Transient Field Calculation of Aperture Antennas

Sergey P. Skulkin and Victor I. Turchin

***Abstract*—The method of transient field calculation for large aperture antennas is proposed. It is shown that in the time domain, the calculation of the transient field is relatively simple for all points in the half-space in front of the aperture. The basic properties of the transient field are discussed.**

***Index Terms*—Aperture antennas, broad-band antennas, transient analysis.**

I. INTRODUCTION

REQUIREMENTS to increase the information content of radar and communication systems result in a bandwidth increase and are a reason to investigate and describe the transient fields from ultra wide-band (UWB) large antennas [1]. Furthermore, a need for such an analysis occurs in near-field time-domain antenna measurements [2]. In these cases, not only is the transient far field of interest. The spatial-temporal near-field distribution is also useful to the optimal antenna arrangement on the complex objects.

Mention must be made that many issues of the theory of an impulse radiating antenna (IRA) are discussed by Baum, Farr, and Giri [3]–[8]. The transient field from the circular focused aperture with uniform field distributions was considered in [3]. Much attention is given to the IRA, which consists of a reflector antenna fed by a large-angle conical TEM feed. It was shown that a radiated field includes three distinct parts. The first of these is the direct radiation from the feed structure (prepulse), which has a low magnitude and lasts for a fairly long time. This is followed by an impulse, which lasts for a brief time and is high in amplitude. Finally, there is a tail expected after the impulse. Closed-form expressions for prepulse were presented in [4]. Analysis was extended to include the diffracted fields from the launcher plates and the circular rim of the reflector [5]. It was shown that there is an exact time-domain match of the spherical and planar TEM waves (inhomogeneous) at the reflector [4]–[7]. The exact expressions for the fields on boresight for a circular aperture were obtained in [8].

While the abovementioned works describe basic properties of transient fields of an IRA, many questions are yet to be investigated. In this paper, the exact expressions for the transient fields radiated from a circular plane aperture are obtained for all observation points in the half-space in front of the aperture.

The properties of UWB antennas can be completely characterized by the radiated field's spatial-frequency dependence $\vec{E}(\omega, \vec{r})$, where $\vec{r}$ is the radius vector of the observation point and ω is the circular frequency. In contrast to the monochromatic antennas where ω is a constant, for the UWB antennas the functional dependence $\vec{E}(\omega, \vec{r})$ as a function of ω is significant because this dependence determines the form of the radiating pulse $\vec{E}(t, \vec{r})$ (where t is the time variable) at the point $\vec{r}$. The latter is the Fourier transform of the former

$$\vec{E}(t, \vec{r}) = \int \vec{E}(\omega, \vec{r}) e^{-i\omega t}\, d\omega. \tag{1}$$

If $\vec{E}(\omega, \vec{r})$ is calculated for a unit complex exciting amplitude at every ω, then $\vec{E}(t, \vec{r})$ can be considered as the totality of the pulse radiating characteristics (PRC) of an "antenna-free space" system for each point $\vec{r}$ of the space around the antenna [9]. In this case, the radiating antenna transient field $S_e(t, \vec{r})$ for the polarization characterized by the unit vector $\vec{e}$ can be defined as a convolution

$$S_e(t, \vec{r}) = \vec{e} \cdot \vec{E}(t, \vec{r}) * \mathrm{S}_0(t) \tag{2}$$

where $S_0(t)$ is the antenna input waveform and $\vec{E}(t, \vec{r})$ is the antenna transfer function in the time domain, respectively. We note that the antenna transfer function in the time domain is calculated more simply than in the frequency domain, where, in the best case, the antenna near field can be represented analytically by a series of special functions [10], [11].

For large antennas, the transfer function calculation can be organized into two independent parts. The first is the transfer function of the primary radiator. The calculated results for such antennas are given, for example, in [12]. The second issue is the calculation of the transfer function of the aperture itself, formed by the reflector or array of primary radiators. In the case of a reflector antenna, the primary radiator pattern can be considered as the aperture illumination function $g(\vec{r}_a, \omega)$ (here $\vec{r}_a$ is the radius-vector of a point on aperture plane). In the case of an antenna array, the primary radiator pattern is introduced into the integral as the integrand factor (provided we ignore the coupling between array elements, etc.).

For large antennas the spatial-field distribution is defined, first of all, by the aperture shape and size. Usually, the main lobe of the primary radiator pattern is quite wide and approximated within the scope of aperture-field theory by model functions [10], [11]. In this approximation, the electrical field

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projection $E_e(\omega, \vec{r})$ on the unit vector $\vec{e}$ can be represented as

$$E_e(\omega, \vec{r}) = \frac{i\omega}{2\pi c} \iint_{S_a} g(\vec{r}_a, \omega)\gamma(\vec{r}, \vec{r}_a) \frac{e^{i(\omega/c)|\vec{r}-\vec{r}_a|}}{|\vec{r}-\vec{r}_a|} dS_a \tag{3}$$

where S_a is the aperture region, $g(\vec{r}_a, \omega)$ is the aperture illumination function, and $\gamma(\vec{r}, \vec{r}_a)$ is the factor defined by the polarization relations. In particular, for a long focal length parabolic antenna, we approximate the aperture surface S_a as a plane and for the basic polarization we assume

$$g(x', y', \omega) \approx f\left(\frac{x'}{F}, \frac{y'}{F}, \omega\right)$$

where x', y' are the Cartesian coordinates on the aperture plane, F is the focal distance, $f(\xi, \eta, \omega)$ is the pattern of the illuminating radiator, ξ, η are the direction cosines, and

$$\gamma(\vec{r}, \vec{r}_a) \cong (\hat{n}, \hat{r}_0)^2 \tag{4}$$

where $\hat{r}_0 = (\vec{r} - \vec{r}_a)/|\vec{r} - \vec{r}_a|$, $\hat{n}$ is the normal to the aperture [13]. Here, for a wide-band illuminating radiator, we will assume that the aperture illumination function and the shape of the illuminator's radiation pattern are independent of the frequency

$$g(\vec{r}_a, \omega) \cong g(\vec{r}_a)w_{ill}(\omega). \tag{5}$$

Based on (5), the radiating antenna transient field $S_e(t, \vec{r})$ for polarization $\hat{e}$ can be represented as

$$S_e(t, \vec{r}) = \mathrm{S}_0(t) * S_{ill}(t) * \left(\frac{d}{dt}\tilde{E}_e(t, \vec{r})\right) \tag{6}$$

where $S_{ill}(t)$ is the pulse radiating characteristic (PRC) of the illuminating radiator, it is defined as the Fourier transformation of $w_{ill}(\omega)$, $d\tilde{E}_e/dt$ is the Fourier transformation of $-i\omega E_e(\omega, \vec{r})$. Taking into account (3) and (6), we can write

$$\tilde{E}_e(t, \vec{r}) = \frac{1}{2\pi c} \iint_{S_a} \frac{g(\vec{r}_a)\gamma(\vec{r}, \vec{r}_a)\delta\left(t - \frac{1}{c}|\vec{r} - \vec{r}_a|\right)}{|\vec{r} - \vec{r}_a|} dS_a \tag{7}$$

where δ is the Dirac delta function. It is essential for representation (7) of $\tilde{E}_e$ that (3) and (5) are correct for all frequencies $-\infty < \omega < \infty$, whereas (3) is suitable for field calculations of antennas with the aperture diameter D_a much larger than the wavelength and (5) does not hold for all frequencies. To overcome this constraint we will assume that the energy of pulse signal S_0 is concentrated essentially in the frequency band $\omega_{\min} < \omega < \omega_{\max}$ and $D_a \gg 2\pi c/\omega_{\min}$. Although (7) would still not be formally correct, in this case will enable us to obtain the right result for $\tilde{E}_e(t, \vec{r})$.

II. Calculation Technique

To calculate the PRC let us take a formula for an integral containing a δ function of complex argument [19]

$$\iint_S f(x, y)\delta[\varphi(x, y)]\, dx\, dy = \int_\Gamma \frac{f[x(\gamma), y(\gamma)]\, d\gamma}{|grad\varphi|_{x=x(\gamma),\, y=y(\gamma)}} \tag{8}$$

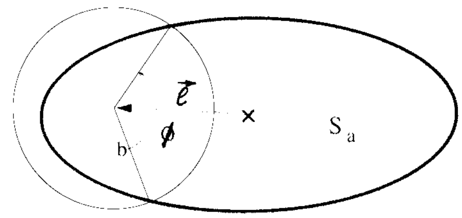

Fig. 1. The curve of intersection of the sphere with radius ct' and the aperture plane. Here S_a is antenna aperture.

where Γ is the curve determined from the equation $\varphi(x, y) = 0$; $x = x(\gamma)$, $y = y(\gamma)$ is a parametric representation of Γ, $d\gamma$ is the element of length of Γ. It is supposed here that the solution of the equation $\varphi(x, y) = 0$ for $x, y \in S$ exists and determines a unique curve Γ. If for all $x, y \in S$, $\varphi > 0$ or $\varphi < 0$, then the integral (8) equals zero. For the integral (8), the equation determining the curve Γ is

$$|\vec{r} - \vec{r}_a| = ct. \tag{9}$$

In three-dimensional space, (9) describes the sphere with its center at the point $\vec{r}$ and radius ct (c is the speed of light). The sphere crosses the aperture plane for $ct > z$, where z is the distance from the point $\vec{r}$ to the aperture plane. Similarly, it is obvious that for $ct < z$, $\tilde{E}_e = 0$. Curve Γ (a locus of intersection of the sphere and the aperture plane) is the circle with radius $b = \sqrt{(ct)^2 - z^2}$ and the center at the point $\vec{\ell}$, where $\vec{\ell}$ is the projection of vector $\vec{r}$ on the aperture plane (see Fig. 1). For this circle, $|\vec{r} - \vec{r}_a||\ \mathrm{grad}(1/c|\vec{r} - \vec{r}_a|)|_{\vec{r}_a \in \Gamma} = b/c$, hence

$$\tilde{E}_e(t, \vec{r}) = \begin{cases} 0, & \text{(a)} \\ \dfrac{1}{2\pi b}\displaystyle\int_{\Gamma_a} \cos^2(\vec{n}, \vec{r}_0) g(\vec{r}_a \in \Gamma_a)\, d\gamma & \text{(b)} \end{cases} \tag{10}$$

Γ_a is the part of Γ, belonging to S_a. In the case (a), we have $\Gamma_a \notin S_a$. For the constant amplitude distribution over the aperture $g(\vec{r}_a) = 1$ in the case (b), there follows the elementary formula for $\tilde{E}_e$:

$$\tilde{E}_e(t, \vec{r}) = \cos^2(\vec{n}, \vec{r}_0)\phi/(2\pi) \tag{11}$$

where angle ϕ is given in Fig. 1.

Such a transformation in the physical sense can be explained the following way. If each element of the aperture radiates a δ pulse at the moment $t = 0$, then, at the moment $t_1 > 0$, the field at the point $\vec{r}$ is defined only by elements lying on the circle or its part where the sphere of ct_1 radius and center at $\vec{r}$ point crosses the aperture plane. The field amplitude is defined by the weighted integral over the given circle; the latter in many cases can be expressed by elementary functions.

III. Near-Field Formulas

Let us now introduce the cylindrical coordinate system (ρ, φ, z), the center of which coincides with the center of the circular aperture of a radius. Using (11) and taking into account that the field depends only on t, ρ, z, we obtain by means of integration $\tilde{E}_e(t, \rho, z) = (z^2/(ct)^2)(1 - \phi(t)/\pi)$.

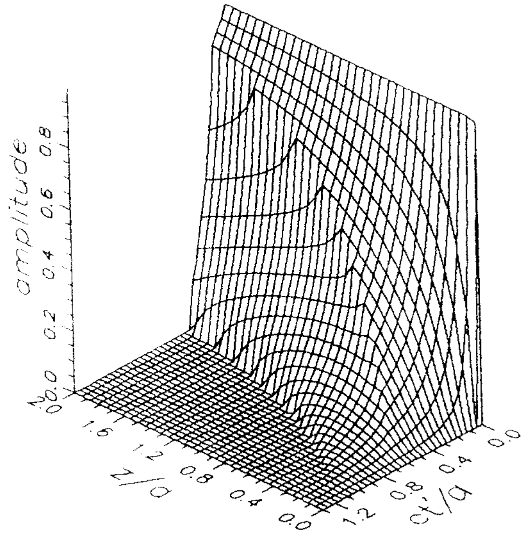

Fig. 2. Spatial-temporal amplitude distribution $\tilde{E}_e(t', \rho = a/4, z)$ for circular plane aperture.

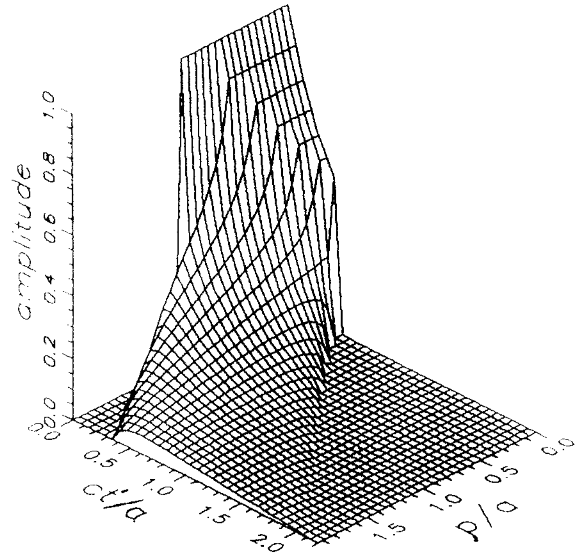

Fig. 3. Spatial-temporal amplitude distribution $\tilde{E}_e(t', \rho, z = a)$ for circular plane aperture. For $\rho/a \leq 1$ we use (12); for $\rho/a > 1$ we use (13).

Substituting the expression for φ, we obtain in the limits of the projector area, i.e., at $\rho < a$,

$$\tilde{E}_e(t, \rho, z) = \begin{cases} 0, & 0 < ct < z \\ \dfrac{z^2}{(ct)^2}, & z < ct < \sqrt{z^2 + (a-\rho)^2} \\ \dfrac{z^2}{\pi (ct)^2} \cdot \left(\pi - \arccos \dfrac{a^2 - \rho^2 - b^2}{2\rho b}\right), & \sqrt{z^2 + (a-\rho)^2} < ct < \sqrt{z^2 + (a+\rho)^2} \\ 0, & \sqrt{z^2 + (a+\rho)^2} < ct. \end{cases} \quad (12)$$

Outside the limits of the projector area, i.e., for $\rho > a$,

$$\tilde{E}_e(t, \rho, z) = \begin{cases} 0, & 0 < ct < \sqrt{z^2 + (a-\rho)^2} \\ \dfrac{z^2}{\pi (ct)^2} \arccos \dfrac{-a^2 + \rho^2 + b^2}{2\rho b}, & \sqrt{z^2 + (a-\rho)^2} < ct < \sqrt{z^2 + (a+\rho)^2} \\ 0, & \sqrt{z^2 + (a+\rho)^2} < ct. \end{cases} \quad (13)$$

The diagrams of these functions are given in Figs. 2 and 3. Here, for simplicity's sake, we introduce new variable $t' = t - z/c$. It can be seen that with small z the PRC turns into the short pulse due to the aperture element pattern $(z/ct)^2$. When $\rho = 0$, i.e., on the axis z, $\tilde{E}_e(t, \rho, z)$ has the form of a rectangle multiplied to $(z/ct)^2$, whereas z increases the delays of the leading and trailing edges vary with different velocity, the pulse duration also decreased, and the PRC form tends to the form of the δ-pulse function.

As ρ increases in the limits of the projector area, the leading edge of the pulse remains invariable, the duration of the pulse trailing edge is increased. As ρ increases outside the limits of the projector area, the pulse edges are broadened, the amplitude drops and the pulse duration tends to the value $2a$. Outside the projector area the steepness of the leading edge is larger than the trailing one; this is due to the nonlinear dependence of the *arccos* function argument on time.

Based on the above described PRC peculiarities, we note that in the near zone the antenna transfer function has two impulses corresponding to the leading and trailing edges of the PRC [2].

IV. Far-Field Formula

At infinity the field at each moment of time is defined as the integral over the line of two plane intersection. The first one is the aperture plane and the second one (viewed as the sphere of radius r, $r \to \infty$) is the plane inclined by the angle θ to the aperture plane and given in the form $\tilde{E}_e(t, \theta, \rho) \to (1/r) f(t'', \theta)$, $r \to \infty$, where $z = r\cos\theta$, $\rho = r \sin\theta$, $ct'' = ct - r$

$$f(t'', \theta) = \begin{cases} 0, & a \sin\theta < |ct''| \\ \dfrac{2}{\sin^2\theta} \cdot \sqrt{(a \sin\theta)^2 - (ct'')^2}, & |ct''| < a \sin\theta \end{cases} \quad (14)$$

for $\theta \to 0 f(t'', \theta) \to \pi a^2 \delta(t'')$. Diagrams depicting the transient far field at three different angles θ are given in Fig. 4.

V. Discussions

The PRC for circular plane apertures have been obtained for all observation points in the half-space in front of the aperture.

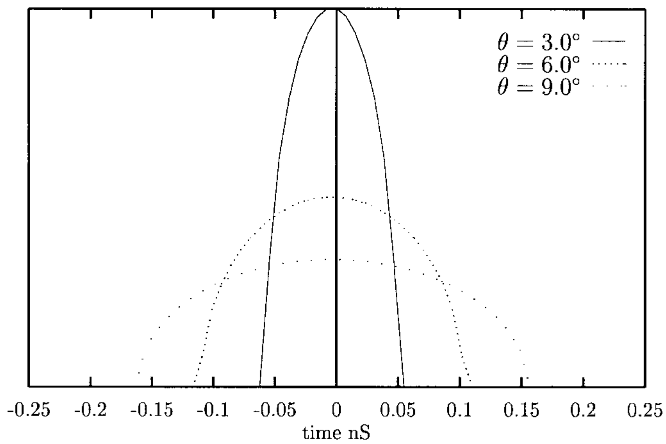

Fig. 4. The far-field time-domain dependence of $f(t'', \theta)$ for the circular plane aperture.

The formulas obtained turn out to be simpler than those for aperture antennas in the monochromatic case [10], [11].

The PRC formulas can be used in the error computation of antenna patterns reconstructed by near-field measurements. These errors are generally associated with near-field peculiarities. The error computation can be done for different probe patterns because this pattern can be introduced to the integrand in (7) and the strict PRC will be obtained by taking the probe pattern into account [17]. The use of the PRC method enables one to understand that far-field time-domain dependence (restored by measurements of the near-field antenna responses to ultrashort pulse using the method of data transformation described in [18]) is defined by the short initial part of this response. This conclusion allows one to reduce distances to surrounding objects to a few wavelengths in the case of near-field time-domain measurements and to reconstruct antenna patterns without the influence of surrounding objects [19]. The use of the PRC method makes a correction to the range distance requirements for large antenna measurements [20] as indicated in [21].

References

[1] C. E. Baum, "Impulse radiating antennas," in *Ultra-Wideband, Short-Pulse Electromagnetics*, Bertroni, L. Carin, L. B. Felsen, Eds. New York: Plenum, 1993.
[2] S. V. Goryonova, S. V. Drozhilkin, V. N. Zhavoronkov, D. M. Ponomarev, L. R. Semenova, S. P. Skulkin, and V. I. Turchin, "The time-pulse method of measuring antenna characteristics in near zone," *Radiophys. Quantum Electron.*, vol. 32, no. 1, pp. 61–70, July 1989.
[3] C. E. Baum, "Focused aperture antennas," Sensor Simulation Note 306, May 19, 1987.
[4] E. G. Farr and C. E. Baum, "Prepulse associated with the TEM feed of an impulse radiating antennas," Sensor Simulation Note 337, Mar. 1992.
[5] D. V. Giri and C. E. Baum, "Reflector IRA design and boresight temporal waveforms," Sensor Simulation Note 365, Feb. 2, 1994.
[6] E. G. Farr and C. E. Baum, "A canonical scatterer for transient scattering range calibration," Sensor Simulation Note 342, June 1992.
[7] C. E. Baum and E. G. Farr, "Hyperboloidal scatterer for spherical TEM waves," Sensor Simulation Note 342, June 1992.
[8] C. E. Baum, "Circular aperture antennas in time domain," Sensor Simulation Note 351, Nov. 2, 1992.
[9] M. M. Al-Halabi and M. G. M. Hussain, "Circular array and nonsinusoidal waves," *Inst. Elect. Eng. Trans. Electromagn. Compat.*, vol. EMC-31, no. 3, pp. 254–261, Aug. 1989.
[10] R. C. Hansen, *Microwave Scanning Antennas.* New York: Academic, 1964, vol. 1.
[11] M. Born and E. Wolf, *Principles of Optics.* New York: Pergamon, 1964.
[12] E. A. Theodorou, M. R. Gorman, P. R. Rigg, and F. N. Kong, "Broadband pulse-optimized antenna," *Proc. Inst. Elect. Eng.*, vol. 128, pt. H, no. 3, pp. 124–129, June 1981.
[13] V. Borovikov and B. Kinber, *Geometrical Theory of Diffraction.* Moscow, USSR: Svaz', 1978 (in Russian).
[14] S. P. Skulkin, "An analysis of aperture antenna near field," in *Proc. JINA'94 Conf.*, Nice, France, Nov. 1994, pp. 340–343.
[15] J. J. H. Wang, "An examination the theory and practices of planar near-field measurements," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 746–753, June 1988.
[16] R. L. Lewis and A. C. Newell, "An efficient and accurate method for representing power density in the near-zone of microwave antennas," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 890–901, June 1988.
[17] S. P. Skulkin, "An estimation of some parameters of near field antenna measurements by using of transient characteristics," in *Proc. ICAP'95 Conf.*, Eindhoven, The Netherlands, Apr. 1995, pp. 118–122.
[18] ——, "Time domain near field antenna measurements," in *Proc. EUROEM'94 Symp.*, Bordeaux, France, May 1994, pt. 2, pp. 1492–1497.
[19] S. P. Skulkin and V. I. Turchin, "Radiation of nonsinusoidal waves by aperture antennas," in *Proc. EUROEM'94 Symp.*, Bordeaux, France, May 1994, pt. 2, pp. 1498–1504.
[20] S. P. Skulkin and Yu. E. Sorpov, "An analysis of range distance requirements for large antennas by the use of the pulse radiating characteristics of aperture," in *Proc. IEEE AP-S/URSI Symp.*, Newport Beach, FL, June 1995, pp. 1758–1761.
[21] T. Uno and S. Adachi "Range distance requirements for large antenna measurements," *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 707–720, June 1989.

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