Faraday Chiral Media Revisited–I: Fields and Sources

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***Abstract*—Faraday chiral media, previously conceptualized as chiroplasmas or chiroferrites, are envisioned to combine the effects of Faraday rotation and chirality. Electromagnetic field representations for arbitrary sources are derived after the recent correct characterization of the constitutive relations of such media. The scalar Hertz potential (SHP) technique is employed and its applicability is thoroughly investigated. In particular, it is shown that all field components can be derived from one scalar Green function (plus so-called auxiliary source potentials) in source problems, whereas one scalar superpotential suffices for source-free problems. Expressions pertaining to radiation from electric and magnetic dipole sources are presented in a simple and compact form. Further generalizations of the results and the actual realizability of Faraday chiral media are discussed.**

***Index Terms*—Bianisotropy, Faraday chiral media, Hertz potentials, scalar Green functions, scalar superpotentials.**

I. Introduction

MACROSCOPIC electromagnetics provides a description of a certain material medium through constitutive relations. Once these relations have been formulated (regardless whether their basis is a concise microscopic theory of matter or a purely phenomenological construct), the solution of the electromagnetic field problem is reduced to a more or less complicated set of partial differential equations with certain initial/boundary/radiation conditions.

Bianisotropy is undoubtedly one of the catchwords in electromagnetics research of the present decade. It describes a type of medium in which the electromagnetic field vectors are coupled in a more complex way beyond the normal type of *anisotropy* [1]. Indeed, the conceptual origin of bianisotropic media is twofold: generalization of the concept of anisotropy [2] on the one hand, extension of the magnetoelectric coupling displayed by isotropic chiral media [3] on the other. Basic theoretical analyses of bianisotropic media have been available for some decades [4], [5]. Yet it is only during the last decade that the rapid advances in materials science—enabling the fabrication of increasingly complex materials in the form of thin films or particulate composites—have generated renewed and significantly more focussed interest in the electromagnetics of bianisotropic media. The huge number of entries in a recent database [6] provides ample proof of these activities with latest research being presented at annual specialist conferences [7]–[9].

In the present paper we consider linear, homogeneous bianisotropic media described by the frequency-domain constitutive relations[1]

$$\boldsymbol{D}(\boldsymbol{x}) = \underline{\underline{\epsilon}} \cdot \boldsymbol{E}(\boldsymbol{x}) + \underline{\underline{\xi}} \cdot \boldsymbol{H}(\boldsymbol{x}) \tag{1}$$

$$\boldsymbol{B}(\boldsymbol{x}) = \underline{\underline{\zeta}} \cdot \boldsymbol{E}(\boldsymbol{x}) + \underline{\underline{\mu}} \cdot \boldsymbol{H}(\boldsymbol{x}). \tag{2}$$

The permittivity dyadic $\underline{\underline{\epsilon}}$, the permeability dyadic $\underline{\underline{\mu}}$, and the magnetoelectric dyadics $\underline{\underline{\xi}}$ and $\underline{\underline{\zeta}}$ are functions of the angular frequency ω and have the following form:

$$\underline{\underline{\epsilon}} = \epsilon_t \underline{\underline{I}} + (\epsilon_u - \epsilon_t)\boldsymbol{uu} - i\epsilon_g \boldsymbol{u} \times \underline{\underline{I}} \tag{3}$$

$$\underline{\underline{\xi}} = \xi_t \underline{\underline{I}} + (\xi_u - \xi_t)\boldsymbol{uu} - i\xi_g \boldsymbol{u} \times \underline{\underline{I}} \tag{4}$$

$$\underline{\underline{\zeta}} = \zeta_t \underline{\underline{I}} + (\zeta_u - \zeta_t)\boldsymbol{uu} - i\zeta_g \boldsymbol{u} \times \underline{\underline{I}} \tag{5}$$

$$\underline{\underline{\mu}} = \mu_t \underline{\underline{I}} + (\mu_u - \mu_t)\boldsymbol{uu} - i\mu_g \boldsymbol{u} \times \underline{\underline{I}}. \tag{6}$$

It is apparent that these dyadics contain a total of 12, in general complex-valued, scalar quantities ϵ_t, ϵ_u, ϵ_g, ξ_t, ξ_u, ξ_g, $\zeta_t, \zeta_u, \zeta_g, \mu_t, \mu_u$, and μ_g, which will be assumed unconstrained for the theoretical analysis to follow.

A note about notational conventions: vectors appear bold while dyadics are double-underlined; $\boldsymbol{u}$ is a unit vector, $\cdot$ symbolizes a dot-product, whereas omission of a symbol between vectors such as in $\boldsymbol{uu}$ indicates a dyadic product; the identity dyadic is written as $\underline{\underline{I}}$. The vectors to observation and source points are denoted by $\boldsymbol{x}$ and $\boldsymbol{x}'$, respectively, and a harmonic time dependence of $\exp(-i\omega t)$ is suppressed throughout wherein ω is the angular frequency.

All four constitutive dyadics in (3)–(6) have a *gyrotropic* structure. Dielectric and magnetic gyrotropic anisotropy (through the permittivity dyadic $\underline{\underline{\epsilon}}$ and the permeability dyadic $\underline{\underline{\mu}}$) are well-known effects and displayed individually by (cold) magnetically biased plasmas and magnetically biased ferrites [2]. In addition, gyrotropic-like bianisotropy is present via the structure of the magnetoelectric dyadics $\underline{\underline{\xi}}$ and $\underline{\underline{\zeta}}$.[2]

At this point the reader may ask what motivates analysis of media described by the constitutive dyadics given in (3)–(6). Some years ago, Engheta *et al.* [11] studied plane-wave propagation in materials they called *Faraday chiral media*. Their aim was to investigate the concept of chirality control by combining the effects of Faraday rotation (as exhibited by a gyrotropic anisotropic medium) and optical activity (as displayed by an isotropic chiral medium) in a novel material. They conceptualized Faraday chiral media in two manifesta-

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[1] As far as frequency-domain analysis is concerned, the use of $\{\boldsymbol{E}(\boldsymbol{x}), \boldsymbol{H}(\boldsymbol{x})\}$ or $\{\boldsymbol{E}(\boldsymbol{x}), \boldsymbol{B}(\boldsymbol{x})\}$ as the fundamental field phasors is equivalent. The first option is chosen here for convenience.

[2] See [10] and [1] for a discussion on why the term *bigyrotropic* is not appropriate for the medium considered here.

tions: a *chiroplasma* consisting of chiral objects embedded in a magnetically biased plasma and a *chiroferrite* consisting of chiral objects immersed in a magnetically biased ferrite [11]. Subsequently, they delineated constitutive relations for such a composite medium in a purely phenomenological way by simply adding the constitutive relations of the two components. A further generalization was provided by Krowne [12] who replaced the isotropic chiral medium by a nonreciprocal bi-isotropic medium.[3]

As a consequence of these approaches, the constitutive dyadics in [11] and [12] remain reasonably simple. For the chiroplasma, the permittivity dyadic is gyrotropic such as in (3), the permeability dyadic is isotropic, and the magnetoelectric dyadics are isotropic too, mimicking those of an isotropic chiral medium (or nonreciprocal bi-isotropic medium in [12], respectively). Likewise, for the chiroferrite, with only the structures of the permittivity and permeability dyadics interchanged from the chiroplasma case.

Since then many applicational studies have been conducted which were based on the chiroplasma/chiroferrite constitutive relations put forth in [11]: from waveguide applications [15]–[17] to dipole radiation studies [18] and cylindrical scattering problems [19]; from propagation problems in stratified geometries [20] to microstrip applications [21]—to provide only a small but representative cross section of published research. It goes without saying that the actual values of the constitutive parameters of these mostly numerical studies were chosen purely on the grounds of plausibility arguments. With hindsight, two questions may appropriately be asked: i) Are the chosen values of the constitutive parameters *realistic* in representing chiroplasmas/chiroferrites based on their conceptualization in [11] as composite media? and ii) is the actual structure of the constitutive dyadics of chiroplasmas/chiroferrites as given in [11] and used henceforth by many authors appropriate to describe media of that nature?

Contrary to [11] and [12], the constitutive dyadics in (3)–(6) are considerably more involved. There is no doubt that the wave propagation studies in [11] broke new ground. However, the constitutive dyadics at their basis now appear oversimplified if indeed Faraday chiral media are conceptualized as composites via the recipes given previously by Engheta *et al.* and Krowne. A more thorough approach using standard homogenization techniques for particulate composites has shown that [22], [23]: i) the definition of constitutive parameters in [11] is open to misinterpretation, and ii) the constitutive relations of chiroplasmas and chiroferrites must, in principle, be of the form (3)–(6). Consequently, on the basis of the theoretical results in [22] and the numerical studies in [23], we therefore propose usage of the term *Faraday chiral media* for a larger class of materials, described by constitutive dyadics (3)–(6), all of which have gyrotropic structure.[4]

[3] It should be mentioned that there are significant theoretical arguments against and no experimental evidence in favor of the recognizable existence of nonreciprocal bi-isotropic media; see [13] and [14] for more detailed references. Isotropic chiral media exist in abundance, on the other hand [3].

[4] We note that investigations of more general types of media with gyrotropic anisotropy and gyrotropic-like bianisotropy have recently become available; most noticeably optical wave propagation studies in Faraday chiral media with constitutive dyadics as in (3)–(6) but for the simplification $\underline{\underline{\xi}} = -\underline{\underline{\zeta}}$ [24].

Returning now to the mathematical apparatus, we eliminate $\boldsymbol{D}$ and $\boldsymbol{B}$ according to (1), (2) from the field equations and obtain Maxwell's equations for the complex-valued frequency-dependent field vectors (phasors, to be exact) $\boldsymbol{E}$ and $\boldsymbol{H}$ in the form

$$i\omega\underline{\underline{\epsilon}} \cdot \boldsymbol{E} + (\nabla \times \underline{\underline{I}} + i\omega\underline{\underline{\xi}}) \cdot \boldsymbol{H} = \boldsymbol{J}_e \quad (7)$$

$$(\nabla \times \underline{\underline{I}} - i\omega\underline{\underline{\zeta}}) \cdot \boldsymbol{E} - i\omega\underline{\underline{\mu}} \cdot \boldsymbol{H} = -\boldsymbol{J}_m \quad (8)$$

where $\boldsymbol{J}_e(\boldsymbol{x})$ and $\boldsymbol{J}_m(\boldsymbol{x})$ are the electric and magnetic current densities, respectively.

The partial differential equations (7) and (8) form the basis of subsequent developments. In Section II, we first provide a general discussion of the scalar Hertz potential (SHP) technique. This is followed by deriving the field representation and its solution in Faraday chiral media. Scalar Green functions are discussed and the concept of a superpotential is outlined, with special emphasis on the representation of all electromagnetic field components in terms of one scalar Green function or one scalar superpotential. Further generalization of the results, the merits and disadvantages of the method and the realizability of Faraday chiral media as particulate composites are discussed in Section III, followed by conclusions in Section IV. While this paper provides the theoretical basis for field analysis, future developments in Part II will be devoted to parametric studies of propagation and radiation in Faraday chiral media.

II. The Scalar Hertz Potential Technique

A. General Remarks

We now analyze the electromagnetic field equations by employing the scalar Hertz potential (SHP) technique. Potential functions in general have a long and distinguished history in electromagnetic theory. Their frequent usage was (and remains to be) based on the recognition that it is often more convenient to express the electromagnetic field vectors in terms of auxiliary functions, scalar and vectorial, and then solve differential equations for these so-called potentials. If these formulations depend on scalar functions only, the potentials are often called (scalar) Hertz potentials (most textbooks provide a thorough introduction into scalar and vector potentials; see also [25] for many historically interesting references).

While original developments pertained to isotropic media only, the technique of SHP's was subsequently extended into the anisotropic and bianisotropic regimes. It appears that no explicit treatment of the simplest type of anisotropic medium, the dielectric/magnetic uniaxial medium, with SHP's has ever appeared in print. Generalizations beyond isotropy were first reported for source-free gyrotropic media [26], [27]. These results were then extended to sources in gyrotropic media [28], [29], thus for the first time providing a scalarization procedure for anisotropic media. Media with magnetoelectric coupling were investigated first by applying scalar Hertz potentials to homogeneous [30] and nonhomogeneous [31] isotropic chiral media. More recently, the technique was used to detail the solution of the field problem in uniaxial bianisotropic media

[32], [33]. The results of these publications all appear as special cases of the formulas obtained here.

Special mention should also be made of a simply moving isotropic medium [2], [1]. This medium is bianisotropic in a frame of reference moving with a relative uniform velocity with respect to the medium. Its constitutive relations appear as special cases of (3)–(6) whereby $\epsilon_g = \mu_g = 0$ and $\xi_g = -\zeta_g$. The last of these parameter relations actually permits a field transformation which completely eliminates the $\boldsymbol{u}\times\underline{\underline{I}}$ terms in the magnetoelectric dyadics $\underline{\underline{\xi}}$ and $\underline{\underline{\zeta}}$ (see [10]).

The key objective of the SHP technique is to obtain an electromagnetic field representation in terms of a small number (which in some instances can be just one) of scalar functions—the SHP's—which are solutions of (systems of) scalar partial differential equations. It was shown in principle [34] that, for any linear bianisotropic medium, Maxwell's equations can be decomposed with respect to an arbitrarily chosen axis. This procedure subsequently permits elimination of the field components parallel to the chosen axis and it, therefore, leads to a system of differential equations for the four remaining components of $\boldsymbol{E}$ and $\boldsymbol{H}$ which are transverse to that direction. Further progress, i.e., another reduction in the number of unknowns depends on certain symmetries of the various constitutive dyadics [34]. The limit of medium complexity for the application of the SHP technique is reached when all of the constitutive medium dyadics are of gyrotropic structure. Therefore, media described by the constitutive dyadics (3)–(6) are indeed the most general ones which can be handled by the SHP technique (in the sense that the method is defined in the literature).[5]

B. Field Decomposition

The initial step in the field analysis consists of a partial scalarization (see [34] and [35] for more background information on scalarization of general bianisotropic media) of the differential equations (7) and (8) with respect to the direction specified by $\boldsymbol{u}$. For all vector quantities we use the decomposition scheme $\boldsymbol{A} = \boldsymbol{A}_t + A_u\boldsymbol{u}$, where $\boldsymbol{A}_t \cdot \boldsymbol{u} = 0$. Thus for the fields we have

$$\boldsymbol{E} = \boldsymbol{E}_t + E_u\boldsymbol{u} \quad \boldsymbol{H} = \boldsymbol{H}_t + H_u\boldsymbol{u}. \tag{9}$$

Similarly, for the current densities

$$\boldsymbol{J}_e = \boldsymbol{J}_{et} + J_{eu}\boldsymbol{u} \qquad \boldsymbol{J}_m = \boldsymbol{J}_{mt} + J_{mu}\boldsymbol{u} \tag{10}$$

and likewise for the derivative operations $\nabla = \nabla_t + \boldsymbol{u}\partial_u$ and $\nabla^2 = \nabla_t^2 + \partial_{uu}^2$ being shorthand notations for $\partial/\partial x_u$ and $\partial^2/\partial x_u^2$, respectively, where $x_u = \mathrm{x}\cdot\mathrm{u}$. Also used, for compactness of writing, is the decomposition of the constitutive dyadics $\underline{\underline{\epsilon}}, \underline{\underline{\xi}}, \underline{\underline{\zeta}}, \underline{\underline{\mu}}$ according to $\underline{\underline{p}} = \underline{\underline{p}}_t + p_u\boldsymbol{u}\boldsymbol{u}$ such that $\boldsymbol{u}\cdot\underline{\underline{p}}_t = \underline{\underline{p}}_t\cdot\boldsymbol{u} = \underline{\underline{\boldsymbol{0}}}$.

In the first instance, manipulation of Maxwell's equations leads to an expression for the components E_u, H_u in terms of $\boldsymbol{E}_t, \boldsymbol{H}_t$ and the current densities in the form[6]

$$\begin{bmatrix} E_u \\ H_u \end{bmatrix} = \frac{1}{i\omega\lambda_u}\begin{bmatrix} \mu_u & -\xi_u \\ -\zeta_u & \epsilon_u \end{bmatrix}\begin{bmatrix} J_{eu} - \boldsymbol{u}\cdot(\nabla_t \times \boldsymbol{H}_t) \\ J_{mu} + \boldsymbol{u}\cdot(\nabla_t \times \boldsymbol{E}_t) \end{bmatrix} \tag{11}$$

where the abbreviation $\lambda_u = \epsilon_u\mu_u - \xi_u\zeta_u$ is introduced. An inversion of (11), to express the transverse components $\boldsymbol{E}_t, \boldsymbol{H}_t$ in terms of E_u, H_u plus terms involving the current densities, is not possible in general. We can, however, derive a system of differential equations for the transverse components in the form

$$-\partial_u\begin{bmatrix} \boldsymbol{E}_t \\ \boldsymbol{H}_t \end{bmatrix} + \frac{1}{i\omega\lambda_u}\begin{bmatrix} \xi_u & \mu_u \\ -\epsilon_u & -\zeta_u \end{bmatrix}\begin{bmatrix} \nabla_t\nabla_t\cdot(\boldsymbol{u}\times\boldsymbol{E}_t) \\ \nabla_t\nabla_t\cdot(\boldsymbol{u}\times\boldsymbol{H}_t) \end{bmatrix} - i\omega\begin{bmatrix} \underline{\underline{\zeta}}_t & \underline{\underline{\mu}}_t \\ -\underline{\underline{\epsilon}}_t & -\underline{\underline{\xi}}_t \end{bmatrix}\begin{bmatrix} \boldsymbol{u}\times\boldsymbol{E}_t \\ \boldsymbol{u}\times\boldsymbol{H}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{q}_{t1} \\ \boldsymbol{q}_{t2} \end{bmatrix}. \tag{12}$$

The source terms in (12) are given by

$$\boldsymbol{q}_{t1} = -\frac{1}{i\omega\lambda_u}(\mu_u\nabla_t J_{eu} - \xi_u\nabla_t J_{mu}) - \boldsymbol{u}\times\boldsymbol{J}_{mt} \tag{13}$$

$$\boldsymbol{q}_{t2} = \frac{1}{i\omega\lambda_u}(\zeta_u\nabla_t J_{eu} - \epsilon_u\nabla_t J_{mu}) + \boldsymbol{u}\times\boldsymbol{J}_{et}. \tag{14}$$

It then becomes possible to obtain the following system for E_u and H_u:

$$\begin{bmatrix} L_1 & L_2 \\ L_3 & L_4 \end{bmatrix}\begin{bmatrix} E_u \\ H_u \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \tag{15}$$

where $L_n(\partial_u, \partial_{uu}, \nabla_t^2)$ are scalar second-order partial differential operators given by

$$L_n(\partial_u, \partial_{uu}, \nabla_t^2) = l_{n1} + l_{n2}\partial_u + l_{n3}\partial_{uu} + l_{n4}\nabla_t^2 \quad (n = 1, 2, 3, 4). \tag{16}$$

Detailed expressions for their coefficients l_{nk} and for the source terms s_1, s_2 can be found in the Appendix in (46)–(58) and (59)–(62), respectively, and in [36].

C. Scalar Hertz Potentials

Instead of solving (12) directly for the fields $\boldsymbol{E}_t, \boldsymbol{H}_t$, scalar potentials may now be introduced by using a two-dimensional version of Helmholtz' theorem according to

$$\boldsymbol{E}_t(\boldsymbol{x}) = \nabla_t\Phi(\boldsymbol{x}) + \nabla_t \times \Theta(\boldsymbol{x})\boldsymbol{u} \tag{17}$$

$$\boldsymbol{H}_t(\boldsymbol{x}) = \nabla_t\Pi(\boldsymbol{x}) + \nabla_t \times \Psi(\boldsymbol{x})\boldsymbol{u}. \tag{18}$$

At this stage, one has simply exchanged the four components of $\boldsymbol{E}_t, \boldsymbol{H}_t$ for four scalar functions Φ, Θ, Π, Ψ. The crucial test as to whether a scalar potential formalism is advantageous to a direct solution of the $\boldsymbol{E}_t, \boldsymbol{H}_t$ problem (12) is this: Can the number of scalar functions defined in (17) and (18) be reduced so that a complete field representation can still be found? This question was investigated previously for general, linear bianisotropic media [34]. Although the actual field representations were not given at that time, the result emerged that the most general type of media for which such a reduction is possible are indeed the Faraday chiral media (3)–(6).

[5] It may, of course, be the case that the gyrotropic structure of the medium dyadics is already a simplification obtained by a field transformation (for example, an affine transformation) itself.

[6] In the following equations, square brackets exclusively denote matrices, their respective dimensions defined by the character of their components. Normal matrix multiplication rules apply.

Substitution of (17) and (18) into (11) and (12), respectively, yields expressions for E_u, H_u as well as differential equations in terms of Φ, Θ, Π, and Ψ. Subsequent manipulations of a somewhat tedious nature permit elimination of these four scalar potentials in favor of only two SHP's $u(\boldsymbol{x})$ and $v(\boldsymbol{x})$ in the form

$$\begin{aligned}\Phi(\boldsymbol{x}) =& (1/\lambda_t)[(a_1 + a_2\partial_u)u(\boldsymbol{x}) \\ &+ (a_3 + a_4\partial_u)v(\boldsymbol{x}) + \mu_t\overline{u}(\boldsymbol{x}) - \xi_t\overline{v}(\boldsymbol{x})] \end{aligned} \tag{19}$$

$$\Theta(\boldsymbol{x}) = i\omega[\zeta_u u(\boldsymbol{x}) + \mu_u v(\boldsymbol{x})] \tag{20}$$

$$\begin{aligned}\Pi(\boldsymbol{x}) =& (1/\lambda_t)[(b_1 + b_2\partial_u)u(\boldsymbol{x}) \\ &+ (b_3 + b_4\partial_u)v(\boldsymbol{x}) - \zeta_t\overline{u}(\boldsymbol{x}) + \epsilon_t\overline{v}(\boldsymbol{x})] \end{aligned} \tag{21}$$

$$\Psi(\boldsymbol{x}) = -i\omega[\epsilon_u u(\boldsymbol{x}) + \xi_u v(\boldsymbol{x})] \tag{22}$$

with $\lambda_t = \epsilon_t\mu_t - \xi_t\zeta_t$. The parameters $a_n, b_n (n = 1,2,3,4)$ are given in (63)–(70) of the Appendix and in [36]. The electromagnetic field representation can then be delineated completely in terms of $u(\boldsymbol{x}), v(\boldsymbol{x})$. We obtain

$$\begin{bmatrix} E_u(\boldsymbol{x}) \\ H_u(\boldsymbol{x}) \end{bmatrix} = -\nabla_t^2 \begin{bmatrix} u(\boldsymbol{x}) \\ v(\boldsymbol{x}) \end{bmatrix} + \frac{1}{i\omega\lambda_u} \begin{bmatrix} \mu_u & -\xi_u \\ -\zeta_u & \epsilon_u \end{bmatrix} \begin{bmatrix} J_{eu}(\boldsymbol{x}) \\ J_{mu}(\boldsymbol{x}) \end{bmatrix} \tag{23}$$

for the components of $\boldsymbol{E}, \boldsymbol{H}$ parallel to $\boldsymbol{u}$ when (19)–(22) are substituted into (11). In addition, (17), (18) with (19)–(22) apply for the components transversal to $\boldsymbol{u}$. It is apparent from (19) and (21) that two auxiliary functions $\overline{u}(\boldsymbol{x})$ and $\overline{v}(\boldsymbol{x})$ have entered the fray. These may be called *auxiliary source potentials*. Their defining differential equations

$$\nabla_t^2\overline{u}(\boldsymbol{x}) = (1/i\omega)\nabla_t \cdot \boldsymbol{J}_{et} \tag{24}$$
$$\nabla_t^2\overline{v}(\boldsymbol{x}) = (1/i\omega)\nabla_t \cdot \boldsymbol{J}_{mt} \tag{25}$$

indicate that they are independent of the medium. Their existence is only due to the components of the current densities transverse to $\boldsymbol{u}$. These functions arise naturally because of the choice of a distinguished direction in the mathematical formalism, see [37] and [33] for a more detailed discussion of their nature.

What remains to provide is the system of second-order differential equations for the SHP's $u(\boldsymbol{x})$ and $v(\boldsymbol{x})$. We obtain

$$\begin{bmatrix} L_1 & L_2 \\ L_3 & L_4 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \tag{26}$$

where the differential operators L_n are as given before in (16) and the source terms t_1, t_2 can be found in (71)–(74) in the Appendix.

At this stage a few comments must be made.

1) The field representation given above is in coordinate-free form for the components and coordinates transverse to $\boldsymbol{u}$. Combining the expressions (17)–(23) into just one formula for $\boldsymbol{E}$ and $\boldsymbol{H}$ each (as is possible for the simpler types of media treated in references given in Section II-A) does not seem possible.

2) The introduction of the scalar Hertz potentials $u(\boldsymbol{x}), v(\boldsymbol{x})$ is not a unique process. New scalar Hertz potentials $u^{\text{new}}(\boldsymbol{x}), v^{\text{new}}(\boldsymbol{x})$ may be defined via

$$\begin{aligned} u^{\text{new}}(\boldsymbol{x}) &= \gamma_1 u(\boldsymbol{x}) + \gamma_2 v(\boldsymbol{x}) \\ v^{\text{new}}(\boldsymbol{x}) &= \gamma_3 u(\boldsymbol{x}) + \gamma_4 v(\boldsymbol{x}) \end{aligned} \tag{27}$$

($\gamma_n, n = 1,2,3,4$, are arbitrary complex constants) if so desired.

3) The auxiliary source potentials $\overline{u}(\boldsymbol{x}), \overline{v}(\boldsymbol{x})$ (and equally, their companion functions $\overline{\overline{u}}(\boldsymbol{x})$ and $\overline{\overline{v}}(\boldsymbol{x})$ defined in (73) and (74) which appear implicitly in t_1 and t_2 in (71) and (72)) depend only on the given current densities $\boldsymbol{J}_{et}, \boldsymbol{J}_{mt}$ and not on the medium at all. As such they are source-specific and do not count as SHP's. The field representation above can thus be regarded as one in truly two SHP's only. Furthermore, if $\boldsymbol{J}_{et} \equiv \boldsymbol{0}$ there is no requirement to introduce $\overline{u}(\boldsymbol{x}), \overline{\overline{u}}(\boldsymbol{x})$ at all and we can, therefore, set $\overline{u}(\boldsymbol{x}) = \overline{\overline{u}}(\boldsymbol{x}) \equiv 0$ in that case. Equally, $\boldsymbol{J}_{mt} \equiv \boldsymbol{0}$ permits the choice $\overline{v}(\boldsymbol{x}) = \overline{\overline{v}}(\boldsymbol{x}) \equiv 0$.

D. Scalar Green Functions and Superpotentials

A set of scalar Green functions $f_1(\boldsymbol{x}, \boldsymbol{x}'), f_2(\boldsymbol{x}, \boldsymbol{x}'), g_1(\boldsymbol{x}, \boldsymbol{x}'), g_2(\boldsymbol{x}, \boldsymbol{x}')$ can now be defined corresponding to the SHP's $u(\boldsymbol{x}), v(\boldsymbol{x})$. Let the scalar Green functions be solutions of the systems of differential equations

$$\begin{bmatrix} L_1 & L_2 \\ L_3 & L_4 \end{bmatrix} \begin{bmatrix} f_{1,2}(\boldsymbol{x}, \boldsymbol{x}') \\ g_{1,2}(\boldsymbol{x}, \boldsymbol{x}') \end{bmatrix} = \begin{bmatrix} \tau_{1,2} \\ \tau_{2,1} \end{bmatrix} \delta(\boldsymbol{x} - \boldsymbol{x}') \tag{28}$$

where $\tau_1 = 1, \tau_2 = 0$ and $\delta(\boldsymbol{x} - \boldsymbol{x}')$ is the Dirac delta function. The complete solution to (26) can then be given in the form

$$\begin{bmatrix} u(\boldsymbol{x}) \\ v(\boldsymbol{x}) \end{bmatrix} = \int_{V'} d^3\boldsymbol{x}' \begin{bmatrix} f_1(\boldsymbol{x}, \boldsymbol{x}') & f_2(\boldsymbol{x}, \boldsymbol{x}') \\ g_1(\boldsymbol{x}, \boldsymbol{x}') & g_2(\boldsymbol{x}, \boldsymbol{x}') \end{bmatrix} \begin{bmatrix} t_1(\boldsymbol{x}') \\ t_2(\boldsymbol{x}') \end{bmatrix} \tag{29}$$

where the integration is over the volume V' in which the source terms t_1, t_2 are nonzero.[7]

The scalar Green functions can now be calculated by solving (28) directly. However, the SHP technique permits a further useful simplification and compactification. Define a scalar Green function $W(\boldsymbol{x}, \boldsymbol{x}')$ and let

$$f_1(\boldsymbol{x}, \boldsymbol{x}') = L_4 W(\boldsymbol{x}, \boldsymbol{x}') \tag{30}$$
$$g_1(\boldsymbol{x}, \boldsymbol{x}') = -L_3 W(\boldsymbol{x}, \boldsymbol{x}'). \tag{31}$$

Substitution of (30) and (31) into (28) (with the first subscript selected) shows that (28) is identically fulfilled provided

$$(L_1L_4 - L_2L_3)W(\boldsymbol{x}, \boldsymbol{x}') = \delta(\boldsymbol{x} - \boldsymbol{x}'). \tag{32}$$

Equally, we can define

$$f_2(\boldsymbol{x}, \boldsymbol{x}') = -L_2 W(\boldsymbol{x}, \boldsymbol{x}') \tag{33}$$
$$g_2(\boldsymbol{x}, \boldsymbol{x}') = L_1 W(\boldsymbol{x}, \boldsymbol{x}') \tag{34}$$

such that once again (28) (with the second subscript selected) is fulfilled provided once again (32) holds. Equation (32)

[7] This volume may differ from the volume in which the original current density distributions $\boldsymbol{J}_e$ and $\boldsymbol{J}_m$ are nonzero due to the presence of the auxiliary source potentials.

constitutes a scalar Green function equation for a fourth-order operator. By tracing one's way backward through the respective equations, it now becomes apparent that the complete electromagnetic field excited by arbitrary sources in Faraday chiral media (3)–(6) can be derived solely from the scalar Green function $W(\boldsymbol{x}, \boldsymbol{x}')$ (and the auxiliary source potentials). In the absence of sources, $W(\boldsymbol{x}, \boldsymbol{x}')$ is a solution of the homogeneous form (right side $= 0$) of (32) and it is then called a *superpotential* [38]. The name is due to the observation that all components of $\boldsymbol{E}$ and $\boldsymbol{H}$, in the representation (17)–(23) with all terms arising from sources set to 0, can be derived directly from it. One scalar Green function or one scalar superpotential thus generates the electromagnetic field representation in the multiparametered Faraday chiral media, see also [36].

It goes without saying that it is a difficult task to invert a fourth-order scalar differential operator. For the general case treated here, no solution to (32) exists in closed form at this time; for the source-free case, the corresponding homogeneous version of (32) leads of course to a standard eigenvalue problem with straightforward algebraic solution. For some general comments on the solvability of fourth-order differential operators (in the context of the somewhat simpler uniaxial bianisotropic media) the reader is referred to [39].

E. Radiation by Axial Sources

It is apparent from the previously presented formulas that the treatment of problems involving electric and magnetic current density distributions becomes somewhat unwieldy due to the presence of the auxiliary source potentials. These are solely due to $\boldsymbol{J}_{et}, \boldsymbol{J}_{mt}$ (i.e., the components of $\boldsymbol{J}_e, \boldsymbol{J}_m$ transverse to $\boldsymbol{u}$). Let us, therefore, for the purpose of applying the SHP technique to a radiation problem, restrict our attention here to the so-called *axial* sources, i.e,

$$\boldsymbol{J}_e^{ax} = J_{eu}^{ax}\boldsymbol{u} \qquad \boldsymbol{J}_m^{ax} = J_{mu}^{ax}\boldsymbol{u}. \tag{35}$$

In particular, let us first consider an electric dipole source

$$J_{eu}^{ax} = J_{e0}\delta(\boldsymbol{x} - \boldsymbol{x}') \qquad J_{mu}^{ax} \equiv 0. \tag{36}$$

Due to the simplification of the source terms $t_{1,2}$ in (29) we immediately obtain

$$u_e^{ax}(\boldsymbol{x}) = (J_{e0}\lambda_t/i\omega\lambda_u)[\mu_u f_1(\boldsymbol{x}, \boldsymbol{x}') - \zeta_u f_2(\boldsymbol{x}, \boldsymbol{x}')] \tag{37}$$
$$v_e^{ax}(\boldsymbol{x}) = (J_{e0}\lambda_t/i\omega\lambda_u)[\mu_u g_1(\boldsymbol{x}, \boldsymbol{x}') - \zeta_u g_2(\boldsymbol{x}, \boldsymbol{x}')]. \tag{38}$$

In analogy, for a magnetic dipole

$$J_{eu}^{ax} \equiv 0 \qquad J_{mu}^{ax} = J_{m0}\delta(\boldsymbol{x} - \boldsymbol{x}') \tag{39}$$

we obtain

$$u_m^{ax}(\boldsymbol{x}) = (J_{e0}\lambda_t/i\omega\lambda_u)[-\xi_u f_1(\boldsymbol{x}, \boldsymbol{x}') + \epsilon_u f_2(\boldsymbol{x}, \boldsymbol{x}')] \tag{40}$$
$$v_m^{ax}(\boldsymbol{x}) = (J_{e0}\lambda_t/i\omega\lambda_u)[-\xi_u g_1(\boldsymbol{x}, \boldsymbol{x}') + \epsilon_u g_2(\boldsymbol{x}, \boldsymbol{x}')]. \tag{41}$$

The expressions (37), (38) and (40), (41) show very instructively that the SHP's for the respective dipole sources are simply linear combinations of the scalar Green functions.

III. Discussion

A. Generalization

The Faraday chiral media defined by the constitutive dyadics (3)–(6) are the most general media for which the SHP technique can be applied. This is not to mean that more general types of media are not amenable to an analysis in terms of scalar potentials. Indeed, Zhuck [40] provided a formalism for scalarization of general, linear anisotropic media (and it appears that his approach can be applied to general, linear bianisotropic media in a straightforward manner). Yet the scalar potentials introduced in [40] are already double Fourier-transformed entities which only retain a dependence on one spatial variable. As a consequence they can *not* be considered as SHP's anymore.

A further generalization of the SHP technique is possible, however. Let $x_u = \boldsymbol{x}\cdot\boldsymbol{u}$ be the Cartesian coordinate in direction of the distinguished axis. Then the results of the preceding section can be extended to nonhomogeneous media of the form

$$\underline{\underline{\epsilon}} = \underline{\underline{\epsilon}}(x_u) \quad \underline{\underline{\xi}} = \underline{\underline{\xi}}(x_u) \quad \underline{\underline{\zeta}} = \underline{\underline{\zeta}}(x_u) \quad \underline{\underline{\mu}} = \underline{\underline{\mu}}(x_u) \tag{42}$$

where the gyrotropic structure of (3)–(6) remains but the constitutive dyadics have become arbitrary functions of x_u. It then becomes possible to analyze stratification problems more directly. The usual approach to such problems is to solve a boundary value problem of multiple plane layers with constitutive parameters constant in each layer. The SHP technique, on the other hand, permits the stratification profile to be implemented directly into the constitutive parameters.

B. Critical Appraisal of the Formalism

For the types of media to which it is applicable, the SHP technique is a very useful mathematical tool, both for field problems in the presence or absence of sources. It also lends itself naturally to stratification problems and the Green functions corresponding to a certain excitation are of a scalar rather than a dyadic nature.

Whenever a mathematical field representation is chosen whereby an arbitrary axis is selected, great care must be taken when evaluating potential functions in order to avoid unphysical singularities of the fields. This is a general observation which does not depend on whether the medium itself is anisotropic or bianisotropic (with a preferred axis as, for example, in the case of the Faraday chiral media discussed here) [41], [37]. General electric and magnetic current densities $\boldsymbol{J}_e(\boldsymbol{x})$ and $\boldsymbol{J}_m(\boldsymbol{x})$ which possess components transverse to the distinguished axis of the medium (the *arbitrary* axis of the mathematical representation) need particular attention. Essentially, the formalism generates a number of auxiliary source potentials; these are the functions $\overline{u}(\boldsymbol{x}), \overline{\overline{u}}(\boldsymbol{x}), \overline{v}(\boldsymbol{x}), \overline{\overline{v}}(\boldsymbol{x})$; which are often singular and which supplement the SHP's. The relevance of the auxiliary source potentials is in the removal of unphysical singularities from the fields [37].

C. Realizability of Faraday Chiral Media

For the theoretical analysis in the previous sections no constraints were imposed on the constitutive dyadics (3)–(6) containing 12 complex-valued parameters. In general, though, any linear bianisotropic medium must fulfil the *Post Constraint* [4], [42]–[44]

$$\mathrm{Trace}\,[\underline{\underline{\mu}}^{-1}\cdot\underline{\underline{\zeta}}+\underline{\underline{\xi}}\cdot\underline{\underline{\mu}}^{-1}]s=0 \tag{43}$$

($^{-1}$ indicates the inverse dyadic) to be in accordance with covariance and consistency requirements of modern electromagnetic theory. Specializing (43) for the constitutive dyadics (3)–(6) leads to

$$\frac{\xi_u+\zeta_u}{\mu_u}+2\frac{\mu_t(\xi_t+\zeta_t)-\mu_g(\xi_g+\zeta_g)}{(\mu_t+\mu_g)(\mu_t-\mu_g)}=0. \tag{44}$$

Thus only 11 independent constitutive parameters remain.

Let us now return to our original motivation, the conceptualization of Faraday chiral media as particulate composites consisting of a dielectric or magnetic gyrotropic medium and an isotropic chiral medium (resulting in a *chiroplasma* or a *chiroferrite,* respectively). Take the chiroplasma, for example (the discussion for the chiroferrite is analogous): the magnetically biased plasma is described by three dielectric parameters $\epsilon_t^P, \epsilon_g^P, \epsilon_u^P$, and a magnetic parameter $\mu^P=\mu_o$ (where μ_o is the vacuum permeability). The isotropic chiral component is characterized through a dielectric ϵ^C, a magnetic μ^C, and a magnetoelectric ξ^C constitutive parameter. We can then use the Maxwell–Garnett or the Bruggeman formalism to estimate the effective properties of a homogenized composite consisting of a plasma and an isotropic chiral medium. Following the derivation of the pointwise singularity of the dyadic Green functions of the most general linear bianisotropic medium in [45], formulas for the most general bianisotropic-in-bianisotropic composite have become available [46] and have since been applied to numerical parameter studies of complex composite media [47]. The relevant expressions in [46] can be simplified straightforwardly for the characteristics of the two components of the composite as specified here. The expressions, as was hinted at in [22], are somewhat unwieldy. In symbolic form, the constitutive parameters of the homogenized composite are of the form

$$a^{\mathrm{HCM}}=a^{\mathrm{HCM}}(\epsilon_t^P,\epsilon_g^P,\epsilon_u^P;\epsilon^C,\mu^C,\xi^C;f) \tag{45}$$

where a^{HCM} is representative of any of the 12 constitutive parameters in (3)–(6). In (45) we have included the volumetric proportion of the inclusion medium f but omitted μ_o as it is a natural constant. We also note that the condition (44) must be automatically fulfilled for the composite because the two component media fulfil their corresponding algebraic constraints individually. The numerical studies on Faraday chiral media, which have meanwhile been reported [23], confirm this result.

IV. Conclusion

In this manuscript we have provided electromagnetic field representations in Faraday chiral media in terms of SHP's and further reductions in terms of scalar Green functions and a superpotential. The Faraday chiral media, described by the constitutive dyadics (3)–(6), are the most general, linear bianisotropic media to which this technique can be applied. The relevant expressions for all media which appear as special cases of Faraday chiral media are contained within the formulas of this manuscript (consistency checks of the derivations with previous results were, of course, performed). This manuscript, therefore, brings to a close the theoretical program spanning the last decade to develop the SHP technique for complex, linear bianisotropic media. As far as Faraday chiral media are concerned, parametric studies of propagation and radiation problems will be reported in Part II.

Appendix
Some Mathematical Expressions

The coefficients of the scalar differential operators L_n in (15), (16) are given by

$$\begin{aligned} l_{11}/\omega^2 =& -\epsilon_u\mu_t(\xi_t\zeta_t+\xi_g\zeta_t)+\epsilon_u\epsilon_t(\mu_t^2-\mu_g^2)\\ &+\xi_t\zeta_u(\zeta_t^2-\zeta_g^2)-\zeta_u\zeta_t(\epsilon_t\mu_t+\epsilon_g\mu_g)\\ &+\epsilon_u\mu_g(\xi_t\zeta_g+\xi_g\zeta_t)+\zeta_u\zeta_g(\epsilon_t\mu_g+\epsilon_g\mu_t) \end{aligned} \tag{46}$$

$$\begin{aligned} l_{12}/\omega =& -\zeta_u(\epsilon_g\mu_t+\epsilon_t\mu_g-2\xi_t\zeta_g)\\ &+\epsilon_u\mu_t(\xi_g-\zeta_g)-\epsilon_u\mu_g(\xi_t-\zeta_t) \end{aligned} \tag{47}$$

$$l_{13} =\epsilon_u\mu_t-\xi_t\zeta_u \tag{48}$$

$$\begin{aligned} l_{21}/\omega^2 =& \mu_u\mu_t(\epsilon_t\zeta_t-\epsilon_g\zeta_g)+\epsilon_t\xi_u(\mu_t^2-\mu_g^2)\\ &+\mu_u\xi_t(\zeta_t^2-\zeta_g^2)-\mu_t\xi_u(\xi_t\zeta_t+\xi_g\zeta_g)\\ &+\mu_g\xi_u(\xi_g\zeta_t+\xi_t\zeta_g)+\mu_u\mu_g(\epsilon_t\zeta_g-\epsilon_g\zeta_t) \end{aligned} \tag{49}$$

$$\begin{aligned} l_{22}/\omega =& -\mu_u(\epsilon_g\mu_t+\epsilon_t\mu_g-2\xi_t\zeta_g)\\ &+\mu_t\xi_u(\xi_g-\zeta_g)-\mu_g\xi_u(\xi_t-\zeta_t) \end{aligned} \tag{50}$$

$$l_{23} =\mu_t\xi_u-\mu_u\xi_t \tag{51}$$

$$\begin{aligned} l_{31}/\omega^2 =& \epsilon_u\epsilon_t(\mu_g\xi_g-\mu_t\xi_t)+(\epsilon_t^2-\epsilon_g^2)\mu_t\zeta_u\\ &+\epsilon_u(\xi_t^2-\xi_g^2)\zeta_t-\epsilon_t\zeta_u(\xi_t\zeta_t+\xi_g\zeta_g)\\ &+\epsilon_g\zeta_u(\xi_g\zeta_t+\xi_t\zeta_g)+\epsilon_u\epsilon_g(\mu_t\xi_g-\mu_g\xi_t) \end{aligned} \tag{52}$$

$$\begin{aligned} l_{32}/\omega =& \epsilon_u(\epsilon_g\mu_t+\epsilon_t\mu_g-2\xi_g\zeta_t)\\ &+\epsilon_t\zeta_u(\xi_g-\zeta_g)-\epsilon_g\zeta_u(\xi_t-\zeta_t) \end{aligned} \tag{53}$$

$$l_{33} =\epsilon_t\zeta_u-\epsilon_u\zeta_t \tag{54}$$

$$\begin{aligned} l_{41}/\omega^2 =& (\epsilon_t\mu_g+\epsilon_g\mu_t)\xi_u\xi_g+(\epsilon_t^2-\epsilon_g^2)\mu_u\mu_t\\ &+\xi_u(\xi_t^2-\xi_g^2)\zeta_t-\epsilon_t\mu_u(\xi_t\zeta_t+\xi_g\zeta_g)\\ &+\epsilon_g\mu_u(\xi_g\zeta_t+\xi_t\zeta_g)-(\epsilon_t\mu_t+\epsilon_g\mu_g)\xi_u\xi_t \end{aligned} \tag{55}$$

$$\begin{aligned} l_{42}/\omega =& \xi_u(\epsilon_t\mu_g+\epsilon_g\mu_t-2\xi_g\zeta_t)\\ &+\epsilon_t\mu_u(\xi_g-\zeta_g)-\epsilon_g\mu_u(\xi_t-\zeta_t) \end{aligned} \tag{56}$$

$$l_{43} =\epsilon_t\mu_u-\xi_u\zeta_t \tag{57}$$

$$l_{14} =l_{44}=\lambda_t=\epsilon_t\mu_t-\xi_t\zeta_t, \quad l_{24}=l_{34}=0. \tag{58}$$

The source terms s_1, s_2 in (15) can be calculated from

$$\begin{aligned} s_1 =& \lambda_t\nabla_t\cdot\boldsymbol{q}_1+(\partial_u-\omega\zeta_g)\alpha_s-\omega\mu_g\beta_s\\ &+(\lambda_t/i\omega\lambda_u)(\mu_u\nabla_t^2+\omega^2\lambda_u\mu_t)J_{eu}\\ &-(\lambda_t/i\omega\lambda_u)(\xi_u\nabla_t^2+\omega^2\lambda_u\zeta_t)J_{mu} \end{aligned} \tag{59}$$

$$\begin{aligned} s_2 =& \lambda_t\nabla_t\cdot\boldsymbol{q}_2+(\partial_u+\omega\xi_g)\beta_s+\omega\epsilon_g\alpha_s\\ &-(\lambda_t/i\omega\lambda_u)(\zeta_u\nabla_t^2+\omega^2\lambda_u\xi_t)J_{eu}\\ &+(\lambda_t/i\omega\lambda_u)(\epsilon_u\nabla_t^2+\omega^2\lambda_u\epsilon_t)J_{mu} \end{aligned} \tag{60}$$

with the abbreviations

$$\alpha_s = i(\xi_t\mu_g - \mu_t\xi_g)J_{eu} + (\mu_t/i\omega)\nabla\cdot\boldsymbol{J}_e + i(\mu_t\epsilon_g - \xi_t\zeta_g)M_{eu} - (\xi_t/i\omega)\nabla\cdot\boldsymbol{J}_m \quad (61)$$

$$\beta_s = -i(\epsilon_t\mu_g - \xi_g\zeta_t)J_{eu} - (\zeta_t/i\omega)\nabla\cdot\boldsymbol{J}_e + i(\epsilon_t\zeta_g - \epsilon_g\zeta_t)M_{eu} + (\epsilon_t/i\omega)\nabla\cdot\boldsymbol{J}_m. \quad (62)$$

The coefficients a_n, b_n in (19) and (21) are given by

$$a_1/\omega = \epsilon_u(\mu_t\xi_g - \mu_g\xi_t) - \zeta_u(\epsilon_g\mu_t - \xi_t\zeta_g) \quad (63)$$

$$a_2 = \epsilon_u\mu_t - \xi_t\zeta_u \quad (64)$$

$$a_3/\omega = \xi_u(\mu_t\xi_g - \mu_g\xi_t) - \mu_u(\epsilon_g\mu_t - \xi_t\zeta_g) \quad (65)$$

$$a_4 = \mu_t\xi_u - \mu_u\xi_t \quad (66)$$

$$b_1/\omega = \epsilon_u(\epsilon_t\mu_g - \xi_g\zeta_t) - \zeta_u(\epsilon_t\zeta_g - \epsilon_g\zeta_t) \quad (67)$$

$$b_2 = \epsilon_t\zeta_u - \epsilon_u\zeta_t \quad (68)$$

$$b_3/\omega = \xi_u(\epsilon_t\mu_g - \xi_g\zeta_t) - \mu_u(\epsilon_t\zeta_g - \epsilon_g\zeta_t) \quad (69)$$

$$b_4 = \epsilon_t\mu_u - \xi_u\zeta_t. \quad (70)$$

Finally, the source terms $t_1(\boldsymbol{x})$ and $t_2(\boldsymbol{x})$, used in (26), are defined as

$$t_1 = (\lambda_t/i\omega\lambda_u)(\mu_u J_{eu} - \xi_u J_{mu}) + \lambda_t\overline{\overline{v}} - (\partial_u - \omega\zeta_g)(\mu_t\overline{u} - \xi_t\overline{v}) + \omega\mu_g(-\zeta_t\overline{u} + \epsilon_t\overline{v}) \quad (71)$$

$$t_2 = (\lambda_t/i\omega\lambda_u)(-\zeta_u J_{eu} + \epsilon_u J_{mu}) - \lambda_t\overline{\overline{u}} - (\partial_u + \omega\xi_g)(-\zeta_t\overline{u} + \epsilon_t\overline{v}) - \omega\epsilon_g(\mu_t\overline{u} - \xi_t\overline{v}) \quad (72)$$

whereas the auxiliary source potentials $\overline{\overline{u}}(\boldsymbol{x})$ and $\overline{\overline{v}}(\boldsymbol{x})$ are solutions of

$$\nabla_t^2\overline{\overline{u}} = \nabla_t\cdot(\boldsymbol{u}\times\boldsymbol{J}_{et}) \quad (73)$$

$$\nabla_t^2\overline{\overline{v}} = \nabla_t\cdot(\boldsymbol{u}\times\boldsymbol{J}_{mt}). \quad (74)$$

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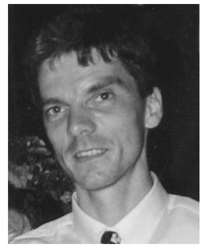

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