Scattering from a Periodic Array of Thin Planar Chiral Structures—Calculations and Measurements

Saïd Zouhdi, *Member, IEEE*, Georges E. Couenon, and Arlette Fourrier-Lamer

***Abstract*—In this paper, as a first approach toward modeling inhomogeneous bianisotropic slabs, the electromagnetic scattering by an infinite regular array of thin planar chiral structures disposed on grounded or ungrounded dielectric slabs is studied. The electromagnetic analysis is based on the Floquet's theorem and the method of moments associated to quadratic wire segments. Experimental results concerning copolarized and cross-polarized reflection coefficients from both grounded and ungrounded finite grids of novel planar chiral particles are reported. Comparisons are made between numerical and experimental results.**

***Index Terms*—Chiral materials, slab waveguide.**

I. Introduction

SINCE the last few years, interaction of electromagnetic waves with bianisotropic media attracts a lot of attention and efforts of researchers since they offer some novel promising applications in microwave and millimeter-wave technology. The bianisotropic medium is a generalization of the well-known isotropic chiral medium and it is described in the general case by four dyadic constitutive parameters

$$\vec{D} = \bar{\bar{\varepsilon}}\vec{E} + \bar{\bar{\xi}}\vec{H} \tag{1}$$

$$\vec{B} = \bar{\bar{\mu}}\vec{H} + \bar{\bar{\zeta}}\vec{E} \tag{2}$$

where $\bar{\bar{\varepsilon}}$, $\bar{\bar{\mu}}$, $\bar{\bar{\xi}}$, and $\bar{\bar{\zeta}}$ are, respectively, the permittivity, the permeability, and the magnetoelectric coupling parameters of the medium [1]. A special type of this medium is the uniaxial bianisotropic chiral (UBC) medium, where the constitutive parameter dyadics are uniaxial. Such materials can be constructed by aligning structures of specific geometry in a homogeneous host medium. These inclusions can be shaped like the capital Greek letter Ω [2]. Various applications have been proposed for the UBC materials: reciprocal phase shifters [3], polarization transformers [4], and nonreflecting shields [5].

In 1991, we patented other shapes named *planar chiral objects* [6]–[8]: a planar chiral object is a planar structure that induces simultaneously electric *and* magnetic dipole moments when it is submitted to an electric *or* a magnetic field. This definition includes structures which might possess less degree of symmetry than the well-known omega objects. Unlike Ω-materials, composite materials formed by asymmetrical planar chiral objects are modeled by more general uniaxial bianisotropic media.

In the literature, a few papers dealing with the reflection and transmission in general uniaxial slabs were recently published. In [9] and [10], numerical procedures were applied to calculate the reflection and transmission coefficients and, more recently in paper [11], authors used the vector transmission line theory. In these papers, the slabs were considered as effective homogeneous media, characterized by four dyadic constitutive parameters.

In this paper, we consider the electromagnetic scattering for arbitrary incidence by an infinite regular array of thin planar chiral structures disposed on grounded or ungrounded dielectric slabs. The motivation of this work is twofold. First, to introduce new planar chiral objects and second, to show the depolarizing character of a periodic array of such objects both theoretically and experimentally.

The problem of electromagnetic scattering from an infinite two-dimensional regular array of conducting patches, has been investigated extensively in the literature and find various applications such as the design of microwave filters, artificial dielectrics, wave polarizers, and antenna reflectors or ground planes [12].

Rigorous methods used for modeling infinite regular arrays of thin wire scatterers are mainly based on the method of moments associated with Floquet's theorem, which reduces the problem to the analysis of a single unit cell [13]–[15]. However, modeling chiral wire structures that generally have a high degree of curvature requires a large number of piecewise linear segments to accurately represent rapid geometrical variations. In order to reduce the number of segments and consequently reduce memory storage required for the moment matrix, we have used quadratic wire segments to discretize the wire geometry, a method proposed by Champagne *et al.* for the analysis of thin wire antennas [16]. In Section IV, experimental results concerning the reflection by finite grids of planar chiral particles are reported. A comparison is made between numerical results and measurements.

II. Formulation

The geometry of an infinite doubly periodic array of two-dimensional chiral scatterers is shown in Fig. 1. The scatterers are assumed perfect electrically conducting thin planar objects, whose width w is smaller than the wavelength. The array is illuminated by a linearly polarized plane wave, with propagation

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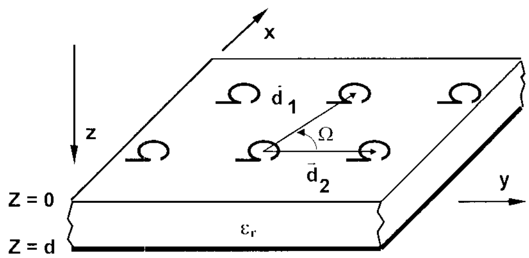

Fig. 1. Geometry of an infinite doubly periodic array of planar chiral elements.

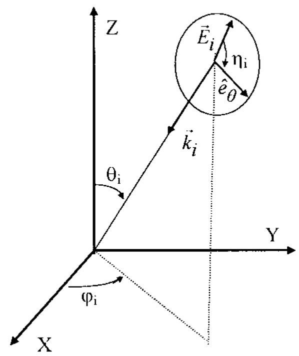

Fig. 2. Specification of the incident plane wave.

vector $\vec{k}_i$ characterized by the standard spherical coordinates θ_i and φ_i and the orientation angle η_i, defined as the angle between $\hat{\mathbf{e}}_\theta$ (θ_i unit vector) and the direction of the electric field (Fig. 2).

In this presentation, we follow the method and the notations presented by Montgomery in [15]. Since the structure has a double periodicity, the electromagnetic field in the region $z > 0$ and $z < 0$, can be expanded in terms of a doubly infinite series of Floquet space harmonics.

Enforcing the condition that the tangential magnetic field is continuous at $z = 0$ except on the scatterers, where it will be discontinuous by the surface induced currents $\vec{J}_s$, we get a vector integral equation for the induced current on a single object

$$\begin{aligned}\sum_{m=1}^{m=2} \left[1 + R_{moo}^{slab}\right] \cdot b_m^{inc} \exp(-j\vec{k}_{Too} \cdot \vec{r}_T) \cdot \vec{\kappa}_{moo} \\ = \frac{1}{\left|\vec{d}_1 \times \vec{d}_2\right|} \cdot \sum_m \sum_p \sum_q \exp(-j\vec{k}_{Tpq} \cdot \vec{r}_T) \\ \cdot \left[\eta_{mpq}^{eq}\right]^{-1} \int_{slot} \vec{\kappa}_{mpq} \cdot \vec{J}_s(\vec{r}_T') \\ \cdot \exp(j\vec{k}_{Tpq} \cdot \vec{r}_T')\, dr_T' \cdot \vec{\kappa}_{mpq}. \qquad (3)\end{aligned}$$

See [15] for the significance of the terms used in (3).

The integral equation is then resolved using the method of moment. The current distribution $\vec{J}_s$ is expanded in a complete set of orthogonal basis functions as

$$\vec{J}_s(\vec{r}_T') = \sum_{n=1}^{N} I_n \vec{\Lambda}_n(\vec{r}_T') \qquad (4)$$

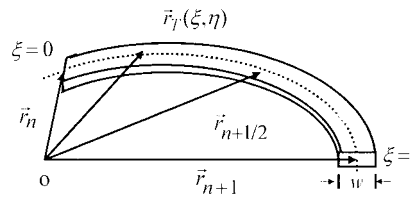

Fig. 3. Quadratic segment defined by three position vectors.

where I_n represent unknown current coefficients, and $\vec{\Lambda}_n(\vec{r}_T')$ the piecewise linear expansion functions, which have support on curved wire segments [16]. A curved segment of length σ is approximated by a quadratic wire segment, which forms a curve that passes through three specified points defined by the position vectors $\vec{r}_n, \vec{r}_{n+1/2}$ and $\vec{r}_{n+1}$. A change of variables is performed to transform the integration over arc length $\sigma \in [\sigma_n, \sigma_{n+1}]$ into an integration over a normalized variable $\xi \in [0, 1]$ (Fig. 3). In this paper, since the structures to be analyzed are not formed by thin cylindrical wires as in [16], but are planar objects of width w, we introduced a new variable $\eta \in [-(w/2), w/2]$.

Since w is too small compared to the wavelength, the current vector $\vec{J}_s$ is assumed tangent to the axis of the strip. Thus, (4) becomes

$$\vec{J}_s(\xi, \eta) = \sum_{n=1}^{N} I_n \Pi(\eta) \vec{\Lambda}_n(\xi). \qquad (5)$$

In order to satisfy the near edges condition, the current distribution $\Pi(\eta)$ in η direction is approximated by

$$\Pi(\eta) = \begin{cases} \dfrac{1}{\sqrt{1 - \left(\dfrac{2\eta}{w}\right)^2}} & |\eta| < \dfrac{w}{2} \\ 0 & |\eta| > \dfrac{w}{2}. \end{cases} \qquad (6)$$

After performing the change of variables in the integral equation and projecting it on a set of testing functions which are chosen identical to the basis functions (Galerkin procedure), we obtain an $N \times N$ matrix equation

$$V_g = \sum_{n=1}^{N} Z_{gn} \cdot I_n, \qquad g = 1, 2, \cdots, N. \qquad (7)$$

The unknown current coefficients I_n are then computed by simple matrix inversion.

III. Measurement System

Measurements were performed in the THOMSON-CSF anechoic chamber. To allow quasi-monostatic radar cross-section (RCS) measurements, the transmit and receive horn antennas are placed on top of each other on a platform (Fig. 4). From the center of this radar platform a laser beam references the sample. The distance between the midpoints in the front planes of horns is 30 cm. This results in a bistatic angle of 1.1°. The horn antennas were aligned using the laser to make their main lobes point toward the middle of the sample. A vertically polarized wave is emitted by the upper horn antenna, which

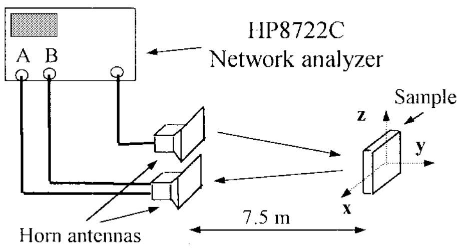

Fig. 4. Free-space reflection measurement setup.

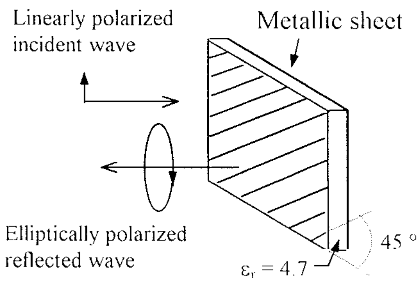

Fig. 5. Polarizer used for calibration.

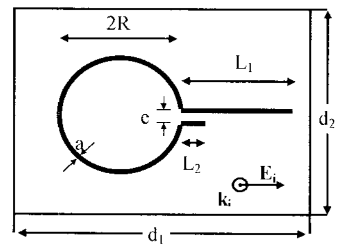

Fig. 6. The cell geometry: "TFH" object.

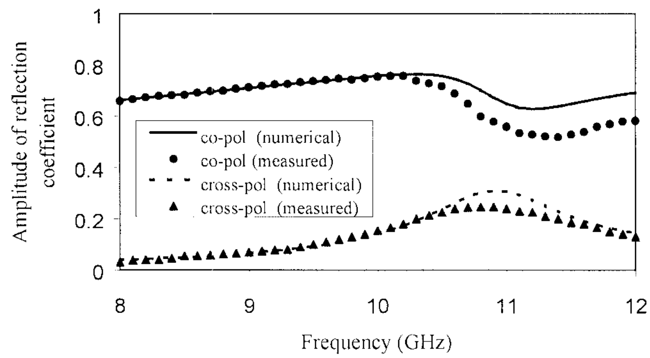

Fig. 7. Measured and simulated co-pol and cross-pol normal reflection for an ungrounded array of TFH-objects: $d = 1.54$ mm, $\varepsilon_r = 4.55$, $d_1 = 4.5$ mm, $d_2 = 2.5$ mm, $\Omega = 90°$, $L_1 = 2.05$ mm, $L_2 = 0.55$ mm, $R = 1.066$ mm, $e = 0.35$ mm, $a = 0.15$ mm.

means that the incident electric field was perpendicular to the horizontal plane of incidence. The copolarized and cross-polarized reflected waves are then transmitted to the network analyzer by the receive antenna.

To correct the measured results from background influence, a measurement without the sample was first performed. Further, measurements have been done on two reference objects for calibration: A metallic flat square plate (160 mm × 160 mm) and a known polarizer. The later is an array of parallel conducting wires (diameter of wires: 0.5 mm; wire spacing: 1 mm) engraved on a 3-mm-thick metal backed substrate (Fig. 5).

After calibration, we estimate that the main measurement error in the reflection coefficient is related to the finite character of the measured sample, while in theory, we assumed an infinite array. Another error is related to the nonuniform character of the waves produced by the horn transmit antenna, while in theory we assumed a uniform plane incident wave. This error has a negligible effect on the results since the variations in amplitude and phase of the incident wave on the sample, are found to be less than ±0.2 dB and ±1.2°, respectively.

IV. Results and Discussion

All results presented thereafter are relative to the electromagnetic far-field scattered by arrays illuminated by a normally incident, linearly polarized plane wave. To check the validity of the code and the underlying theory, the program was applied to an array of rectangular, perfectly conducting patches and comparison were made with results found in the literature [17]. Very good agreement was observed between the results. Concerning arrays of complex shaped structures, comparisons were made with results simulated by numerical electromagnetics code (NEC-2) [18]. Good agreement was observed in the comparisons.

For the measurements, a 160 mm × 160 mm sample with 2205 (63 × 35) chiral structures has been machined using photolithography. The used chiral objects, called two-finger hand (TFH) objects are represented in Fig. 6. The electric incident field was taken parallel to the "fingers" of the TFH objects. Both copolarization and cross-polarization measurements were performed for the reflected electric field. Figs. 7 and 8 show the measured reflection coefficients of an ungrounded and a grounded array, respectively, with each compared to the computed curve. In each case, we note quite good agreement for both copolarization and cross-polarization. However, the amplitude of the measured reflection coefficient is smaller than the computed amplitude near resonance and, in less proportions, beyond resonance. This might be explained in part by insertion losses in the substrate and imperfections in the array (structures losses, air gap between the substrate and ground plane ...).

Numerical results, as well as experimental results, show clearly the phenomenon of polarization transformation by both grounded and ungrounded arrays of the planar chiral structures. As expected, for the cross-polarized wave, the maximum observed value of the reflection coefficient occurs around the first resonance frequency of a single particle, where $L = \lambda/2$. Here, L is the total length of the structure and λ is the wavelength in the effective medium (air + substrate). This phenomenon in chiral composites is well known and was discussed in detail in [19] and [20]. It is interesting to note that this depolarization happens even with the presence of a close ground plane.

Figs. 9 and 10 show, respectively, the calculated axial ratio (minor axis/major axis) of the polarization ellipse and

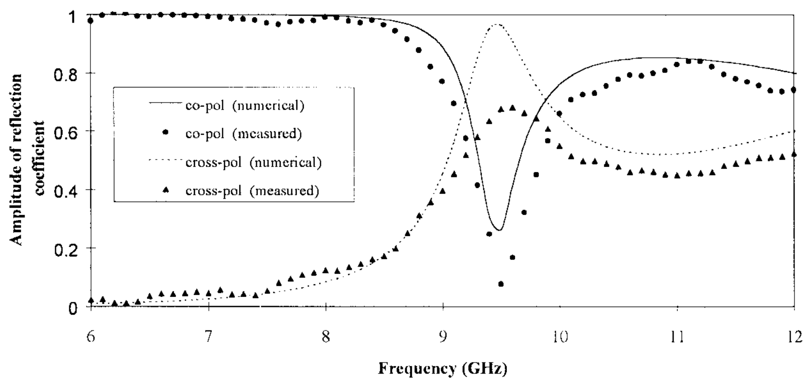

Fig. 8. Measured and simulated co-pol and cross-pol normal reflection for a grounded array of TFH-objects: $d = 1.54$ mm, $\varepsilon_r = 4.55$, $d_1 = 4.5$ mm, $d_2 = 2.5$ mm, $\Omega = 90°$, $L_1 = 2.05$ mm, $L_2 = 0.55$ mm, $R = 1.066$ mm, $e = 0.35$ mm, $a = 0.15$ mm.

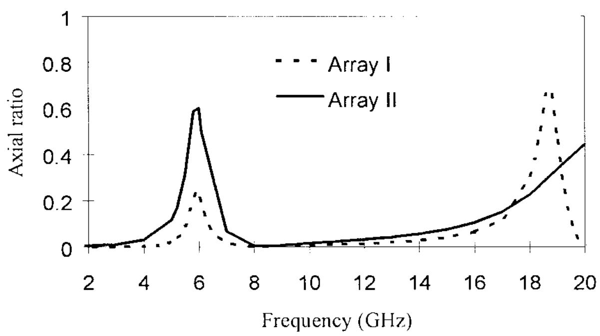

Fig. 9. Calculated axial ratio for two types of free-standing arrays of TFH objects, with $L_1 = 5$ mm, $L_2 = 2$ mm, $R = 3$ mm, $e = 1$ mm, $a = 0.1$ mm. Array I: $\Omega = 90°$, $d_1 = 10$ mm, $d_2 = 15$ mm. Array II: $\Omega = 60°$ and $d_1 = d_2 = 8$ mm.

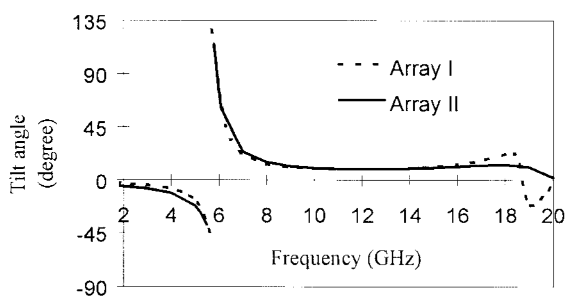

Fig. 10. Calculated tilt angle for two types of free-standing arrays of TFH objects, with $L_1 = 5$ mm, $L_2 = 2$ mm, $R = 3$ mm, $e = 1$ mm, $a = 0.1$ mm. Array I: $\Omega = 90°$, $d_1 = 10$ mm, $d_2 = 15$ mm. Array II: $\Omega = 60°$ and $d_1 = d_2 = 8$ mm.

tilt angle of major axis of the reflected wave from two ungrounded freestanding arrays, as a function of frequency. Arrays I and II are composed by the same TFH objects (Fig. 6) with different arrangements ($\Omega = 90°$ and $\Omega = 60°$, respectively). The electric incident field is taken parallel to the "fingers" of arrays elements. A notable "electromagnetic activity" (i.e., polarization transformation and rotation of the plane of polarization) is observed around 6 GHz, which approximately corresponds to the first resonance of a single particle of the array. We also note from Fig. 9, that by reducing the Ω angle, we increase the concentration of particles, which results in a broadened resonance bandwidth and an increased depolarization. Another resonance is observed at about 19 GHz. This second resonance, as discussed in [19], happens when the loop diameter equals to half the wavelength.

V. Conclusion

In this paper, the electromagnetic scattering by an infinite regular array of novel thin planar chiral structures, disposed on grounded or ungrounded dielectric slabs, was considered. A computer code based on the resolution of the vector integral equation by Galerkin method associated to quadratic segments has been developed.

Both experimental and computed results concerning arrays of planar chiral objects, show a notable electromagnetic activity (polarization transformation and rotation of the plane of polarization), in frequency bands centered on the resonance frequencies of a single element of the array. These results suggest a potential application of the studied slabs in the design of polarization transformers.

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Georges E. Couenon, photograph and biography not available at the time of publication.

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