Robust Adaptive Beamforming for Wide-Band, Moving, and Coherent Jammers via Uniform Linear Arrays

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***Abstract*—The problem of providing robustness to the conventional narrow-band uniform linear array configuration so as to handle wide-band and moving jammers is addressed here. This robustness is achieved via the use of derivative constraints in jammer directions. However, since the jammer directions are not known *a priori*, these constraints are incorporated with a maximum likelihood characterization of the so-called jammer subspace. This formulation does not need to assume the availability of signal-free observations, as stipulated in earlier work. Computer simulation results are presented, which show that the algorithms proposed here yield significantly better performance as compared to the previous algorithms of Gershman *et al.* [1], [2] and Hung and Turner [3] in a variety of situations required to handle wide-band, moving, and coherent jammers.**

***Index Terms*—Adaptive arrays, array signal processing.**

I. Introduction

One of the important applications of optimum adaptive beamforming is to suppress jamming signals, which would normally be strong enough to leak through sidelobes of the beam pattern. There is a very large body of literature on such optimum beamforming techniques. Typically, in a narrow-band optimum beamformer, the sensor outputs are combined by a weight vector to pass a desired (look direction) signal without distortion, while maximizing the attenuation of the interference signals or jammers (see [4] and references cited there).

The performance of such optimum beamformers, however, degrades significantly in the presence of jammers, which are wide-band and/or moving. The problem of rejecting wide-band jammers via antenna arrays is rendered particularly difficult when the signal of interest is also wide-band in nature. It is possible, in principle, to use wide-band arrays [5]–[7], which have an finite impulse response (FIR) filter associated with each sensor or use frequency-domain wide-band beamforming algorithms to provide additional degrees of freedom via space–time processing. However, such solutions are expensive in terms of hardware and software complexity. An interesting approach, proposed recently in [8], attempts to make the narrow-band beamformer reject wide-band as well as moving jammers via the use of certain derivative constraints in the jamming directions [1], [2]. Since these directions are not known *a priori*, the algorithm proposed by Gershman *et al.* [1] for implementing the required derivative constraints is based on a modification of the Hung–Turner [3] algorithm for estimating the so-called jammer-plus-noise subspace. The Hung–Turner algorithm assumes the interference-to-signal ratio to be large and uses the observed data vectors directly to characterize the jammer-plus-noise subspace. Alternatively, as suggested in [1] and [8], it may be possible to observe signal-free snapshots in some applications such as radar where an appropriate temporal windowing of data is used so as to have a more accurate characterization of the so-called jammer-plus-noise subspace.

These conditions, however, are restrictive and limit the usefulness of the algorithm in applications like communications and other passive listening systems. Also, in a moving jammer scenario, it is difficult to separate or categorize the observations as signal free or otherwise. Finally, many smart jammers would attempt to cancel the main beam signal via the so-called coherent jamming—a situation that is difficult to handle via conventional adaptive beamforming.

In this paper, we obtain a formulation of a robust optimum beamforming technique that does not suffer from these restrictions. More specifically, it is aimed that: 1) the signal-free jammer-plus-noise subspace (which is essentially orthogonal complement of the desired signal subspace) may be characterized from the observed data without any application restriction; 2) the array should be effective, both against wide-band as well as moving jammers; and 3) it should be able to handle coherent jammers as effectively as uncorrelated jammers without undue signal cancellation.

Our approach is based on a very elegant characterization of the orthogonal complement of the signal subspace for the uniform linear arrays due to Bresler and Macovski [9]. Such a characterization is robust, does not depend on assumptions regarding the input signal-to-noise ratio (SNR), and is equally applicable to active and passive systems, including the communications scenarios. Bresler *et al.* [10] used this characterization earlier to show that it is possible to steer deep notches in the directions of interferences, irrespective of their coherent or incoherent relationship with the desired signal. In this paper, we extend this formulation of the beamforming problem to handle wide-band and moving sources and jammers by using a narrow-band array.

Taking a cue from Gershman *et al.* [1], it is shown that this robustness can be achieved by incorporating derivative constraints with the maximum likelihood (ML) characterization

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of the jammer subspace. The resulting beamformer exhibits all the desirable properties and is shown to yield significant performance improvement, not only in a correlated jammer environment, but also for the classical uncorrelated case in the presence of wide-band and moving jammers.

The paper is organized as follows. Section II formulates the beamforming problem with derivative constraints assuming jammer directions to be known. These constraints are subsequently expressed in terms of the signal subspace spanned by jammers (also called here jammer subspace). Maximum likelihood estimation of this jammer subspace and solution of the beamforming problem are presented in Section III. Section IV briefly comments on some implementation, complexity, and performance issues. Simulation studies are presented in Section V, where it is demonstrated that the technique proposed here yields significant performance benefits in a variety of situations incorporating wide-band and moving jammers.

II. Problem Formulation

Consider a uniform linear array of M sensors. Let a desired signal impinge on the array from a known direction θ_0 along with $P-1$ jammer signals from unknown directions $\{\theta_1, \theta_2, \cdots, \theta_{P-1}\}$, respectively. Therefore, the tth snapshot of the $M\times 1$ received vector at the sensors can be written as

$$\boldsymbol{y}(t) = \boldsymbol{a}(\theta_0)s_0(t) + \tilde{\boldsymbol{A}}\tilde{\boldsymbol{s}}(t) + \boldsymbol{n}(t) \qquad t = 0, 1, 2, \cdots \quad (1)$$

where $s_0(t)$ is the scalar signal waveform of the desired signal. Here the $\tilde{\boldsymbol{s}}(t) \triangleq [s_1(t), s_2(t), \cdots, s_{P-1}(t)]^T$ is the $(P-1)\times 1$ vector of jammer waveforms and $\tilde{\boldsymbol{A}}$ is an $M\times(P-1)$ array matrix of jammer direction array steering vectors given by

$$\tilde{\boldsymbol{A}} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_{P-1})]^T \quad (2)$$

where the $M\times 1$ array steering vector $\boldsymbol{a}(\theta)$ is given by

$$\boldsymbol{a}(\theta) = [1, \exp(j\tau_\theta), \cdots, a(j(M-1)\tau_\theta)]^T \quad (2a)$$

with $\tau_\theta = \omega_0\Delta\sin\theta/c$, Δ being the sensor spacing, and c denoting the velocity of propagation. The $M\times 1$ vector $\boldsymbol{n}(t)$ denotes the additive sensor noise in the tth snapshot, assumed here to be spatially white and Gaussian with power p_N at each sensor. Define

$$\boldsymbol{A} = [a(\theta_0), \tilde{\boldsymbol{A}}] \quad (3)$$

such that (1) can be written as

$$\boldsymbol{y}(t) = \boldsymbol{A}\begin{bmatrix} s_0(t) \\ \tilde{\boldsymbol{s}}(t)\end{bmatrix} + \boldsymbol{n}(t). \quad (3a)$$

Consider next the output of the beamformer associated with a weight vector $\boldsymbol{w}$. This can be written as

$$\boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{y}(t) = \boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{a}(\theta_0)s_0(t) + \boldsymbol{w}^{\boldsymbol{H}}(\tilde{\boldsymbol{A}}\tilde{\boldsymbol{s}}(t) + \boldsymbol{n}(t)) \quad (4)$$

or more succinctly as

$$\boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{y}(t) = \boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{a}(\theta_0)s_0(t) + e(t) \quad (4a)$$

where $e(t)$ denotes the undesired contribution to the output due to jammers and noise. It is desired to form deep nulls in the directions of *moving* and *wide-band* interferences regardless of their mutual correlation or correlation with the desired source.

It has been established that flat nulls provide robustness to wide-band [8] as well as moving jammers [1]. It is this property of the "flat" or "broad" nulls which is exploited in the sequel to formulate a beamforming problem, which enables elimination of wide-band and moving jammers by using a narrow-band array configuration. This flatness can be achieved by incorporating derivative constraints in the directions of the jammers. This implies, therefore, that the robustness is achieved by paying some price in terms of an increased number of required sensors.

To start with, let us assume that the jammer directions are known. The beamforming problem of interest here can then be formulated as follows:

$$\min_w E\left[e(t)^H e(t)\right] \quad (5a)$$

subject to the constraints

$$\boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{a}(\theta_0) = 1 \quad (5b)$$

$$\boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{a}(\theta_p) = 0 \qquad p = 1, 2, \cdots, P-1 \quad (5c)$$

and

$$\left.\frac{\boldsymbol{d}^{\boldsymbol{m}}(\boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{a}(\theta))}{d\tau^{\boldsymbol{m}}}\right|_{\theta=\theta_p} = 0 \qquad m = 1, 2, \cdots, q; \quad p = 1, 2, \cdots, P-1. \quad (5d)$$

Combining (2) and (3) with (5c) and (5d) these two constraint sets can be written more compactly as

$$\boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{C}^{\boldsymbol{m}}\boldsymbol{a}(\theta_p) = 0 \qquad m=0, 1, \cdots, q; \quad p=1, 2, \cdots, P-1 \quad (6)$$

where

$$\boldsymbol{C} = \mathrm{diag}\{0, 1, \cdots, M-1\} \quad (6a)$$

is an $M\times M$ diagonal matrix of space coordinates of the sensors. From (6), it follows that the desired weight vector $\boldsymbol{w}$ lies in the orthogonal complement to the $(P-1)(q+1)$-dimensional subspace spanned by vectors $\boldsymbol{C}^{\boldsymbol{m}}\boldsymbol{a}(\theta_p)$, $m = 0, 1, \cdots, q$, $p = 1, 2, \cdots, P-1$. It is easy to see then that the weight vector $\boldsymbol{w}$, which satisfies the constraints (5c) and (5d) will equivalently satisfy the following identity:

$$\boldsymbol{w} = \boldsymbol{D}\boldsymbol{w} \quad (7)$$

where $\boldsymbol{D}$ is the $M\times M$ projection matrix onto the orthogonal complement of the subspace spanned by the vectors $\boldsymbol{C}^{\boldsymbol{m}}\boldsymbol{a}(\theta_p)$, $m = 0, 1, \cdots, q$, $p = 1, 2, \cdots, P-1$, and is given by

$$\boldsymbol{D} = \boldsymbol{I} - \boldsymbol{F}\left(\boldsymbol{F}^{\boldsymbol{H}}\boldsymbol{F}\right)^{-1}\boldsymbol{F}^{\boldsymbol{H}}. \quad (8a)$$

Here $\boldsymbol{F}$ is a matrix of size $M\times(q+1)(P-1)$ given by

$$\boldsymbol{F} = \left[\tilde{\boldsymbol{A}}, \boldsymbol{C}\tilde{\boldsymbol{A}}, \cdots, \boldsymbol{C}^{\boldsymbol{q}}\tilde{\boldsymbol{A}}\right] \quad (8b)$$

with $\tilde{\boldsymbol{A}}$ given in (2). *It is clear that with* $(P-1)$ *jammers and* q *derivative constraints, the array would need* $M > (q+1)(P-1)+1$ *sensors.*

It is straightforward to show that the solution of the optimization problem (5) can be written as [1]

$$\boldsymbol{w} = \frac{\boldsymbol{D}\boldsymbol{a}(\theta_0)}{\boldsymbol{a}(\theta_0)^{\boldsymbol{H}}\boldsymbol{D}\boldsymbol{a}(\theta_0)}. \tag{9}$$

This solution, unfortunately, requires knowledge of the jammer directions as embedded in $\boldsymbol{D}$. In the next section, we consider a method for the maximum likelihood characterization of this matrix from the observed data, even when the latter contains both the desired signal and noise.

III. Characterization of the Jammer Subspace and Solution to the Beamforming Problem

A. Jammer Subspace Characterization

Bresler *et al.* [10] have suggested a method for estimating the signal-free snapshot vector from the observed data vector. It has been shown in [9] that for the array matrix $\boldsymbol{A}$ defined in (3), there exists a unique generating polynomial $b(z)$ of the form

$$b(z) = b_0 z^P + b_1 z^{P-1} + \cdots + b_P \tag{10a}$$

whose P roots are $\{\exp(-j\tau_{\theta i}),\, 0 \le i \le P-1\}$. Define the vector $\boldsymbol{b}$ associated with the coefficients of $b(z)$ as follows:

$$\boldsymbol{b} = [b_0,\, b_1,\, \cdots,\, b_P]^T. \tag{10b}$$

Lemma [9]: Let $\boldsymbol{A}$ be a Vandermonde matrix as defined in (3), with $\{\theta_i,\ 0 \le i \le P-1\}$ distinct, and let $b(z)$ be its generating polynomial. Then $\boldsymbol{R}^{\perp}(\boldsymbol{A})$, the orthogonal complement to the space spanned by the columns of $\boldsymbol{A}$, is spanned by the columns of $\boldsymbol{B}$ given as $M \times (M-P)$-dimensional Toeplitz matrix

$$\boldsymbol{B} = \begin{bmatrix} \mathbf{b}_P^* & \cdot & \cdot & \mathbf{0} \\ \mathbf{b}_{P-1}^* & \mathbf{b}_P^* & \cdot & \mathbf{0} \\ \cdot & \cdot & \cdot & \mathbf{b}_P^* \\ \mathbf{b}_0^* & \mathbf{b}_1^* & \cdot & \cdot \\ \mathbf{0} & \mathbf{b}_0^* & \cdot & \cdot \\ \mathbf{0} & \cdot & \cdot & \mathbf{b}_0^* \end{bmatrix}. \tag{11}$$

In other words, we can write the following projector relationships:

$$\boldsymbol{I} - \boldsymbol{A}\left(\boldsymbol{A}^{\boldsymbol{H}}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^{\boldsymbol{H}} = \boldsymbol{B}\left(\boldsymbol{B}^{\boldsymbol{H}}\boldsymbol{B}\right)^{-1}\boldsymbol{B}^{\boldsymbol{H}}. \tag{12}$$

□

Clearly, a knowledge of $\boldsymbol{B}$ is sufficient to characterize the so-called noise subspace, which is essentially the orthogonal complement of the signal-plus-jammer subspace. An ML method of estimating the vector $\boldsymbol{b}$ (called IQML) and, hence, constructing $\boldsymbol{B}$ from one or more observed data vectors is also outlined in [10].

However, we may recall that our objective here is to characterize and estimate the orthogonal complement only of the jammer subspace, spanned by the columns of $\tilde{\boldsymbol{A}}$. This can be done as follows.

A Toeplitz matrix $\tilde{\boldsymbol{B}}$ of dimension $M \times (M-P+1)$ for $\boldsymbol{R}^{\perp}(\tilde{\boldsymbol{A}})$, similar to $\boldsymbol{B}$ of (11), can be constructed by replacing $b(z)$ with the $(P-1)$ degree polynomial

$$\tilde{b}(z) = \tilde{b}_0 z^{P-1} + \tilde{b}_1 z^{P-2} + \cdots + \tilde{b}_{P-1} \tag{13}$$

whose roots are given by $\{\exp(-j\tau_{\theta i}),\, 1 \le i \le P-1\}$. Hence, the columns of $\tilde{\boldsymbol{B}}$ are orthogonal to all but the direction vector of the desired signal, i.e.,

$$\begin{aligned} \tilde{\boldsymbol{B}}^{\boldsymbol{H}}\boldsymbol{a}(\theta) &\neq 0 \qquad \theta = \theta_0 \\ &= 0 \qquad \theta = \theta_i \quad i = 1,\, 2,\, \cdots,\, P-1. \end{aligned} \tag{14}$$

The $\tilde{\boldsymbol{b}}$ can be calculated from $\boldsymbol{b}$ in a least-square sense as in [10]. Let $b_1(z)$ be polynomial of degree one, whose root is $\exp(-j\tau_{\theta_0})$, i.e.,

$$b_1(z) = z - \exp(-j\tau_{\theta_0}). \tag{15}$$

Then it has been shown in [10], that $\tilde{\boldsymbol{b}}$ can be obtained from $\boldsymbol{b}$ via

$$\tilde{\boldsymbol{b}} = \left(\boldsymbol{B_1}^{\boldsymbol{H}}\boldsymbol{B_1}\right)^{-1}\boldsymbol{B_1^H}\boldsymbol{b} \tag{16}$$

where $\boldsymbol{B_1}$ is a $(P+1) \times P$ dimensional Toeplitz matrix constructed from $b_1(z)$, in the same manner as $\boldsymbol{B}$ was constructed from $b(z)$ in (12).

B. Solution of the Beamforming Problem

The main idea here is to use the ML estimate of the span of orthogonal complement of $\tilde{\boldsymbol{A}}$ defined in (2) for use in (9), in order to place flat nulls in the unknown jammer directions. Since

$$\boldsymbol{R}(\tilde{\boldsymbol{A}}) = \boldsymbol{R}^{\perp}(\tilde{\boldsymbol{B}}) \tag{17}$$

it follows that

$$\boldsymbol{I} - \boldsymbol{P}_{\tilde{\boldsymbol{A}}} = \boldsymbol{P}_{\tilde{\boldsymbol{B}}} \tag{18a}$$

where

$$\boldsymbol{P}_{\tilde{\boldsymbol{A}}} = \tilde{A}\left(\tilde{\boldsymbol{A}}^{\boldsymbol{H}}\tilde{\boldsymbol{A}}\right)^{-1}\tilde{\boldsymbol{A}}^{\boldsymbol{H}} \tag{18b}$$

and

$$\boldsymbol{P}_{\tilde{\boldsymbol{B}}} = \tilde{\boldsymbol{B}}\left(\tilde{\boldsymbol{B}}^{\boldsymbol{H}}\tilde{\boldsymbol{B}}\right)^{-1}\tilde{\boldsymbol{B}}^{\boldsymbol{H}}. \tag{18c}$$

Similarly, it can be shown that

$$\boldsymbol{R}(\boldsymbol{C}^{\boldsymbol{m}}\tilde{\boldsymbol{A}}) = \boldsymbol{R}^{\perp}\left(\boldsymbol{C}^{-\boldsymbol{m}}\tilde{\boldsymbol{B}}\right) \tag{19a}$$

and

$$\boldsymbol{I} - \boldsymbol{P}_{(\boldsymbol{C}^{\boldsymbol{m}}\tilde{\boldsymbol{A}})} = \boldsymbol{P}_{(\boldsymbol{C}^{-\boldsymbol{m}}\tilde{\boldsymbol{B}})}. \tag{19b}$$

It follows that the span of the columns of $\boldsymbol{F}$ defined in (8b) can also be written as

$$\begin{aligned} \mathrm{span}(\boldsymbol{F}) &= \mathrm{span}([\boldsymbol{P}_{\tilde{A}},\, \boldsymbol{P}_{C\tilde{A}},\, \cdots,\, \boldsymbol{P}_{C^q\tilde{A}}]) && (20a) \\ &= \mathrm{span}([\boldsymbol{I} - \boldsymbol{P}_{\tilde{\boldsymbol{B}}},\, \boldsymbol{I} - \boldsymbol{P}_{C^{-1}B},\, \cdots,\, \boldsymbol{I} - \boldsymbol{P}_{C^{-q}\tilde{B}}]). && (20b) \end{aligned}$$

$\boldsymbol{D}$ defined in (8a) can, therefore, be equivalently calculated from (20). Define

$$\boldsymbol{Q} = [\boldsymbol{I} - \boldsymbol{P}_{\tilde{\boldsymbol{B}}},\, \boldsymbol{I} - \boldsymbol{P}_{C^{-1}B},\, \cdots,\, \boldsymbol{I} - \boldsymbol{P}_{C^{-q}\tilde{B}}] \tag{21}$$

and

$$\boldsymbol{U} = \boldsymbol{I} - \boldsymbol{Q}\left(\boldsymbol{Q}^{\boldsymbol{H}}\boldsymbol{Q}\right)^{-1}\boldsymbol{Q}^{\boldsymbol{H}} \tag{22a}$$

which is equivalent to $\boldsymbol{D}$ in the sense that

$$\text{span}\{\boldsymbol{D}\} = \text{span}\{\boldsymbol{U}\} \tag{22b}$$

and, therefore, $\boldsymbol{U}$ serves as projection matrix onto the same subspace as $\boldsymbol{D}$.

As discussed earlier, $\boldsymbol{b}$, $\tilde{\boldsymbol{b}}$ and, hence, $\boldsymbol{U}$ can be estimated from one or more observed data snapshots. Since in a moving jammer situation, the background scenario changes with time, it is desirable to update $\boldsymbol{b}$ and $\boldsymbol{U}$ with each snapshot. $\boldsymbol{U}$ then becomes a function of t. This update can be carried out via the IQML algorithm discussed in [9] or its variants, such as the IQML with norm constraint [11] and, more recently, the modified IQML (MIQML) proposed by Kristensson *et al.* [12]. The weight vector defined in (9), is now given by

$$\boldsymbol{w}(t) = \frac{\boldsymbol{U}(t)\boldsymbol{a}_s}{\boldsymbol{a}_s^{\boldsymbol{H}}\boldsymbol{U}(t)\boldsymbol{a}_s} \tag{23}$$

in terms of the matrix $\boldsymbol{U}$.

IV. Implementation Issues and Remarks

A. Computational Complexity and Related Issues

The implementation of (23) in real time would require updating of $\boldsymbol{b}$, $\tilde{\boldsymbol{b}}$, and the resulting $\boldsymbol{U}$ with every observation, especially for the case of moving jammers. The computational complexity associated with the calculation of $\tilde{\boldsymbol{b}}$ is $O((M+1)(3M+4)(M-P)/2)$ flops per iteration [13]. The number of iterations required for this method to converge is not fixed, making the computational complexity of this technique to be somewhat variable. But our experience shows that three to five iterations are usually sufficient for convergence. Calculation of the weight vector requires an additional $O(M^3q)$ multiplications and additions and $O(Mq)$ square root and division operations. In contrast the complexity of the robust sample matrix inversion (SMI) [2] and the robust Hung–Turner algorithms are $O(N^2Mq)$ and $O(NM^2q)$, respectively, making our method computationally more complex. However, the performance gain justifies the additional computational cost. Also, the complexity is well within the ability of modern day digital signal processor (DSP) devices for moderate values of M, P, and q.

In some applications, it may not be possible to tolerate the delay incurred in completing the above computation within the current snapshot interval. Assuming that the scenario change is insignificant over two successive snapshots, it is possible to implement a suboptimal beamformer where the weight vector for the current snapshot is computed from the immediate past snapshot. The simulation results reported in the next section show that the performance loss in doing so is indeed marginal, even in the case of moving jammers.

B. Remarks on Performance of the Proposed Algorithm

It may be noted that the approach of Hung and Turner [3] and Gershman *et al.* [1] for estimation of the jammer subspace is *ad-hoc* and heuristic. As a consequence, their simulation studies show that even though the use of derivative constraints leads to improved performance in terms of the SINR for both wide-band and moving jammers, the performance remains well below the optimal value. This may be attributed to a noisy and imperfect estimation of the jammer subspace even when the weight vector is calculated on the basis of a signal-free snapshot. This noisy estimate prevents the derivative constraints to be obeyed satisfactorily. The situation is expected to become considerably worse when such signal-free snapshots are not available.

The algorithm proposed in this paper, on the other hand, is based on ML estimation of the jammer subspace (within the limitations of the IQML algorithm). The approach outlined here may, therefore, be considered to be *much closer to optimal* in this sense. The simulation studies reported in Section V indeed demonstrate this superiority of the proposed algorithm, even though it remains a heuristic so far as wide-band jamming in concerned. Finally, the approach suggested here will work equally well for correlated (smart) as well as uncorrelated jammers, whereas the approach of [1] may be expected to suffer in a correlated jamming scenario.

C. Limitations

We now briefly discuss a few limitations of the approach proposed in this paper. First and foremost, the algorithm proposed here is applicable only to uniform linear arrays. This, of course, is a limitation associated with the IQML formulation. It may be mentioned here, though, that the approach suggested here can be extended to the case of uniform planar arrays. Clark and Scharf [9], [15] have, in fact, extended the simple IQML algorithm to the case of 2-D damped harmonic signals, which can be easily adapted to direction of arrival (DOA) estimation via uniform planner arrays. Their approach can, therefore, be modified to include the derivative constraints in the manner proposed in this paper.

It may be mentioned here that deviations of the array from the uniform linear array (ULA) configuration, or the assumption of perfectly decoupled omnidirectional antenna elements or nonplanarity of the wavefront (say from a near-field jammer) may also adversely affect the performance of the proposed algorithm. Some of these issues have been taken up in our simulation studies and, fortunately, there appears to be a generally graceful degradation in performance in the presence of many of these imperfections.

Next, a few words of caution are in order when using the IQML algorithm. IQML estimates have been shown to be biased and inconsistent [13], [14]. Also, sometimes the number of iterations required for convergence may be large. Recently proposed modifications, viz., IQML with the norm constraint [11] and modified IQML [12] largely mitigate some of these deficiencies.

Finally, we would like to point out that use of “spatial only” processing to deal with wide-band sources via derivative

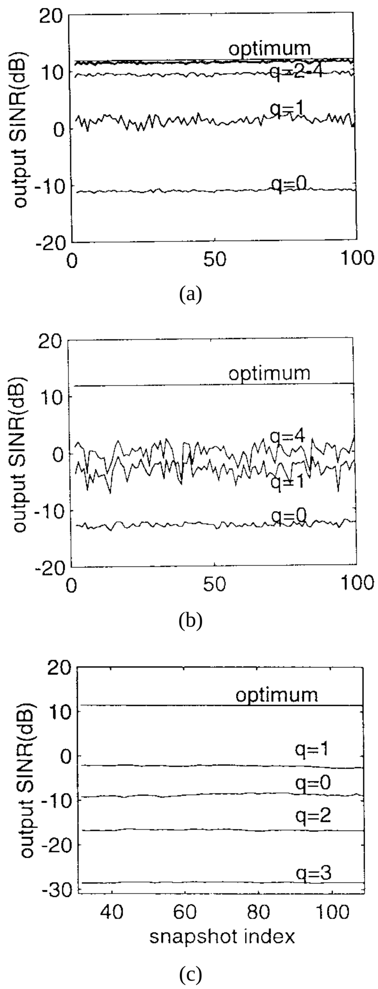


Fig. 1. Output SINR versus snapshot index in the presence of a wide-band jammer at 40° with 5% bandwidth. (a) Robust IQML beamformer. (b) Robust HT beamformer. (c) Robust SMI beamformer.

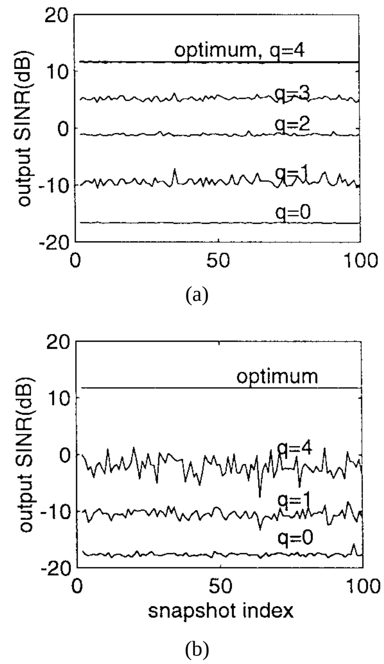


Fig. 2. Output SINR versus snapshot index in the presence of a wide-band jammer at 40° with 20% bandwidth. (a) Robust IQML beamformer. (b) Robust HT beamformer.

constraints, is associated with a cost. We are clearly decreasing the available degrees of freedom to satisfy these additional constraints, which, in turn, would affect performance. In contrast, use of space-time processing is more suited to wide-band scenarios and would yield superior performance at the cost of some increased processing complexity.

In the next section, we present the results of simulation studies, which support the remarks and observations made in this section.

V. Simulations Studies

Simulation studies have been carried out separately for both wide-band and moving jammer scenarios. For ease of comparison and also for bringing out the salient features of our algorithm, several scenarios discussed in the sequel have been chosen for study. In each of these cases, ambient noise at each sensor is assumed to be spatially uncorrelated and to have a reference power level of unity ($p_N = 1$ or 0 dB) at each sensor. The desired signal is also assumed to have a power of 0 dB with one or more jammers, each of 30-dB strength introduced as relevant. A uniform linear array of 16 sensors ($M = 16$) was considered. For the robust SMI algorithm 32 snapshots [2] are used to estimate the covariance matrix. An optimal window length of six (as per the guidelines suggested in [17]) is used to study the performance of the robust Hung–Turner (HT) algorithm.

The performance of the algorithms has been evaluated by computing the output SINR, which can be written as

$$\text{SINR}(t) = \frac{p_s|\boldsymbol{w}(t)^H\boldsymbol{a}_s|^2}{\boldsymbol{w}(t)^H\boldsymbol{R}(t)\boldsymbol{w}(t)} \tag{24}$$

where

$$\boldsymbol{R}(t) = E\left[\tilde{\boldsymbol{y}}(t)\tilde{\boldsymbol{y}}(t)^H\right] \tag{24a}$$

with

$$\tilde{\boldsymbol{y}}(t) = \tilde{\boldsymbol{A}}\tilde{\boldsymbol{s}}(t) + \boldsymbol{n}(t). \tag{24b}$$

The optimal SINR defined by

$$\text{SINR}_0(t) = p_s\boldsymbol{a}_s^{\boldsymbol{H}}\boldsymbol{R}^{-\mathbf{1}}(t)\boldsymbol{a}_s \tag{25}$$

serves as a benchmark for performance comparison [16].

A. Wide-Band Jammers

The following scenarios are typical of those studied.

1) Single uncorrelated jammer at 40° with a bandwidth of 5% of the center frequency.
2) Single wide-band jammer at 40° with a bandwidth of 20% of center frequency.
3) Single uncorrelated jammer at 5° (i.e., close to the desired signal direction) with a bandwidth of 5% of the center frequency.
4) Two uncorrelated wide-band jammers (5% bandwidth) at 40° and 20°, respectively.

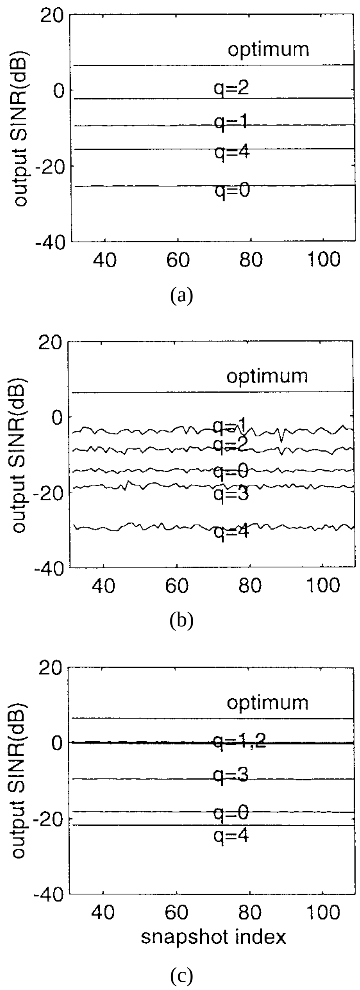


Fig. 3. Output SINR versus snapshot index in the presence of a wide-band jammer at 5° with 5% bandwidth. (a) Robust IQML beamformer. (b) Robust HT beamformer. (c) Robust IQML-norm beamformer.

5) Two wide-band jammers correlated with the desired signal, (5% bandwidth) at 40° and 20°, respectively.
6) Single uncorrelated jammer at 40° with a bandwidth of 5% of the centre frequency with random perturbations in sensor positions.

In each case, the desired source has been taken to lie in the broadside direction.

Figs. 1–6 summarize the results obtained for these scenarios by plotting the output SINR as a function of the snapshot index, both for the algorithm proposed here and that of [1]. The following general conclusions can be drawn from these results.

The case of $q = 0$ (i.e., without any derivative constraints) yields equally poor SINR ($= -11$ to -13 dB for the 5% bandwidth case and degrading to about -17 dB for the 20% bandwidth case, against a benchmark optimal value of 12 dB) for both sets of algorithms. Also shown in Fig. 1(c) is the performance of the robust SMI algorithm proposed recently [2], which appears to have a similar performance for $q = 0$. However, as the derivative constraints are introduced, there is a phenomenal improvement in the performance of our algorithm over those of Gershman *et al.* [1], [2], viz., over both the robust HT and the robust SMI. For the 5% bandwidth case

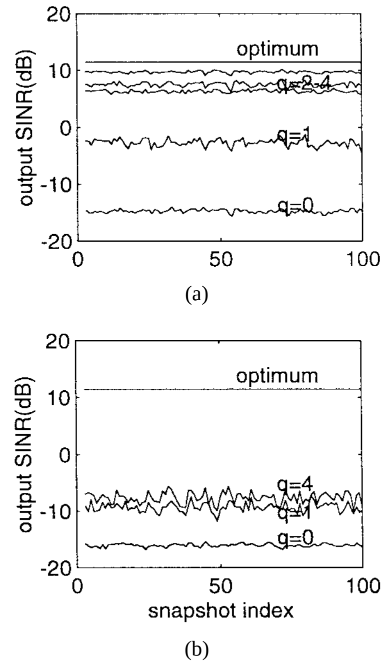
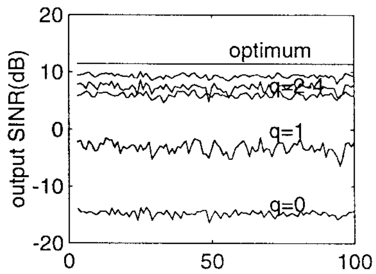


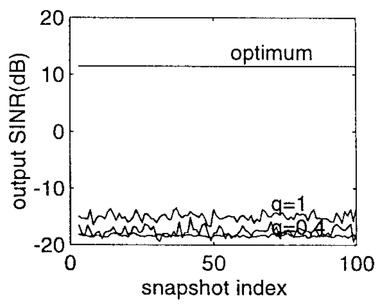
Fig. 4. Output SINR versus snapshot index in the presence of two uncorrelated wide-band jammer with 5% bandwidth. (a) Robust IQML beamformer. (b) Robust HT beamformer.

(Fig. 1), $q = 2$ yields a near optimum performance in our case, with an improvement of nearly 7 dB over the robust Hung–Turner (HT) [1] algorithm. In contrast, the performance of the robust SMI [2] method improves initially with the first derivative constraint but deteriorates below the unconstrained algorithm for higher values of q. This trend was observed almost consistently in all simulation studies with robust SMI and, hence, comparison with SMI has not been considered any further. Higher bandwidth (Fig. 2) seems to require a somewhat higher value of q to approach the optimal performance, but performance of our approach remains significantly better for all values of q. *This is a significant observation, since* q, *the number of derivative constraints has a direct bearing on the minimum number of sensors required in the array.* Further, the presence of more than one wide-band jammer causes a significant degradation in the performance of robust HT approach, whereas it has a relatively minor impact on the performance of the algorithm proposed here (Fig. 4).

Fig. 3 shows the behavior of the various algorithms when a jammer lies close to the desired signal direction. In all cases, it is seen that the use of derivative constraints causes some initial improvement in performance (mostly with $q = 1$ or sometimes with $q = 1, 2$), but soon the performance begins to degrade rather than improving with higher values of q. This is because the broadening of the null (and, hence, of bandwidth) achieved via derivative constraints now also tends to eliminate the desired signal. This observation also points to a general weakness that may be expected to be shared by all robust algorithms based on derivative constraints. The unconstrained beamformers in all cases, of course, have very poor performance for closely spaced jammers. In fact, at low SNR's and closely spaced jammers, the unconstrained IQML beamformer is seen to be much worse than the corresponding HT algorithm.

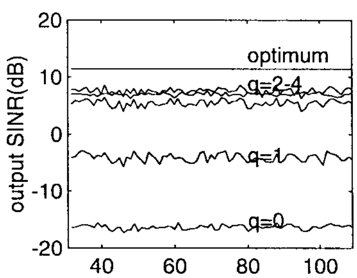


(a)

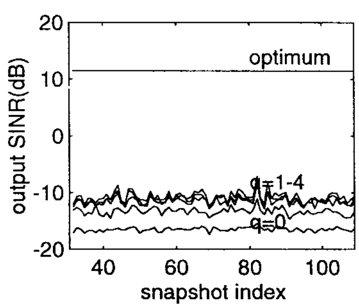


(b)

Fig. 5. Output SINR versus snapshot index in the presence of two correlated wide-band jammer with 5% bandwidth. (a) Robust IQML beamformer. (b) Robust HT beamformer.



(a)

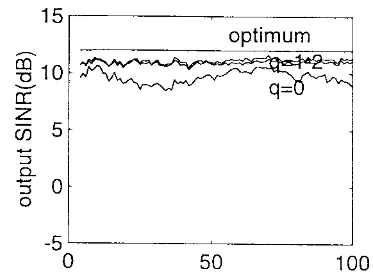


(b)

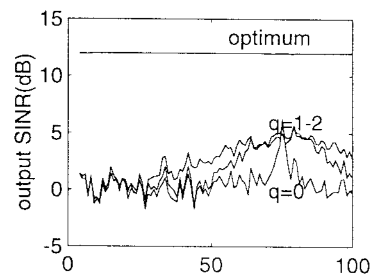
Fig. 6. Study of perturbations in array sensor position: output SINR versus snapshot index in the presence of a wide-band jammer at 40° with 5% bandwidth. (a) Robust IQML beamformer. (b) Robust HT beamformer.

This may be ascribed partially to the bias and inconsistency of the IQML algorithm. In fact, a considerable improvement is observed if IQML is replaced by its modification such as the IQML with the norm constraint [11]

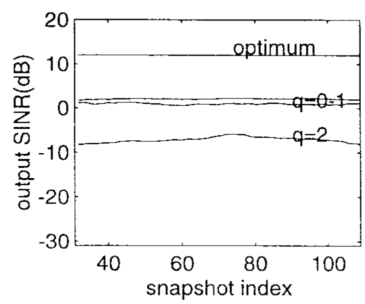
Next, it is seen that the robust HT algorithm completely breaks down in the presence of correlated jammers, whereas these seem to have little effect on the performance of the proposed beamformer (Fig. 5).



(a)



(b)



(c)

Fig. 7. Output SINR versus snapshot index in the presence of two uncorrelated moving jammer moving with $20^\circ + 0.1^\circ t$ and $40^\circ - 0.1^\circ t$ profile. (a) Robust IQML beamformer. (b) Robust HT beamformer. (c) Robust SMI beamformer.

Finally, Fig. 6 shows the effect of perturbations in the array manifold. The sensor locations were perturbed randomly around their nominal positions using a uniform distribution over $\pm 10\%$ of sensor spacings. With $q = 0$, the SINR obtained from both the algorithms is of the order of -17 dB, indicating a loss of about 6–7 dB in performance over the unperturbed array. Incorporation of the derivative constraints leads to significant improvement in the performance of the proposed algorithm, whereas the robust HT algorithm fails to yield similar results.

B. Moving Jammers

We consider two moving jammers with their azimuthal trajectories given by

$$\theta_1(t) = 20 + 0.1^\circ t; \qquad \theta_2(t) = 40 - 0.1^\circ t \qquad (26)$$

where t is the snapshot index. Again experiments have been carried out for the following scenarios:

1) jamming signals and the desired signal mutually uncorrelated;
2) same as 1) with correlated jammers;

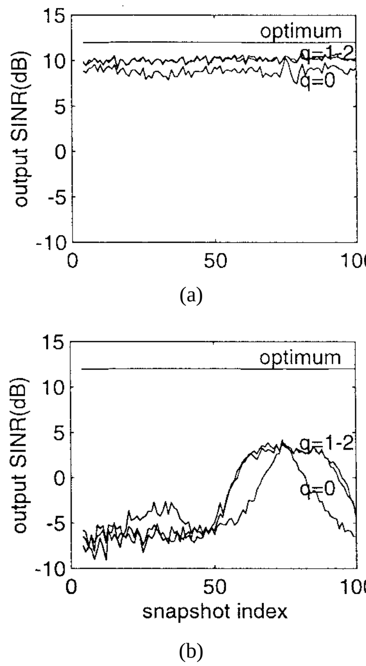


Fig. 8. Output SINR versus snapshot index in the presence of two correlated moving jammer moving with $20° + 0.1° t$ and $40° - 0.1° t$ profile. (a) Robust IQML beamformer. (b) Robust HT beamformer.

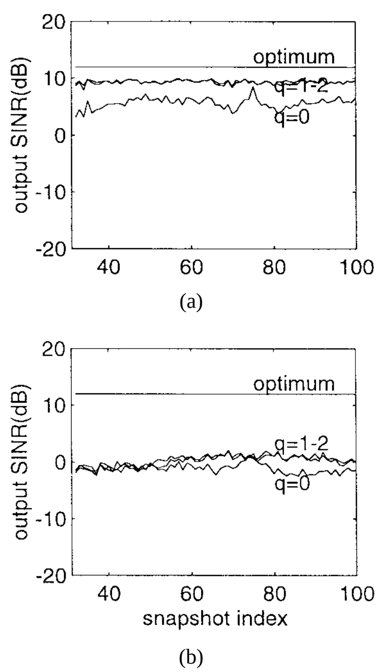


Fig. 9. Study of perturbations in array sensor position: output SINR versus snapshot index in the presence of two uncorrelated moving jammer moving with $20° + 0.1° t$ and $40° - 0.1° t$ profile. (a) Robust IQML beamformer. (b) Robust HT beamformer.

3) study of the effect of perturbations in sensor position.

The results are summarized in Figs. 7–9. The improvements obtained by our algorithms are remarkable even for the uncorrelated case. With one or two derivative constraints, the performance is seen to be nearly optimal. This is quite unlike the algorithms of [1] and [2], which yield SINR considerably below the optimal value, even when "averaging over dimension" is carried out by taking an optimal window [17] of size six. Moreover, the robust SMI algorithm [2] yields only marginal performance improvement on incorporation of the first derivative constraint and, to our surprise, causes performance degradation for higher values of q, just as observed in the wide-band case. Further, the robust HT algorithm [1] once again breaks down completely for correlated jammers, whereas there is little or no effect on the algorithm proposed here. Although not shown here, it is seen that in both cases 1) and 2), *use of the immediate past snapshot for calculation of the weight vector instead of current snapshot, appears to cause no degradation.* Finally, the robust IQML is seen to be much superior compared to the robust HT in the presence of array manifold perturbations.

VI. Conclusion

It is shown that for uniform linear arrays, it is possible to reject wide-band, moving, and correlated jammers, even when the jammer directions are unknown and separate signal-free observations are not available. The robust IQML beamforming algorithm described here yields considerably superior performance as compared to that proposed by Gershman *et al.* [1] in all situations of wide-band and moving jammers considered here.

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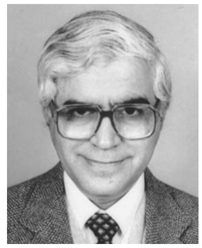

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