Diffraction of a Normally Incident Plane Wave at a Wedge with Identical Tensor Impedance Faces

Mikhail A. Lyalinov and Ning Yan Zhu, *Member, IEEE*

***Abstract*—Diffraction of a normally incident plane wave by a wedge with identical tensor impedance faces is studied in this paper and an exact solution is obtained by reducing the original problem to two decoupled and already solved ones. A uniform asymptotic solution then follows from the exact one and agrees excellently with numerical results due to the method of parabolic equation.**

***Index Terms*—Anisotropic impedance wedge, diffraction, exact solution, uniform asymptotic solution.**

I. Introduction

The electromagnetic properties of a number of new materials such as chiral and composite ones, are no longer isotropic. A tensor surface impedance thus recommends itself as an appropriate way to take into account the influence of this kind of materials on exterior electromagnetic fields [1]–[4].

This paper studies the diffraction of a normally incident plane wave by a wedge whose two faces are characterized by the *same* impedance tensor. The four elements of the tensor are restricted solely by the passivity and the nondefectiveness of a related matrix $\overline{\overline{A}}$ (Section II). To the authors' knowledge, only numerical methods are capable of solving this problem ([5], where *obliquely* incident cases have been studied [6]). This situation has not been improved since the first submission of this paper in April 1997.

To satisfy just this type of boundary conditions the electric and magnetic field components parallel to the edge of the wedge are forced to depend on each other. A suitable linear combination of these components meets nevertheless *independent* boundary conditions. With the known solution to the latter problem [7] and the above mentioned linear transformation, one obtains the exact closed-form solution to the problem under study. By asymptotically evaluating the exact solution for large distances from the edge of the wedge, a uniform asymptotic solution has been established for this canonical problem and agrees excellently with the one due to the parabolic equation method (PEM) [6].

The problem under study is formulated in Section II in which the reader will find a detailed description of the decoupling procedure. The exact solution and its uniform asymptotic expansion are obtained immediately by making use of [3] and [7]. An example is given, where the PEM results are also shown. Finally, Section III summarizes the main points and future work is also discussed.

A time dependence $\exp(-i\omega t)$ is used in this study and suppresssed in the following as usual.

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M. A. Lyalinov is with the Department of Mathematical Physics, Institute of Physics, St. Petersburg University, Petrodvorec–St. Petersburg, 198904 Russia.

N. Y. Zhu is with the Institut für Hochfrequenztechnik, Universität Stuttgart, D-70550 Stuttgart, Germany.

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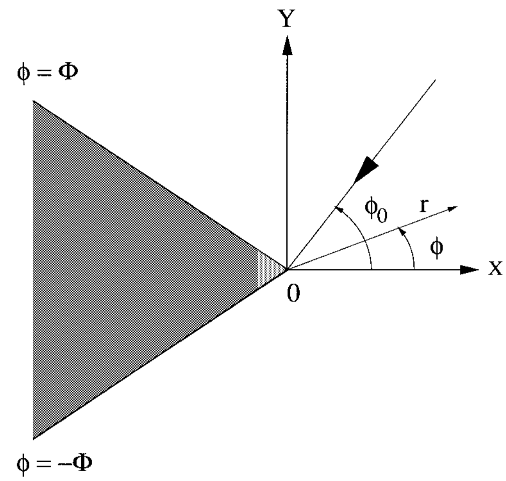

Fig. 1. An anisotropic impedance wedge illuminated by a plane wave.

II. Analysis

A tensor impedance wedge is shown in Fig. 1. Its faces are given by $\phi = \pm\Phi$ in a cylindrical coordinate system (r, ϕ, z). The z axis coincides with the edge of the wedge. A normally incident plane wave whose field components parallel to the edge are given by

$$[Z_0 H_z^i(r, \phi) \;\; E_z^i(r, \phi)]^T = \overline{U}_0 \exp[-ik_0 r \cos(\phi - \phi_0)]$$
$$\overline{U}_0 = [U_{1,0} \;\; U_{2,0}]^T \tag{1}$$

illuminates the wedge. Without sacrificing generality it is assumed that the wedge is embedded in vacuum. k_0 and Z_0 denote the wave number and the intrinsic wave impedance there. $U_{1,0}$ and $U_{2,0}$ are complex amplitudes of the incident wave.

The total field

$$[Z_0 H_z(r, \phi) \;\; E_z(r, \phi)]^T = \overline{U}(r, \phi) = [U_1(r, \phi) \;\; U_2(r, \phi)]^T \tag{2}$$

which satisfies a tensor impedance boundary condition at the wedge faces (e.g., [2])

$$\frac{i}{k_0 r} \frac{\partial \overline{U}(r, \pm\Phi)}{\partial \phi} = \mp \overline{\overline{A}}\, \overline{U}(r, \pm\Phi) \tag{3}$$

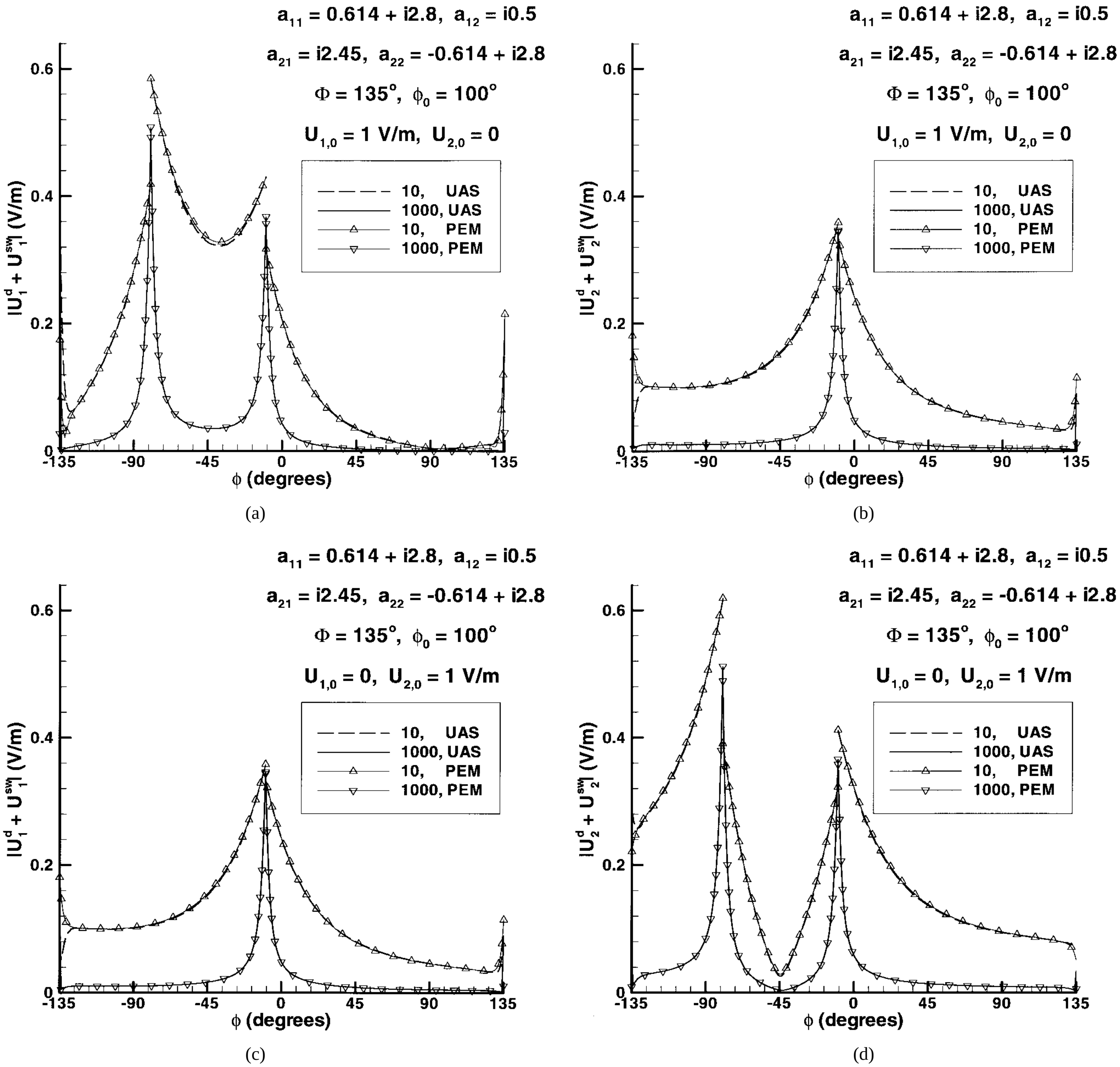

Fig. 2. Diffraction fields by a tensor impedance wedge with lossless faces. (a) Copolarized components excited by a TE_z plane wave. (b) Cross-polarized components excited by a TE_z plane wave. (c) Cross-polarized components excited by a TM_z plane wave. (d) Copolarized components excited by a TM_z plane wave.

the Meixner edge condition at $k_0 r \to 0$ and the Sommerfeld radiation condition at $k_0 r \to \infty$ is sought. The matrix $\overline{\overline{A}}$ is given by

$$\overline{\overline{A}} = \begin{pmatrix} a_{12} - a_{11}a_{22}/a_{21} & a_{11}/a_{21} \\ -a_{22}/a_{21} & 1/a_{21} \end{pmatrix}$$

where a_{jk} with $j,\ k = 1,\ 2$ are the elements of a with respect to Z_0 normalized impedance tensor and are constrained for passive surfaces in the following way [2], [6]:

$$\mathrm{Re}.a_{12} \geq 0, \quad \mathrm{Re}.a_{21} \geq 0$$
$$4\mathrm{Re}.a_{12}\,\mathrm{Re}.a_{21} \geq |a_{11} + a_{22}^*|^2$$

where $\mathrm{Re}.\ a$ stands for the real part of a and a^* denotes the complex conjugate of a. The impedance tensors used in this paper and [6] are related to each other in the following way:

$$\eta_{11} = Z_0 a_{21}, \quad \eta_{12} = -Z_0 a_{11}, \quad \eta_{21} = -Z_0 a_{22}$$
$$\eta_{22} = Z_0 a_{12}.$$

If the matrix $\overline{\overline{A}}$ is diagonal, then U_1 and U_2 (H_z and E_z) become independent to each other and the exact solution can be given immediately [7].

For a general, nondiagonal but *nondefective* $\overline{\overline{A}}$ to which this work is confined, there always exists a matrix $\overline{\overline{P}}$ such that [8]

$$\overline{\overline{P}}^{-1}\overline{\overline{A}}\,\overline{\overline{P}} = \mathrm{diag}(\sin\theta_1, \sin\theta_2).$$

The matrix $\overline{\overline{P}}$ is given by

$$\overline{\overline{P}} = \frac{1}{2a_{21}} \begin{pmatrix} -2a_{11} & -2a_{11} \\ -1+d-\sqrt{D} & -1+d+\sqrt{D} \end{pmatrix} \quad (4)$$

with

$$d = a_{12}a_{21} - a_{11}a_{22}, \quad D = (1+d)^2 - 4a_{12}a_{21}$$

and diag(a, b) means a matrix where a, b are its diagonal elements and all off-diagonal entries vanish.

The nondefective matrix $\overline{\overline{A}}$ has different eigenvalues $\sin\theta_{1,2}$ that are given by

$$\sin\theta_{1,2} = \frac{1+d\pm\sqrt{D}}{2a_{21}}.$$

Clearly, the passivity of the impedance tensor implies $0 \leq \mathrm{Re}.\ \theta_{1,2} \leq \pi/2$.

It is thus advantegeous to introduce new unknown functions defined by

$$\overline{V}(r, \phi) = [V_1(r, \phi)\ \ V_2(r, \phi)]^T = \overline{\overline{P}}^{-1}\,\overline{U}(r, \phi). \quad (5)$$

As a linear combination of the z components of the electric and magnetic field $V_{1,2}(r, \phi)$ satisfy naturally the Helmholtz wave equation.

The boundary condition (3) takes in turn the simple form

$$\frac{i}{k_0 r}\frac{\partial \overline{V}(r, \pm\Phi)}{\partial \phi} = \mp \mathrm{diag}(\sin\theta_1, \sin\theta_2)\overline{V}(r, \pm\Phi). \quad (6)$$

Hence, the original problem is decoupled and two *independent* Malyuzhinets' problems result. The exact solutions to them can be written as

$$\overline{V}(r, \phi) = \mathrm{diag}[v_1(r, \phi), v_2(r, \phi)]\overline{V}_0 \quad (7)$$

where $v_{1,2}(r, \phi)$ are Sommerfeld integrals given explicitly in [7]. Evidently, $\theta_{1,2}$ are the complex Brewster angles of the decoupled problems.

Therefore, the rigorous solution of the original diffraction problem is obtained in a closed form

$$\begin{aligned}&[Z_0 H_z(r, \phi)\ \ E_z(r, \phi)]^T \\ &\quad = \overline{\overline{P}}\,\mathrm{diag}[v_1(r, \phi), v_2(r, \phi)]\,\overline{\overline{P}}^{-1}\,\overline{U}_0. \end{aligned} \quad (8)$$

By making use of a standard procedure as described in [3], a uniform asymptotic solution (UAS) for the posed problem is given by

$$U(r, \phi) \sim \overline{U}^{go}(r, \phi) + \overline{U}^{sw}(r, \phi) + \overline{U}^{d}(r, \phi) \quad (9)$$

where $\overline{U}^{go}(r, \phi)$, $\overline{U}^{sw}(r, \phi)$, and $\overline{U}^{d}(r, \phi)$ represent the geometrical optics fields, the surface wave fields, and the first term of a uniform asymptotic expansion of the diffracted fields, respectively.

As an example, the diffraction behavior of a wedge with lossless impedance faces which are characterized in general by $\mathrm{Re}.\ a_{12} = \mathrm{Re}.\ a_{21} = 0$; and $a_{11} = -a_{22}^*$ are shown. Fig. 2 also depicts the PEM results, which are obtained by means of a program described in [6]. This method as well as the program based on it have been developed for diffracted and slope-diffracted *space* waves generated by tensor impedance wedges illuminated by normally incident plane and cylindrical waves and are capable of dealing with such diffraction bodies with two *different* tensor impedance faces. For the PEM results reported here, the following discretization parameters have been used: $\Delta\phi = 1/\pi$ and $\Delta r = 0.1/k_0$.

Also for such a case the UAS and PEM results agree very well except near the wedge faces, where surface waves predominate (Fig. 2). In this way, the PEM reported in [6] is confirmed for the first time with the aid of a UAS of an *exact* solution.

III. Conclusion

This paper derived an exact solution and from it a uniform asymptotic formula for a wedge with identical tensor impedance faces excited by a normally incident plane wave. The exact solution was enabled by introducing a suitable linear transformation for the two field components parallel to the edge of the wedge and then use is made of the known solution to the resulted decoupled Malyuzhinets' problems [7]. An application of a standard technique (for instance [3]) led eventually to a uniform asymptotic solution.

This study can and will be continued in two directions; that is, wave diffraction by a wedge with the same tensor impedance faces excited by a normally incident cylindrical wave and by a skewly incident plane wave.

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Mikhail A. Lyalinov was born on April 7, 1962, in Kirovsktown, Russia. He received the Diploma degree (honors) and the Candidate of Science degree (Ph.D.) in physics and mathematics, both from the University of Saint Petersburg, USSR, in 1985 and 1988, respectively.

In 1988, after receiving the Ph.D., he joined the Department of Mathematical Physics, Institute of Physics, University of Saint Petersburg, and became an Associate Professor in 1997. His research interests include mathematical theory of diffraction, mathematical physics, and electromagnetic diffraction theory. In these fields, he has had one monograph and 30 papers in refereed journals published, and has given 17 conference presentations. In 1994 he worked under a contract with LCTAR Thompson Radars, France, on diffraction by a coated wedge. Now he deals with the problem of electromagnetic diffraction by an impedance cone for the French Atomic Agency.

Dr. Lyalinov was a URSI Young Scientist in 1992 and 1995 and spent one year at the Cukurova University, Turkey, under a Research Fellowship from TUBITAK and NATO CP.

Ning Yan Zhu (S'89–M'91) was born on August 2, 1960, in Nanjing, China. He received the B.S. degree from the East China Engineering Institute, Nanjing, China, in 1982, the Dipl.-Ing. degree from the Technische Universität München, Germany, in 1986, and the Dr.-Ing. degree from the Universität Stuttgart, in 1990, all in electrical engineering.

He is currently a permanent Staff Member of the Institut für Hochfrequenztechnik of the Universität Stuttgart. His interests are in solving electromagnetic problems encountered in antennas, wave propagation, and microwaves.