{n}_{1D}^i &= k_1\left[\sin\theta_1^i\cos\phi^i\vec{a}_x - \cos\theta_1^i\vec{a}_y + \sin\theta_1^i\sin\phi^i\vec{a}_z\right] \\ &= k_1\left[s_1^ic_\phi^i\vec{a}_x - c_1^i\vec{a}_y + s_1^is_\phi^i\vec{a}_z\right] \end{aligned} \tag{6f}$$

$$\vec{k}_{1D} = k_1\vec{n}_{1D} = k_1[s_1c_\phi\vec{a}_x + c_1\vec{a}_y + s_1s_\phi\vec{a}_z]. \tag{6g}$$

In (6), the complex sines and cosines of the incident and scatter angles in medium one and two are related by Snell's

$$\begin{aligned} E_{SD}^{PQ}(\vec{r}) = \frac{E_0^{Qi}}{(2\pi)^2}\iint\iint &\frac{e^{i(\vec{k}_0-\vec{k}_{1D}^i)\cdot\vec{r}_{s2}-i\vec{k}_0\cdot\vec{r}}F_{11U}^{PQ}(\vec{k},\vec{k}_{1D}^i)T_{10}^{Qi}T_{10}^P}{(s_1^ic_\phi^i - s_1c_\phi)\left[1-R_{01}^{Qi}R_{21}^{Qi}e^{-i2v_1^iH_D}\right]\left[1-R_{01}^PR_{21}^Pe^{-i2v_1H_D}\right]} \\ &\times N^{PQ}\frac{dh_{s2}}{dx_s}\,dx_s\,dz_s\frac{k_0}{u}\,dv_0\,dw \end{aligned} \tag{5b}$$

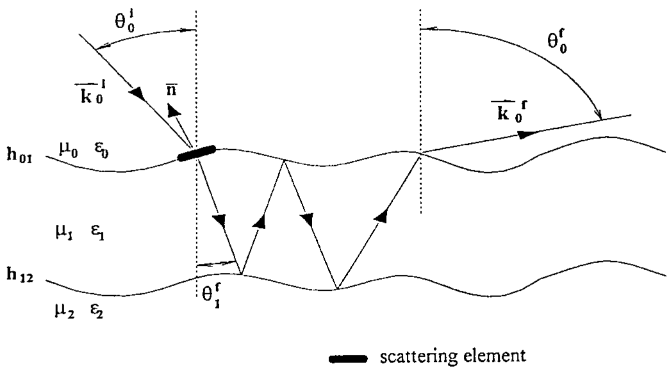

Fig. 4. Scattering upon transmission (across upper interface) from medium zero to medium one.

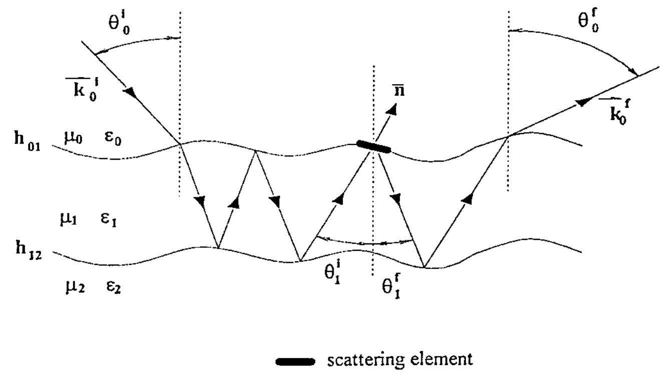

Fig. 5. Scattering upon reflection (in medium one) below the upper interface.

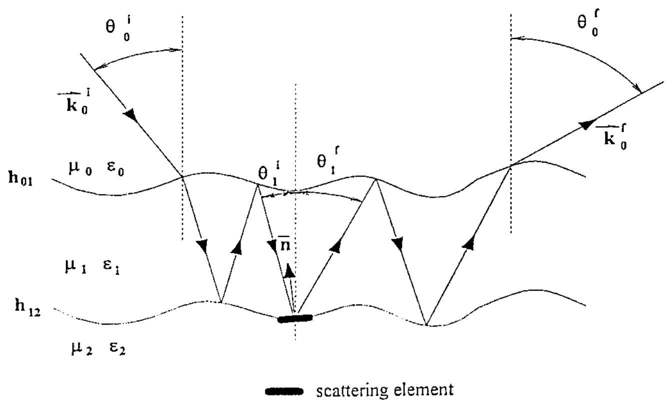

Fig. 6. Scattering upon reflection (in medium one) above the lower interface.

law

$$k_0 s_0 = k_1 s_1 = k_2 s_2; \quad c_j = \sqrt{1 - s_j^2}, \quad \mathrm{Im}[k_j c_j] \leq 0 \qquad j = 0, 1, 2. \tag{6h}$$

The Fresnel reflection ($R_{\alpha\beta}^P$) and transmission ($T_{\alpha\beta}^P$) coefficients for vertically and horizontally polarized waves, the wave impedance η, and refractive index n were defined earlier [6].

The physical interpretation of (5) is illustrated in Fig. 2–6 [1], [2], [5], [6]. Equation (5a) represents scattering due to the upper interface and (5b) represents scattering due to the lower interface. The first term on the right-hand side of (5a) associated with the scattering coefficient F_{00U}^{PQ} accounts for scattering upon reflection from above the rough upper interface (see Fig. 2). The second term in (5a) associated with F_{01U}^{PQ} accounts for waves that undergo multiple reflections in medium one before scattering upon transmission from medium one back to zero (see Fig. 3). The third term in (5a) associated with F_{10U}^{PQ} accounts for scattering upon transmission from medium zero into one followed by multiple reflections in medium one before wave transmission back to the medium zero (see in Fig. 4). The fourth term in (5a) associated with F_{11U}^{PQ} accounts for multiple reflections in medium one before scattering upon reflection in medium one from below the upper interface, followed by multiple reflections in medium one before transmission back to medium zero (see Fig. 5). The single term in (5b) associated with the scattering coefficient F_{11D}^{PQ} accounts for multiple reflections in medium one before scattering upon reflection in medium one from above the lower interface, followed by multiple reflections in medium one before transmission back to medium zero (see Fig. 6). It is shown that for uniform layered structures, the full-wave solutions sum up to the classical solutions [1], [5].

The diffuse scattered fields are evaluated at a point in the far-field region above the upper interface. The stationary phase method is used to evaluate the integrals over the scatter wave-vector variables v_0 and w in (5). Thus, the scattered far fields at $\vec{r}^f$ (the position vector from origin to the receiver) are expressed as follows:

$$\begin{aligned} E_{SU}^{PQ}(\vec{r}) &= E_0^{Qi} G_0 \int_{-L}^{L}\int_{-l}^{l} \frac{e^{i(\vec{k}_0^f - \vec{k}_0^i)\cdot\vec{r}_{s1}} - e^{i(\vec{k}_0^f - \vec{k}_0^i)\cdot\vec{r}_{s10}}}{c_0^f + c_0^i} \\ &\times \Bigg[F_{00U}^{PQ}(\vec{k}_0^f, \vec{k}_0^i) + T_{10}^{Qi} R_{21}^{Qi} F_{10U}^{PQ}(\vec{k}_0^f, \vec{k}_0^i) R^{Qi} e^{-i2v_1^i H_D} \\ &\quad + \frac{c_0^f}{c_1^f} F_{10U}^{PQ}(\vec{k}_1^f, \vec{k}_0^i) R_{21}^{Pf} T_{01}^{Pf} R^{Pf} e^{-i2v_1^f H_D} \\ &\quad - n_r \frac{c_0^f}{c_1^f} T_{10}^{Qi} R_{21}^{Qi} F_{11U}^{PQ}(\vec{k}_1^f, \vec{k}_1^i) R_{21}^{Pf} T_{01}^{Pf} R^{Qi} \\ &\quad \times R^{Pf} e^{-i2(v_1^i + v_1^f) H_D} \Bigg] dx_s\, dz_s \\ &= S_U^{PQ} E_0^{Qi} G_0 \end{aligned} \tag{7a}$$

and

$$\begin{aligned} E_{SD}^{PQ}(\vec{r}) &= E_0^{Qi} G_0 \int_{-L}^{L}\int_{-l}^{l} \frac{e^{i(\vec{k}_{1D}^f - \vec{k}_{1D}^i)\cdot\vec{r}_{s2}} - e^{i(\vec{k}_{1D}^f - \vec{k}_{1D}^i)\cdot\vec{r}_{s20}}}{c_1^f + c_1^i} \\ &\times F_{11D}^{PQ}(k_1^f, k_1^i) T_{10}^{Qi} T_{01}^f N^{PQ} R^{Qi} R^{Pf}\, dx_s\, dz_s \\ &= S_D^{PQ} E_0^{Qi} G_0 \end{aligned} \tag{7b}$$

where the position vectors to the mean upper and lower surfaces are

$$\vec{r}_{s10} = x_s \vec{a}_x + h_{01} \vec{a}_y + z_s \vec{a}_z \tag{7c}$$

$$\vec{r}_{s20} = x_s \vec{a}_x + h_{12} \vec{a}_y + z_s \vec{a}_z \tag{7d}$$

and

$$G_0 = -\frac{i k_0}{2\pi r^f} e^{-i k_0 r^f}. \tag{7e}$$

The wave vectors associated with scattering in the media are

$$\begin{aligned}\vec{k}_j^f &= k_j\vec{n}_j^f = k_j\left[\sin\theta_j^f\cos\phi^f\vec{a}_x \pm \cos\theta_j^f\vec{a}_y + \sin\theta_j^f\sin\phi^f\vec{a}_z\right]\\ &= k_j\left[s_j^f c_\phi^f\vec{a}_x \pm c_j^f\vec{a}_y + s_j^f s_\phi^f\vec{a}_z\right] \quad \pm\text{for } j=0,1 \qquad (7f)\\ \vec{k}_{1D}^f &= k_1\vec{n}_{1D}^f = k_1\left[\sin\theta_1^f\cos\phi^f\vec{a}_x + \cos\theta_1^f\vec{a}_y + \sin\theta_1^f\sin\phi^f\vec{a}_z\right]\\ &= k_1\left[s_1^f c_\phi^f\vec{a}_x + c_1^f\vec{a}_y + s_1^f s_\phi^f\vec{a}_z\right]. \qquad (7g)\end{aligned}$$

and the terms associated with multiple bounces in the coating material are

$$R^{Qi} = \left[1 - R_{01}^{Qi}R_{21}^{Qi}e^{-i2v_v^iH_D}\right]^{-1} \qquad (7h)$$

$$R^{Pf} = \left[1 - R_{01}^{Pf}R_{21}^{Pf}e^{-i2v_1^fH_D}\right]^{-1}. \qquad (7i)$$

The normalization coefficients are as in

$$N^{PQ} = \begin{cases}\eta_{ri}, & \mathrm{P} = Q = V\\ 1, & \mathrm{P} \neq Q\\ 1/\eta_{ri}, & \mathrm{P} = Q = V\end{cases} \qquad (7j)$$

where

$$\eta_i = \sqrt{\frac{\mu_i}{\varepsilon_i}}, \quad (i=0,1,2), \quad \eta_{r0} = \eta_1/\eta_0, \quad \eta_{r1} = \eta_2/\eta_1.$$

III. Matrix Notation for the Diffuse Scattered Fields

The incident and diffusely scattered vertically and horizontally polarized fields are expressed by the following 2 × 1 matrices:

$$\mathbf{E}^i = \begin{bmatrix}E_1^i\\ E_2^i\end{bmatrix} = \begin{bmatrix}E_0^{Vi}\\ E_0^{Hi}\end{bmatrix} \qquad (8a)$$

$$\mathbf{E}^f = \begin{bmatrix}E_1^f\\ E_2^f\end{bmatrix} = \begin{bmatrix}E_s^{Vf}\\ E_s^{Hf}\end{bmatrix}. \qquad (8b)$$

Since the fields diffusely scattered by the irregular stratified media are associated with the upper and lower interfaces, they are expressed as follows for the vertically (V) and horizontally (H) polarized waves:

$$\begin{cases}E_s^{Vf} = E_s^{VV} + E_s^{VH} = \left(E_{sU}^{VV} + E_{sD}^{VV}\right) + \left(E_{sU}^{VH} + E_{sD}^{VH}\right)\\ E_s^{Hf} = E_s^{HV} + E_s^{HH} = \left(E_{sU}^{HV} + E_{sD}^{HV}\right) + \left(E_{sU}^{HH} + E_{sD}^{HH}\right).\end{cases} \qquad (9)$$

Thus, using (7a) and (7b)

$$\begin{cases}E_s^{Vf} = \left(S_U^{VV}E_0^{Vi} + S_D^{VV}E_0^{Vi} + S_U^{VH}E_0^{Hi} + S_D^{VH}E_0^{Hi}\right)G_0\\ E_s^{Hf} = \left(S_U^{HV}E_0^{Vi} + S_D^{HV}E_0^{Vi} + S_U^{HH}E_0^{Hi} + S_D^{HH}E_0^{Hi}\right)G_0.\end{cases} \qquad (10)$$

The above equations are expressed in the following matrix rotation:

$$\begin{aligned}\mathbf{E}^f &= \begin{bmatrix}S_U^{VV} + S_D^{VV} & S_U^{VH} + S_D^{VH}\\ S_U^{HV} + S_D^{HV} & S_U^{HH} + S_D^{HH}\end{bmatrix}\begin{bmatrix}E_0^{Vi}\\ E_0^{Hi}\end{bmatrix}G_0\\ &= [\mathbf{S}_U + \mathbf{S}_D]\mathbf{E}^iG_0 \qquad (11)\end{aligned}$$

where the 2 × 2 scattering matrices are

$$\mathbf{S}_m = \begin{bmatrix}S_m^{VV} & S_m^{VH}\\ S_m^{HV} & S_m^{HH}\end{bmatrix}; \quad m = U, D. \qquad (12)$$

The scattering matrices $\mathbf{S}_U$ and $\mathbf{S}_D$ are associated with scattering at the upper and lower interfaces. They are given by the following integrals:

$$\begin{aligned}S_U = \int_{-L}^{L}\int_{-l}^{l}\frac{1}{c_0^f + c_0^i}&\left[e^{i(\vec{k}_0^f - \vec{k}_0^i)\cdot\vec{r}_{s1}} - e^{i(\vec{k}_0^f - \vec{k}_0^i)\cdot\vec{r}_{s10}}\right.\\ &\left\{\mathbf{F}_{00U} + \mathbf{F}_{01U}\mathbf{R}^i\mathbf{R}_{21}^i\mathbf{T}_{10}^ie^{-i2v_v^iH_D} + \frac{c_0^f}{c_1^f}\mathbf{T}_{01}^f\mathbf{R}_{21}^f\mathbf{R}^f\mathbf{F}_{01U}\right.\\ &\times e^{-i2v_1^fH_D} - n_r\frac{c_0^f}{c_1^f}\mathbf{T}_{01}^f\mathbf{R}_{21}^f\mathbf{R}^f\mathbf{F}_{11U}\mathbf{R}^i\mathbf{R}_{21}^i\mathbf{T}_{10}^i\\ &\left.\times e^{-i2v_v^fH_D}\right\}dx_s\,dz_s \qquad (13a)\end{aligned}$$

$$\begin{aligned}\mathbf{S}_D = \int_{-L}^{L}\int_{-l}^{l}&\frac{e^{i(\vec{k}_{1D}^f - \vec{k}_{1D}^i)\cdot\vec{r}_{s2}} - e^{i(\vec{k}_{1D}^f - \vec{k}_{1D}^i)\cdot\vec{r}_{s20}}}{c_1^i + c_1^f}\\ &\times\mathbf{T}_{10}^f\mathbf{R}^f\mathbf{F}_{11D}\mathbf{R}^i\mathbf{T}_{10}^i\,dx_s\,dz_s \qquad (13b)\end{aligned}$$

where

$$\mathbf{F}_{mnU} = \begin{bmatrix}F_{mnU}^{VV} & F_{mnU}^{VH}\\ F_{mnU}^{HV} & F_{mnU}^{HH}\end{bmatrix} \quad m,n = 0,1 \qquad (14a)$$

$$\mathbf{F}_{11D} = \begin{bmatrix}F_{11D}^{VV}N^{VV} & F_{11D}^{VH}N^{VH}\\ F_{11D}^{HV}N^{HV} & F_{11D}^{HH}N^{HH}\end{bmatrix} \qquad (14b)$$

$$\mathbf{T}_{mn}^i = \begin{bmatrix}T_{mn}^{Vi} & 0\\ 0 & T_{mn}^{Hi}\end{bmatrix}$$

$$\mathbf{T}_{mn}^f = \begin{bmatrix}T_{mn}^{Vf} & 0\\ 0 & T_{mn}^{Hf}\end{bmatrix}; \quad m,n = 0,1 \qquad (14c)$$

$$\mathbf{R}^f = \begin{bmatrix}R^{Vf} & 0\\ 0 & R^{Hf}\end{bmatrix} \quad \mathbf{R}^i = \begin{bmatrix}\mathbf{R}^{Vi} & 0\\ 0 & R^{Hi}\end{bmatrix} \qquad (14d)$$

$$\mathbf{R}_{21}^f = \begin{bmatrix}R_{21}^{Vf} & 0\\ 0 & R_{21}^{Hf}\end{bmatrix} \quad \mathbf{R}_{21}^i = \begin{bmatrix}R_{21}^{Vi} & 0\\ 0 & R_{21}^{Hi}\end{bmatrix}. \qquad (14e)$$

On setting

$$\mathbf{S}_0 = \mathbf{S}_U + \mathbf{S}_D \qquad (15)$$

the diffuse scattered fields $\mathbf{E}^f$ can be represented as

$$\mathbf{E}^f = \mathbf{S}_0\mathbf{E}^iG_0. \qquad (16)$$

IV. Concluding Remarks

The full-wave solutions for the first-order diffusely scattered fields from uniformly coated (constant thickness) two-dimensional (2-D) rough surfaces consist of two parts. The first part is associated with the diffuse scattered fields due to the roughness of the upper rough interface. The second part corresponds to the diffuse scattered fields due to the roughness of the lower rough interface. There are four distinct contributions to the first part and one contribution to the second part of the expression for the diffuse scattered field. Each term has a different physical interpretation. For parallel stratified structures (no roughness), the full-wave solutions reduce to the exact classical solution. The solutions for the like and cross-polarized diffuse scattered fields presented here can be applied to scattering from deterministic 2-D rough surfaces with known profiles and from random rough surfaces with known surface height statistical characteristics.

The full-wave (first-order) diffuse scatter solutions (at a single rough interface) reduce to the perturbation solution when the surface root mean square (rms) heights (in wavelengths) and slopes are very small and of the same order of smallness. When the surface radii of curvature (related to correlation length) and rms heights are very large (compared to the wavelength) the full-wave (first-order) scatter solutions reduce to the physical optics solutions. The polarimetric solutions can be applied to remote sensing of dielectric coating materials on rough surfaces, to radiowave propagation over irregular stratified media, and to the detection of buried objects in the presence of signal clutter from rough interfaces.

APPENDIX A
THE FIRST-ORDER DIFFUSE SCATTERED FIELDS FROM IRREGULAR LAYERED STRUCTURES

For irregular layered structures, the surface element forward and backward scattering coefficients are defined as [7]

$$S_{\alpha\beta}^{PQ}(v', v, w'w) = \left[\mp C_P^Q(v', v, w', w) - D_P^Q(v', v, w', w)\right]. \tag{A.1}$$

The upper and lower signs correspond to $\alpha \neq \beta$ (forward) and $\alpha = \beta$ (backward), respectively, where C_P^Q and D_P^Q are proportional to the derivatives of the surface heights at the rough interfaces. Thus, for the three medium problem, the scattering coefficients are expressed as follows:

$$\begin{aligned} S_{\alpha\beta}^{PQ}(v', v, w', w) &= -\left[\pm C_{PU}^Q + D_{PU}^Q\right] - \left[\pm C_{PD}^Q + D_{PD}^Q\right] \\ &= S_{\alpha\beta U}^{PQ} + S_{\alpha\beta D}^{PQ}. \end{aligned} \tag{A.2}$$

where the coefficients C_{PU}^Q, D_{PU}^Q, $S_{\alpha\beta U}^{PQ}$, C_{PD}^Q, D_{PD}^Q, and $S_{\alpha\beta D}^{PQ}$ are associated with the upper (U) and lower (D) interfaces, respectively, and the upper and lower signs are interpreted as in (A.1).

The first-order iterative solution for the diffuse scattered wave amplitudes are solutions to the following approximate forms of the generalized telegraphists' equations a_{0s}^Q and b_{0s}^Q in which the scattered forward and backward traveling-wave amplitudes are neglected on the right-hand side of the equations [8]–[11]:

$$-\left(\frac{d}{dx} + iu\right)a_{0s}^P(x, v', w') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(S_{FFU}^{PQ} + S_{FFD}^{PQ}\right) \times a_P^Q(x, v, w)\, dw\, dv_0 \tag{A.3a}$$

$$-\left(\frac{d}{dx} - iu\right)b_{0s}^P(x, v', w') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(S_{BFU}^{PQ} + S_{BFD}^{PQ}\right) \times a_P^Q(x, v, w)\, dw\, dv_0. \tag{A.3b}$$

Thus, multiple scatter is ignored in (A.3), $b_{0s}^P(x, v', w')$ and $a_{0s}^P(x, v', w')$ are first-order diffuse scattered-wave amplitudes, and $a_P^Q(x, v, w)$ is the primary wave amplitude. It is seen from (A.3) that the first-order solution for the diffuse scattered fields in the three media structure is a superposition of the solutions for the earlier models [1]–[5].

APPENDIX B
SCATTERING COEFFICIENTS IN THE REFERENCE AND THE LARGE-SCALE LOCAL COORDINATE SYSTEMS

The scattering coefficients F_{mnU}^{PQ} $(m, n = 0, 1)$ and F_{11D}^{PQ} for like $(P = Q)$ and cross $(P \neq Q)$ polarized waves at the upper and lower interfaces (7a), (7b) are given here. The superscripts Q and P denote the polarization of the incident and scattered waves $(Q, P = V, H)$ and the subscripts n and m indicate that the incident wave is in medium n $(=0, 1)$ and scattered wave is in medium m $(=0, 1)$, respectively.

For the upper rough interface the reflection scattering coefficients for waves incident in medium zero and reflected back to medium zero are [12] (B.1a) and (B.1b), shown at the bottom of the page

$$F_{00U}^{VH}(\vec{k}_0^f, \vec{k}_0^i) = \frac{-2c_0^i c_0^f n_r \sin(\phi^f - \phi^i)\left[\left(1 - \frac{1}{\mu_r}\right)c_1^i - \left(1 - \frac{1}{\varepsilon_r}\right)c_1^f\right]}{\left(c_0^i + \frac{c_1^i}{\eta_r}\right)\left(c_0^f + \eta_r c_1^f\right)} \tag{B.1c}$$

$$F_{00U}^{HV}(\vec{k}_0^f, \vec{k}_0^i) = \frac{2c_0^i c_0^f n_r \sin(\phi^f - \phi^i)\left[\left(1 - \frac{1}{\varepsilon_r}\right)c_1^i - \left(1 - \frac{1}{\mu_r}\right)c_1^f\right]}{\left(c_0^i + \eta_r c_1^i\right)\left(c_0^f + \frac{c_1^f}{\eta_r}\right)} \tag{B.1d}$$

where

$$\begin{aligned} &\varepsilon_r = \frac{\varepsilon_1}{\varepsilon_0}, \quad \mu_r = \frac{\mu_r}{\mu_0}, \\ &n_r = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_0\varepsilon_0}}, \quad \text{and} \quad \eta_r = \frac{\eta_1}{\eta_0} = \sqrt{\frac{\mu_1\varepsilon_0}{\mu_0\varepsilon_1}}. \end{aligned} \tag{B.2}$$

In (B.1) the quantities c_a^i and c_a^f $(a = 0, 1)$ are the cosines of the incident (i) and scatter (f) angles and s_a^i and s_a^f $(a = 0, 1)$ are the sines of the incident (i) and scatter (f) angles.

For the upper rough interface, the transmission scattering coefficients for waves incident in medium zero and transmitted into medium one are [5], [7] as shown in (B.3a) and (B.3b)

$$F_{00U}^{VV}(\vec{k}_0^f, \vec{k}_0^i) = \frac{2c_0^i c_0^f(1 - \mu_r)\cos(\phi^f - \phi^i) + \left(1 - \frac{1}{\varepsilon_r}\right)\left[\mu_r c_1^i c_1^f \cos(\phi^f - \phi^i) - s_0^i s_0^f\right]}{\left(c_0^i + \eta_r c_1^i\right)\left(c_0^f + \eta_r c_1^f\right)} \tag{B.1a}$$

$$F_{00U}^{HH}(\vec{k}_0^f, \vec{k}_0^i) = \frac{2c_0^i c_0^f(1 - \varepsilon_r)\cos(\phi^f - \phi^i) + \left(1 - \frac{1}{\mu_r}\right)\left[\varepsilon_r c_1^i c_1^f \cos(\phi^f - \phi^i) - s_0^i s_0^f\right]}{\left(c_0^i + \frac{c_1^i}{\eta_r}\right)\left(c_0^f + \frac{c_1^f}{\eta_r}\right)} \tag{B.1b}$$

at the bottom of this page

$$F_{10U}^{VH}\left(\vec{k}_1^f, \vec{k}_0^i\right) = \frac{2c_0^i c_0^f \eta_r \sin(\phi^f - \phi^i)\left[\left(1-\epsilon_r\right)c_0^f - n_r\left(1-\frac{1}{\mu_r}\right)c_1^i\right]}{\left(c_0^i + \frac{c_1^i}{\eta_r}\right)\left(c_0^f + \eta_r c_1^f\right)} \tag{B.3c}$$

$$F_{10U}^{HV}\left(\vec{k}_1^f, \vec{k}_0^i\right) = \frac{-2c_0^i c_0^f \sin(\phi^f - \phi^i)\left[\left(1-\frac{1}{n_r}\right)c_1^f - n_r\left(1-\mu_r\right)c_1^i\right]}{\left(c_0^i + \eta_r c_1^i\right)\left(c_0^f + \frac{c_1^f}{\eta_r}\right)}. \tag{B.3d}$$

For the upper rough interface the transmission scattering coefficients F_{01U}^{PQ} for waves incident in medium one and transmitted into medium zero, are similar to F_{10U}^{PQ} except that the roles of medium zero and one are interchanged.

For the upper rough interface the reflection scattering coefficients F_{11U}^{PQ} for waves incident (from below) in medium one and reflected back to medium one, are similar to except that the roles of medium zero and one are interchanged.

For the lower rough interface, the reflection scattering coefficients F_{11D}^{PQ} for waves incident in medium one and reflected back to medium one [1], [2], are similar to F_{00U}^{PQ} except that the roles of medium zero and one are replaced by medium one and two. Thus, ε_r, μ_r, n_r, and η_r are replaced by

$$\varepsilon_{r1} = \frac{\varepsilon_2}{\varepsilon_1}, \quad \mu_{r1} = \frac{\mu_2}{\mu_1}, \quad n_{r1} = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}, \quad \text{and} \quad \eta_{r1} = \frac{\eta_2}{\eta_1}. \tag{B.4}$$

To obtain the scattering coefficients (in the local coordinate system) associated with large-scale rough surfaces, the following transformations are introduced [12]:

$$\mathbf{F}_{00U} \rightarrow \mathbf{T}_0^f \mathbf{F}_{00U}^n \mathbf{T}_0^i \tag{B.5a}$$

$$\mathbf{F}_{01U} \rightarrow \mathbf{T}_0^f \mathbf{F}_{01U}^n \mathbf{T}_1^i \tag{B.5b}$$

$$\mathbf{F}_{10U} \rightarrow \mathbf{T}_1^f \mathbf{F}_{10U}^n \mathbf{T}_0^i \tag{B.5c}$$

$$\mathbf{F}_{11U} \rightarrow \mathbf{T}_1^f \mathbf{F}_{11U}^n \mathbf{T}_1^i \tag{B.5d}$$

$$\mathbf{F}_{11D} \rightarrow \mathbf{T}_{1D}^f \mathbf{F}_{11D}^n \mathbf{T}_{1D}^i \tag{B.5e}$$

where $\mathbf{T}_a^i$ $(a = 0, 1)$ and $\mathbf{T}_{1D}^i$ are 2×2 matrices (associated with the upper and lower interfaces) that transform the vertically and horizontally polarized incident waves in the reference coordinate system to the vertically and horizontally polarized incident waves in the (large scale) local coordinate system. Similarly, T_a^f $(a = 0, 1)$ and T_{1D}^f are the matrices that transform the vertically and horizontally polarized scattered waves in the local (large scale) coordinate system back to the vertically and horizontally polarized scattered waves in the reference coordinate system. The subscript a denotes the medium of propagation.

For the upper interface the 2×2 matrices associated with medium zero and one that transform the incident vertically and horizontally polarized waves in the reference coordinate system to the incident vertically and horizontally polarized waves in the local coordinate system are

$$\mathbf{T}_0^i = \begin{bmatrix} C_{\Psi 0}^i & S_{\Psi 0}^i \\ -S_{\Psi 0}^i & C_{\Psi 0}^i \end{bmatrix} \tag{B.6a}$$

$$\mathbf{T}_1^i = \begin{bmatrix} C_{\Psi 1}^i & -S_{\Psi 1}^i \\ S_{\Psi 1}^i & C_{\Psi 1}^i \end{bmatrix}. \tag{B.6b}$$

Similarly, for the upper interface, the matrices associated with medium zero and one that transform the scattered vertically and horizontally polarized waves in local coordinate system back to the scattered vertically and horizontally polarized waves in reference coordinate system are

$$\mathbf{T}_0^f = \begin{bmatrix} C_{\Psi 0}^f & -S_{\Psi 0}^f \\ S_{\Psi 0}^f & C_{\Psi 0}^f \end{bmatrix} \tag{B.7a}$$

$$\mathbf{T}_1^f = \begin{bmatrix} C_{\Psi 1}^f & S_{\Psi 1}^f \\ -S_{\Psi 1}^f & C_{\Psi 1}^f \end{bmatrix}. \tag{B.7b}$$

For the lower interface, the 2×2 matrices that transform the incident and scattered vertically and horizontally polarized waves in the reference coordinate system to the incident and scattered vertically and horizontally polarized waves in the local coordinator system are

$$\mathbf{T}_{1D}^i = \begin{bmatrix} C_{\Psi 1D}^i & S_{\Psi 1D}^i \\ -S_{\Psi 1D}^i & C_{\Psi 1D}^i \end{bmatrix} \tag{B.8a}$$

$$\mathbf{T}_{1D}^f = \begin{bmatrix} C_{\Psi 1D}^f & -S_{\Psi 1D}^f \\ S_{\Psi 1D}^f & C_{\Psi 1D}^f \end{bmatrix}. \tag{B.8b}$$

In (B.6)–(B.8), the real quantities $C_{\Psi a}^t$, $S_{\Psi a}^t$, $C_{\Psi 1D}^t$, and $S_{\Psi 1D}^t$ $(t = i, f$ and $a = 0, 1)$ are defined as follows [2], [6], [12]:

$$C_{\Psi a}^t = \cos\Psi_a^t = \frac{\left(\vec{n}_a^t \times \vec{a}_y\right)\cdot\left(\vec{n}_a^t \times \vec{n}\right)}{\left|\vec{n}_a^t \times \vec{a}_y\right|\left|\vec{n}_a^t \times \vec{n}\right|} \tag{B.9a}$$

$$S_{\Phi a}^t = \sin\Psi_a^t = \frac{\left(\vec{n}_a^t \times \vec{a}_y\right)\times\left(\vec{n}_a^t \times \vec{n}\right)\cdot\vec{n}_a^t}{\left|\vec{n}_a^t \times \vec{a}_y\right|\left|\vec{n}_a^t \times \vec{n}\right|} \tag{B.9b}$$

$$F_{10U}^{VV}\left(\vec{k}_0^f, \vec{k}_0^i\right) = \frac{2c_0^i c_1^f(1-\mu_r)\cos(\phi^f - \phi^i) + \left(1-\frac{1}{\varepsilon_r}\right)\left[n_r c_1^i c_0^f \cos(\phi^f - \phi^i) + s_0^i s_0^f\right]}{\left(c_0^i + \eta_r c_1^i\right)\left(c_0^f + \eta_r c_1^f\right)/\eta_r} \tag{B.3a}$$

$$F_{10U}^{HH}\left(\vec{k}_1^f, \vec{k}_0^i\right) = \frac{2c_0^i c_0^f(1-\varepsilon_r)\cos(\phi^f - \phi^i)\left(1-\frac{1}{\mu_r}\right)\left[n_r c_1^i c_0^f \cos(\phi^f - \phi^i) + s_0^i s_0^f\right]}{\left(c_0^i + \frac{c_1^i}{\eta_r}\right)\left(c_0^f + \frac{c_1^f}{\eta_r}\right)} \tag{B.3b}$$

$$\sin(\phi^{fn}-\phi^{in}) = \frac{(\vec{n}_0^f\times\vec{n})\cdot(\vec{n}_0^i\times\vec{n})}{|\vec{n}_0^f\times\vec{n}||\vec{n}_0^i\times\vec{n}|} = \frac{\sin\gamma\left[c_0^i s_0^f\sin(\phi^f-\delta)+s_0^i c_0^f\sin(\phi^f-\delta)\right]+\cos\gamma s_0^i s_0^f\sin(\phi^f-\phi^i)}{s_0^{in}s_0^{fn}} \tag{B.12b}$$

$$C_{\Psi 1D}^t = \cos\Psi_{1D}^t = \frac{(\vec{n}_{1D}^t\times\vec{a}_y)\cdot(\vec{n}_{1D}^t\times\vec{n})}{|\vec{n}_{1D}^t\times\vec{a}_y||\vec{n}_{1D}^t\times\vec{n}|} \tag{B.9c}$$

$$S_{\Psi 1D}^t = \sin\Psi_{1D}^t = \frac{(\vec{n}_{1D}^t\times\vec{a}_y)\times(\vec{n}_{1D}^t\times\vec{n})\cdot\vec{n}_{1D}^t}{|\vec{n}_{1D}^t\times\vec{a}_y||\vec{n}_{1D}^t\times\vec{n}|}. \tag{B.9d}$$

The unit vector normal to the large-scale rough surface is given by

$$\vec{n} = \frac{-h_x\vec{a}_x+\vec{a}_y-h_z\vec{a}_z}{\sqrt{1+h_x^2+h_z^2}}. \tag{B.10a}$$

If the medium one is dissipative, the unit vector $\vec{n}_1^t$ and $\vec{n}_{1D}^t$ (associated with the upper and lower interfaces) are taken to be in the direction of the time average Poynting vector. Thus

$$\vec{n}_{1r}^t = \frac{\mathrm{Re}[\vec{k}_1^t]}{|\mathrm{Re}[\vec{k}_1^t]|} \quad (t=i,f) \tag{B.10b}$$

$$\vec{n}_{1rD}^t = \frac{\mathrm{Re}[\vec{k}_{1D}^t]}{|\mathrm{Re}[\vec{k}_1^t]|} \quad (t=i,f). \tag{B.10c}$$

These unit vectors (associated with the upper and lower interfaces) are expressed as follows:

$$\vec{n}_{1r}^i = s_{1r}^i c_\phi^i\vec{a}_x + c_{1r}^i\vec{a}_y + s_{1r}^i s_\phi^i\vec{a}_z \tag{B.11a}$$

$$\vec{n}_{1rD}^i = s_{1rD}^i c_\phi^i\vec{a}_x - c_{1rD}^i\vec{a}_y + s_{1rD}^i s_\phi^i\vec{a}_z \tag{B.11b}$$

$$\vec{n}_{1r}^f = s_{1r}^f c_\phi^f\vec{a}_x - c_{1r}^f\vec{a}_y + s_{1r}^f s_\phi^f\vec{a}_z \tag{B.11c}$$

$$\vec{n}_{1rD}^f = s_{1rD}^f c_\phi^f\vec{a}_x + c_{1rD}^f\vec{a}_y + s_{1rD}^f s_\phi^f\vec{a}_z. \tag{B.11d}$$

On defining the unit vectors in dissipative media, the elements of the transformation matrices can be evaluated. From (B.7) we know that Ψ_a^t is the angle between the planes of incidence (or scatter) $(t=i,f)$ in the reference coordinate system and the local coordinate system (associated with the large-scale rough surface). The subscript *a* denotes the medium $(a=0,1)$. The explicit expressions for the elements in the transformation matrices associated with the upper interfaces in medium zero or one are given elsewhere [12].

To obtain the scattering coefficients (in the local coordinates system) $F_{\alpha,\beta U}^{PQn}$ and F_{11D}^{PQn} (B.5) the incident scatter angles θ_α^i, θ_α^f in the expression for $F_{\alpha,\beta U}^{PQ}$ (see B.3d) and F_{11D}^{PQ} (associated with the reference system coordinate system) are replaced by the corresponding angles associated with the local coordinate system [12]. Furthermore, $\cos(\phi^f-\phi^i)$ and $\sin(\phi^f-\phi^i)$ are replaced by

$$\cos(\phi^{fn}-\phi^{in}) = \frac{(\vec{n}_0^f\times\vec{n})\cdot(\vec{n}_0^i\times\vec{n})}{|\vec{n}_0^f\times\vec{n}||\vec{n}_0^i\times\vec{n}|} = \frac{c_0^{in}c_0^{fn}-c_0^i c_0^f+s_0^i s_0^f\cos(\phi^f-\phi^i)}{s_0^{in}s_0^{fn}} \tag{B.12a}$$

and (B.12b), shown at the top of the page.

ACKNOWLEDGMENT

The authors would like to thank R. Vietz and J. Craig for their work in preparing this manuscript.

REFERENCES

[1] E. Bahar, "Physical interpretation of the full wave solutions for electromagnetic fields scattered from irregular stratified media," *Radio Sci.*, vol. 23, no. 5, pp. 749–759, Sept./Oct. 1988.
[2] S. M. Haugland, "Scattering of electromagnetic waves from coated rough surfaces full wave approach," Ph.D. dissertation, Univ. Nebraska, Lincoln, NE, May 1991.
[3] E. Bahar, S. M. Haugland, and A. H. Carrieri, "Full-wave solutions for Mueller matrix elements used to remotely sense irregular stratified structures," in *Proc. Int. Geosci. Remote Sensing Symp. (IGARSS'91) Remote Sensing: Global Monitoring Earth Management*, Espoo, Finland, 1991, vol. 1, pp. 1479–1482.
[4] S. M. Haugland, E. Bahar, and A. H. Carrieri, "Identification of contaminant coatings over rough surfaces using polarized IR scattering," *Appl. Opt.*, vol. 31, no. 19, pp. 3847–3852, July 1992.
[5] E. Bahar and M. Fitzwater, "Full wave physical models of nonspecular scattering in irregular stratified media," *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 1609–1616, Dec. 1989.
[6] R. D. Kubik, "Scattering and depolarization upon transmission and reflection from multilayered structures," Ph.D. dissertation, Univ. Nebraska, Lincoln, NE, 1993.
[7] E. Bahar, "Full wave—Copolarized nonspecular transmission and reflection scattering matrix elements for rough surfaces," *J. Optical Soc. Amer.*, vol. A5, pp. 1873–1882, Nov. 1988.
[8] ——, "Depolarization of electromagnetic waves excited by distributions of electric and magnetic sources in inhomogeneous multilayered structures of arbitrarily varying thickness—Generalized field transforms," *J. Math. Phys.*, vol. 14, no. 11, pp. 1502–1509, Nov. 1973.
[9] ——, "Depolarization of electromagnetic wave propagation in inhomogeneous multilayered structures of arbitrary thickness—Full wave solutions," *J. Math. Phys.*, vol. 14, no. 11, pp. 1510–1515, Nov. 1973.
[10] ——, "Electromagnetic wave propagation in inhomogeneous multilayered structures of arbitrary thickness—Generalized field transforms," *J. Math. Phys.*, vol. 14, no. 8, pp. 1024–1029, Aug. 1973.
[11] ——, "Electromagnetic wave propagation in inhomogeneous multilayered structures of arbitrary thickness—Full wave solutions," *J. Math. Phys.*, vol. 14, no. 8, pp. 1030–1036, Aug. 1973.
[12] ——, "Full-wave solutions for the depolarization of the scattered radiation fields by rough surfaces of arbitrary slope," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 443–454, May 1981.

Ezekiel Bahar (S'63–M'64–SM'72–F'85) received the B.S. and M.S. degrees from the Israel Institute of Technology, in 1958 and 1960, respectively, and the Ph.D. degree from the University of Colorado, Boulder, in 1964.

He joined the Department of Electrical Engineering, University of Nebraska, Lincoln, as an Associate Professor in 1967. In 1971 he became a Professor of electrical engineering. He designed electromagnetic laboratory models to investigate problems of propagation in nonuniform terrestrial waveguides. He developed transform techniques to obtain full-wave solutions to problems of depolarization, diffraction, and scattering of radio waves in nonuniform layered structures. He has developed generalized Wentzel, Kramers, and Brillouin (WKB) techniques and generalized characteristic vectors to solve problems of propagation in inhomogeneous anisotropic media.

Dr. Bahar is a member of Commissions B, C, and F of the International Union of Radio Science. He is a member of the IEEE Antennas and Propagation Society.

Yuzhi Zhang received the B.S. and M.S. degrees in electrical engineering from Xi'an Jaotong University, Xi'an, China, in 1983 and 1986, respectivley, and the Ph.D. degree in electrical engineering from the University of Nebraska, Lincoln, in 1994.

He is currently a Senior Software Engineer with Wheb Systems, Inc., San Diego, CA. His research interests include electromagnetic wave interaction with irregular layered media, pattern recognition, and document image analysis and processing.