Application of On-Surface MEI Method on Wire Antennas

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***Abstract*—In this paper, the formulas of on-surface measured equation of invariance (OSMEI) for wire antennas are derived. The unknowns of each node on the antenna surface are expressed by the vector potential function and surface current density. The unknowns in the vicinity of each node are coupled in a linear equation and the coefficients of the linear equation are determined by the measured equation of invariance (MEI) method. The final impedance matrix obtained by the OSMEI is a highly sparse matrix. It demonstrates that the currents on thin wire antennas may also be solved by a differential equation-based formulation in addition to the conventional integral equations.**

***Index Terms*—Absorbing boundary conditions, wire antennas.**

I. Introduction

As a fast algorithm, the measured equation of invariance (MEI) method has been successfully used in electromagnetic computational problems [1]–[7]. The development of the MEI method makes it possible to solve very large-scale scattering problems [8] that are not so practical for method of moment (MoM). However, there is difficult to apply MEI with the frequency-domain (FD) mesh in the thin wire antenna problems. The reason is that the thin wire structures cannot be accurately simulated by the FD mesh in their radial directions. In order to overcome this difficulty, one of the possible methods is to use the recently developed on-surface MEI method (OSMEI) [9]–[10] where the FD mesh is no longer required so that the thin wire antennas can be accurately modeled by the nodes sampled on the wire surface. However, it is found that the OSMEI formulas for the two-dimensional (2-D) scattering problems [9], [10], which describe the relationship of electric field and magnetic field $\vec{H}$, are not numerically suitable for the thin wire antennas, hence, the relationship of vector potential function $\vec{A}$ and its normal derivative $\partial\vec{A}/\partial n$ is formulated to meet our need in this paper. Following the procedure used for deriving the OSMEI formulas for the scattering problems in [10], the novel formulas for the thin wire antennas are deduced. These formulas describe such a fact that there exists a linear relationship of the vector potential functions and surface current densities between each node and its neighboring nodes, and the MEI method can be used to numerically determine such a relationship. Since the real current at the feed-region and the ends-region has a very rapid variation [12], [13], a set of metrons satisfying the end conditions of the zero currents should include high-order sinusoidal functions. Since the unknown at each node is only linked with those of its neighbors, a highly sparse matrix are generated. Furthermore, if we let the vector potentials in the MEI equations satisfy the real boundary conditions produced by a real excitation source, the current distributions of the wire antennas can be directly obtained by solving the sparse matrix.

The MoM is the most efficient numerical methods to solve wire antenna problems. This paper shows that the OSMEI method can also work well in this region with a highly sparse matrix rather than a dense matrix in the MoM. Although time-saving of the OSMEI approach compared with the MoM is not obvious in a single linear antenna problem in this paper, it is expected that great time-saving can be obtained in large-scale wire arrays. It should be noted that the OSMEI also uses the same approximation of the Green's function traditionally used by the MoM for the thin wire antennas, i.e., $|\vec{r}-\vec{r'}| \approx [a^2+(z-z')^2]^{1/2}$.

II. OSMEI Formulas for Wire Antennas

For simplicity, only linear wire antennas are discussed in this paper. The symmetrical axis of the wire is placed along the z axis, the two ends of the wire are located at $z=0$ and $z=L$, and the radius α of the wire is thin enough so that only z-direction component of the vector potential A_z and z-direction component of the current J_z need to be considered. We start to discretize the following wave equation of the vector potential in cylindrical coordinate system:

$$\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}+\kappa^2\right)A_z(\rho,z)=0 \tag{1}$$

where ρ is the coordinate in radial direction. For the ith node among the total N discretized nodes, the second-order partial differential with respect to z in (1) can be approximated as

$$\left(\frac{\partial^2}{\partial z^2}+\kappa^2\right)A_{zi} \approx \sum_{j=i-(m-1/2)}^{i+(M+1/2)} b_j A_{zj}, \qquad M\geq 3, \quad i=1,2,\cdots,N. \tag{2}$$

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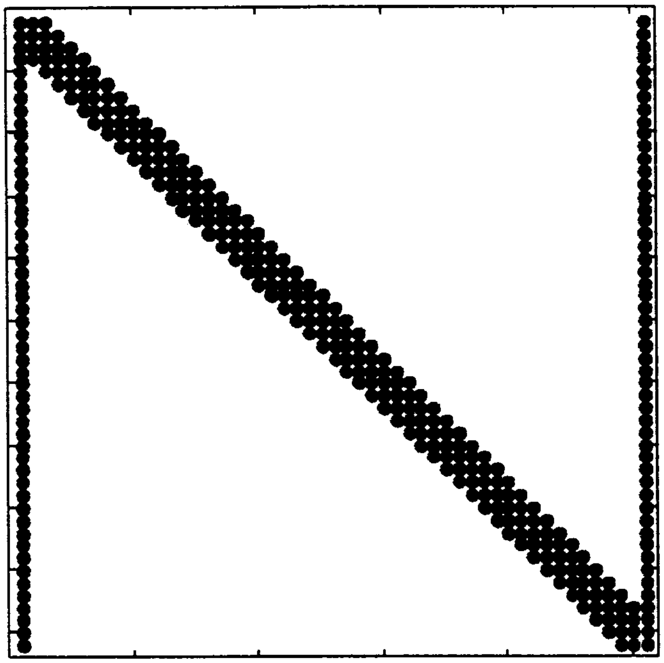

Fig. 1. The highly sparse impedance matrix obtained by the OSMEI method.

The second-order partial differential with respect to ρ in (1) can be approximately written as

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial A_{zi}}{\partial\rho}\right) = \frac{1}{\mu\rho}\frac{\partial}{\partial\rho}(\rho H_{\phi i}) \approx \sum_{j=i-(M-1/2)}^{1+(M+1/2)} a_j H_{\phi j} \qquad M \ge 3, \quad i = 1, 2, \cdots, N. \tag{3}$$

In (3), $\vec{H} = \nabla \times \vec{A}$ is used and $\partial H_\phi/\partial\rho$ is approximately substituted by a linear combination of H_ϕ and nearby fields [11]. Substituting (2) and (3) into (1) and considering $J_z(z) = H_\phi(z)$ on the surface of the wire antenna, we get the on-surface MEI equation as follows:

$$\sum_{j=i-(M-1/2)}^{i+(M+1/2)} a_j J_{zj} + \sum_{j=i-(M-1/2)}^{i+(M+1/2)} b_j A_{zj} = 0 \qquad M \ge 3, \quad i = 1, 2, \cdots N. \tag{4}$$

The coefficients $\{a_j\}$ and $\{b_j\}(j = (M-1)/2, \cdots, (M+1)/2$ and $i = 1, 2, \cdots, N)$ in (4) are the MEI coefficients. Only $(2M-1)$ coefficients are independent for each i. By means of $T(T \ge (2M-1))$ possible currents I called metrons, solutions of J_z and A_z, which are called measuring functions can be obtained, as stated in Section III. If follow that the MEI coefficients $\{a_j\}$ and $\{b_j\}$ can be determined.

What remains is to impose the boundary conditions of A_z in (4) for solving the real currents J_z. An existing analytical solution for A_z with two unknown constants can be used for this purpose [12]

$$A_z(z) = c\cos kz + d\sin kz - \frac{j}{\eta}\int_0^z E_z^{\rm inc}(z')\sin k(z-z')\,dz' \tag{5}$$

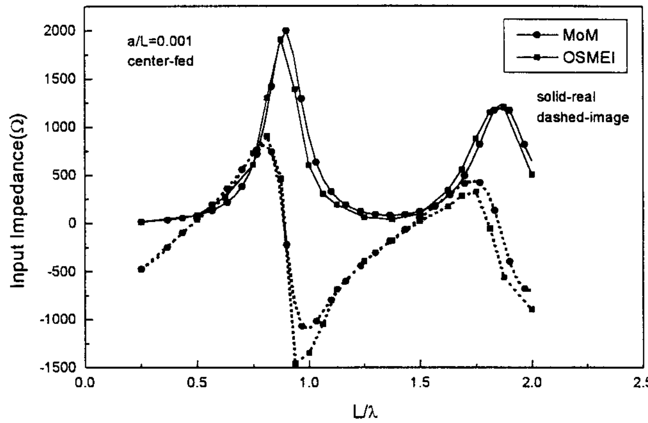

Fig. 2. Input impedance of a center-fed linear antenna with versus (solid line: real part; dashed line: image part).

where η is the intrinsic impedance of the free-space, $E_z^{\rm inc}$ is the driving field in the feed region, c and d are constants to be determined. Equation (5) is the boundary conditions that (4) should satisfy. Substituting (5) into (4), we obtain the following linear system of equations:

$$[Z] \cdot \boldsymbol{I} = \boldsymbol{U} \tag{6}$$

where the impedance matrix $[Z]$ is determined by (4) and (5), the exciting voltage vector $\boldsymbol{U}$ comes from the last term in (5). The unknown vector $\boldsymbol{I}$ includes the surface current on each node with the zero current conditions at the ends and the constants **c** and **d**. At first glance, (6) is very similar to that obtained by the MoM for Hallen's equation [13]. However, the computation of the matrix $[Z]$ is significantly different from that of the MoM in that matrix $[Z]$ is highly sparse, while the matrix for the MoM is full.

III. Metrons to Determine the MEI Coefficients

Finding the MEI coefficients is the key for the OSMEI method. The MEI coefficients in (4) are determined by a series of the measuring function pairs of $\{A_{zm}\}$ and $\{J_{zm}\} \cdot J_{zm}$ in (4) are directly approximated by the assumed metrons, $I_m = 2\pi a J_{zm}$, and $\{A_{zm}\}$ can be found by

$$A_{zm}(z) = \mu\int_0^L G(z,z') I_m(z')\,dz' \tag{7}$$

where $G(z,z') = (e^{-jkR}/4\pi R)$ is the Green's function in free-space, $R = ((z-z')^2 + a^2)^{1/2}, k$ is the wavenumber, and μ is the permeability of free-space.

A series of possible current distributions $\{J_{zm}\}$ is chosen for determining the MEI coefficients $\{a_j\}$ and $\{b_j\}$ in (4). The principle of choosing $\{J_{zm}\}$ is that $\{J_{zm}\}$ must satisfy the end conditions of $J_{zm}(0) = 0$ and $J_{zm}(L) = 0$. Thus, the metrons should be

$$J_{zm}(z) = \sin\left(\frac{m\pi}{L}z\right). \tag{8}$$

Since the real currents in the ends and feed region vary rapidly [14], we have to represent this behavior in the set of metrons. The simplest way is to include high order sinusoidal functions. One technique we have used for choosing $\{m\}$ (the

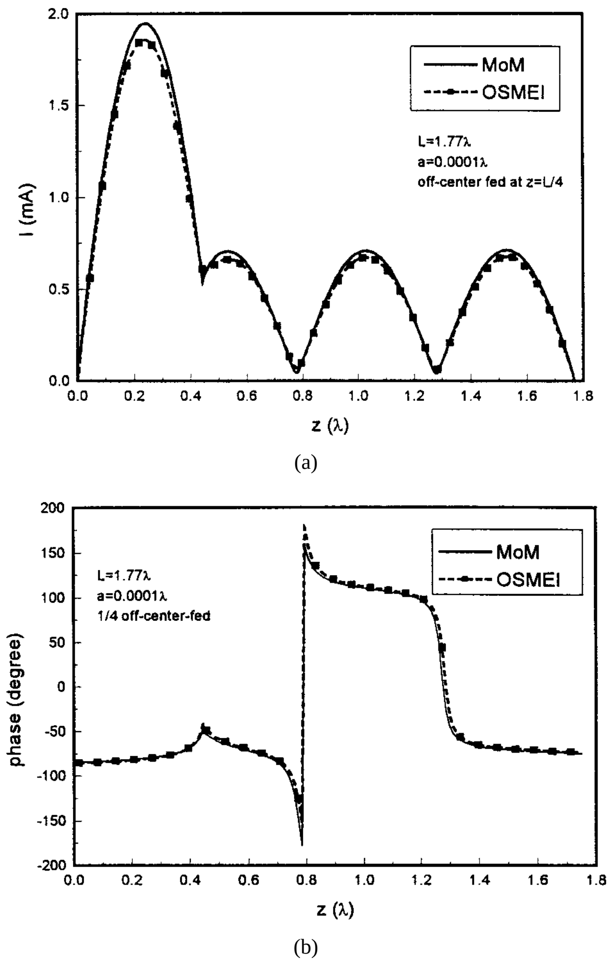

Fig. 3. Current distributions on a off-center fed linear antenna with $L = 1.77\lambda$, $a = 0.0001\lambda$, and feeding point $z = L/4$. (a) Amplitude. (b) Phase.

order of metrons) is to combine a sequence of the lowest order $\{m_l\}$ with a few of the discretely high order $\{m_h\}$, i.e.,

$$\{m\} = \{m_l\} \oplus \{m_h\} \tag{9}$$

where $\{m_l\} = \{1, 2, 3, \cdots, m_l\}, \{m_h\} \subseteq \{m_l, \cdots, m_h\}$.

For a more exacting theoretic consideration, we choose the amplitude of the metron of (9) to be paired with m as

$$J_{zm}(z) = \sin\left(\frac{m\pi}{L}z\right) J_0(a\sqrt{k^2 - (m\pi/L)^2}) \tag{10}$$

where J_0 is the zeroth-order Bessel function. This choice of metron simplifies the calculations of (4) because the equivalent currents $I_m = 2\pi a J_{zm}$ along the z axis produce the same fields as excited by the real current [15]. The number of the metrons needed for robust MEI coefficients is about 15.

IV. Computational Examples

In this section, some numerical results obtained by the OSMEI method are given and compared with the results of the Hallen's integration equation solved by the MoM method with pulse base and point matching. In the following computations, the wire antennas are driven by delta gap voltage source, the five node scheme $(M = 5)$ in the MEI equation (4) is used, which produces highly sparse matrix $[Z]$, as shown in Fig. 1. Fig. 2 gives the input impedance of a center-fed linear antenna with $a/L = 0.001$ versus L/λ. Because of the rapid variation of the current densities near the driving point, it is difficult to model the current in this region accurately. However, the impedances calculated by the MEI method are still reasonable compared with those of the MoM except near the anti-resonant frequency. Fig. 3 shows the comparison of the current distribution of the OSMEI with that of the MoM for an off-center fed dipole antenna with $L/\lambda = 1.77$, $a/\lambda = 0.0001$, and feeding at $z = L/4$.

V. Discussion

In this paper, the OSMEI is used to analyze the wire antennas. The FD mesh is no longer necessary, the wire structures can be accurately modeled by the nodes sampled on the wire surface. The MEI equations, which describe the relationship between the surface currents and vector potential functions, are applied to characterize the electromagnetic property of the wire surface. The examples of center-fed and off-center-fed dipoles show that reasonable numerical results can be obtained by the OSMEI method.

The formulas presented in this paper are limited to the straight wires, however, they can be directly extended to the case of a general curved wire, as long as the generalized integral formulas are used [12]. It is noted that the same thin wire approximation of the Green's function is used in MEI as in the MoM, there is no need for more accurate integration of the Green's function near its singularity. The method can also be applied for the case of wires with series distributed loads if the analogous modifications have been done as in the extension of Hallen's integral equation [13].

The advantage of the method discussed in this paper compared with the MoM is that the highly sparse matrix can be obtained. It should be pointed out that although time-saving is not observed in the computations presented in this paper, it is expected that great time-saving can be obtained in large-scale wire arrays or computer-aided antenna synthesis using optimization procedure.

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