Analysis of Electromagnetic Scattering by Periodic Strip Grating on a Grounded Dielectric/Magnetic Slab for Arbitrary Plane Wave Incidence Case

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***Abstract*— A new type of integral equation that is coupled with strip-electric and slot-magnetic currents is applied to the analysis of electromagnetic scattering by a periodic strip grating on a grounded dielectric/magnetic slab for an oblique incident plane wave with arbitrary polarization. In the analysis, the electric and magnetic currents are expanded into a product of a series of cosine functions and a function satisfying the edge condition. Coupled linear equations for the unknown electric and magnetic currents are obtained. From the coupled linear equations, explicit expressions for the reflection coefficients are derived by the use of a single edge-mode expansion. The validity of the method is examined by numerically calculated boundary conditions. A comparison between the calculated results from the present method and a previous by measured ones shows that the accuracy of the method is excellent. Numerical calculations show that the method converges very rapidly with resonable accuracy.**

***Index Terms*— Boundary integral equations, gratings, slab waveguide.**

I. Introduction

A periodic strip grating on a grounded dielectric/magnetic slab is used to rotate or twist the polarization of the incident plane wave after reflection in a scanned reflector antenna system as a twist reflector [1]–[3], and is used to reduce the cross-polarization and sidelobe level in a horn antenna as an artificially soft or hard surface [4]–[6].

Because of the former applications in antenna engineering, electromagnetic scattering by a grating has been investigated by numerous researchers [1]–[13]. Many different formulations have been considered in their analysis. In [4] and [7]–[11], however, the direction of the incident plane wave has been assumed to be in the plane perpendicular to the grating axis and the polarization direction to H or E polarization; that is, actually a one-dimensional problem. Electromagnetic wave scattering by a free-standing strip grating in the case of oblique incidence and arbitrary polarization was analyzed in [12]. In this analysis, the spectral-domain method combined with the sampling theorem was applied. In [1] and [2], an analysis of this problem as a twist reflector was presented by decomposing the electromagnetic fields into E- and H-type modes and using the equivalent network representation under the assumption that the period of the grating was smaller than a half-wavelength ($\lambda/2$) in free-space, i.e., only one propagating order mode ($n = 0$) was present. This model, however, was the first-order approximation of the strip-grating discontinuity because the equivalent circuit parameters [13] for a free-standing strip grating were used, thus, the theoretical prediction widely deviated from the experimental result. Analytical expressions for reflection coefficients of the structure as artificially soft and hard surfaces were derived in [5] and [6] by use of the variational method. In this derivation, it was assumed that the period p was much smaller than the wavelength and that the dielectric thickness h was large enough to neglect the reflected higher order mode due to the ground plane located at $z = -h$. So the validity of these expressions was confined to the case of a small period and a thick dielectric/magnetic slab.

The purposes of this paper are to propose a rigorous and complete analysis methods without the above constraints, and to obtain analytical expressions for the reflection coefficients that are very useful for the small period case. Since the grating is periodic in one direction and uniform in the other direction, the fields in each region (half-space and dielectric/magnetic slab) can be expressed in terms of Floquet modes. The unknown strip-electric and slot-magnetic currents in the direction perpendicular to the grating axis are expanded into a product of a series of cosine functions and a function satisfying the edge condition. Boundary conditions are imposed at the interfaces, and coupled linear equations for the unknown electric and magnetic currents are obtained. All the expressions in the coupled linear equations are given in closed form. From the coupled linear equations, simple closed-form expressions for the reflection coefficients of the specular order mode are derived under the assumption that the period is much smaller than the free-space wavelength.

The validity of the present method is examined by numerically calculated boundary conditions and power conservation. Numerical results are compared with previous results in [1], [2], [5], and [6], when the structure is used as a twist reflector and as an artificially soft and hard surface. The calculated result shows very good agreement with the measured data in [1]. The results obtained by the simple closed-form expressions are

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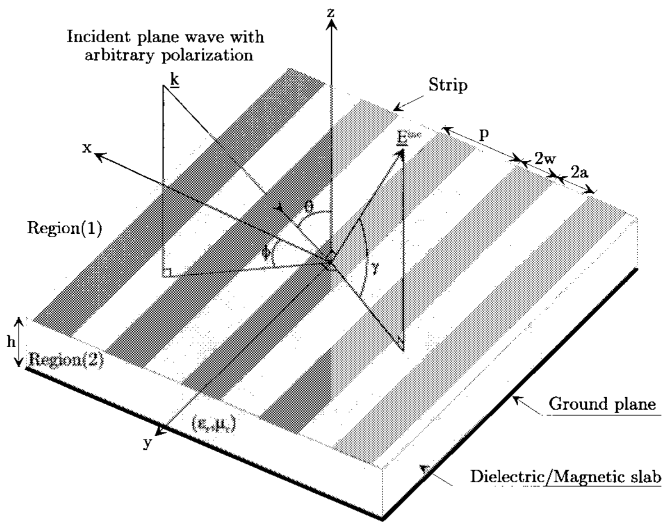

Fig. 1. Geometry of the problem.

compared with those of the complete solution. The comparison shows that the accuracy of the approximate expressions is good for the small period case ($p < 0.5\lambda$).

II. Formulation of the Problem

A. Complete Solution

The geometry of the problem is shown in Fig. 1 where a periodic, infinite strip grating in the x–y plane is placed on a grounded dielectric/magnetic slab of thickness h. The strips of width $2w$ are assumed to be perfect conductors with a negligible thickness and are arranged along the y direction at $z = 0$ with period p. The upper-half free-space and the dielectric/magnetic slab with parameters (ϵ_r, μ_r) regions are denoted by regions (1) and (2), respectively. It is assumed that a linearly polarized plane wave with an arbitrary plane of polarization is incident on the grating as shown in Fig. 1. The incident wave with unit amplitude of electric field is expressed in [12]. The structure is uniform along the y direction and periodic along the x direction. The uniformity of the grating structure does not affect the spatial dependence with respect to the y direction, therefore, the tangential components of the fields in each region are expressed in terms of a summation of Floquet modes. The fields in region (1) are expressed as

$$E_y^{(1)}(x, y, z) = \sum_{n=-\infty}^{\infty} R_n Y(y) X_n(x) e^{-jk_{zn}z} + Y(y) X_0(x) e^{jk_{z0}z} E_y^i \quad (1)$$

$$H_y^{(1)}(x, y, z) = \sum_{n=-\infty}^{\infty} S_n Y(y) X_n(x) e^{-jk_{zn}z} + Y(y) X_0(x) e^{jk_{z0}z} H_y^i \quad (2)$$

$$E_x^{(1)}(x, y, z) = -\frac{Y(y)}{\varkappa_0^2} \sum_{n=-\infty}^{\infty} X_n(x) e^{-jk_{zn}z} \cdot (k_y k_{xn} R_n - \eta_0 k_0 k_{zn} S_n) + Y(y) X_0(x) e^{jk_{z0}z} E_x^i \quad (3)$$

$$H_x^{(1)}(x, y, z) = -\frac{Y(y)}{\varkappa_0^2} \sum_{n=-\infty}^{\infty} X_n(x) e^{-jk_{zn}z} \cdot (k_0/\eta_0 k_{zn} R_n + k_y k_{xn} S_n) + Y(y) X_0(x) e^{jk_{z0}z} H_x^i \quad (4)$$

by decomposing the field into TE$_y$ and TM$_y$ components. In region (2) ($-h \leqslant z \leqslant 0$), the waves satisfying an appropriate boundary condition at $z = -h$ are represented as

$$E_y^{(2)}(x, y, z) = \sum_{n=-\infty}^{\infty} P_n Y(y) X_n(x) \cdot \left(e^{-jk_{1zn}z} - e^{j2k_{1zn}h} e^{jk_{1zn}z}\right), \quad (5)$$

$$H_y^{(2)}(x, y, z) = \sum_{n=-\infty}^{\infty} Q_n Y(y) X_n(x) \cdot \left(e^{-jk_{1zn}z} + e^{j2k_{1zn}h} e^{jk_{1zn}z}\right) \quad (6)$$

$$E_x^{(2)}(x, y, z) = -\frac{Y(y)}{\varkappa_1^2} \sum_{n=-\infty}^{\infty} X_n(x) \cdot (k_y k_{xn} P_n - \mu_r \eta_0 k_0 k_{1zn} Q_n) \cdot \left(e^{-jk_{1zn}z} - e^{j2k_{1zn}h} e^{jk_{1zn}z}\right) \quad (7)$$

$$H_x^{(2)}(x, y, z) = \frac{Y(y)}{\varkappa_1^2} \sum_{n=-\infty}^{\infty} X_n(x) \cdot (\epsilon_r k_0/\eta_0 k_{1zn} P_n + k_y k_{xn} Q_n) \cdot \left(e^{-jk_{1zn}z} + e^{j2k_{1zn}h} e^{jk_{1zn}z}\right) \quad (8)$$

where

$$\begin{aligned} &Y(y) = e^{jk_y y}, \qquad k_y = k_0 \sin\theta \sin\phi \\ &X_n(x) = e^{jk_{xn}x}, \qquad k_{xn} = k_0 \sin\theta \cos\phi + 2n\pi/p \\ &k_{zn} = \sqrt{k_0^2 - k_y^2 - k_{xn}^2}, \qquad k_{1zn} = \sqrt{k_1^2 - k_y^2 - k_{xn}^2} \\ &\varkappa_0^2 = k_0^2 - k_y^2, \qquad \varkappa_1^2 = k_1^2 - k_y^2 \\ &k_1 = \omega\sqrt{\epsilon_0 \epsilon_r \mu_0 \mu_r}, \qquad k_0 = \omega\sqrt{\epsilon_0 \mu_0} \\ &\eta_0 = \sqrt{\mu_0/\epsilon_0}. \end{aligned} \quad (9)$$

Here R_n, S_n, P_n, and Q_n are the unknown complex amplitudes of the space harmonics to be determined and $X_n(x)$ is the Floquet mode due to the periodicity along the x direction. To determine the unknown amplitudes (R_n, S_n, P_n, and Q_n), the boundary conditions that will be imposed at $z = 0$ are summarized as

$$E_y^{(1)}(x, y, 0) = E_y^{(2)}(x, y, 0) \quad (10)$$

$$E_x^{(1)}(x, y, 0) = E_x^{(2)}(x, y, 0) \quad (11)$$

$$H_y^{(1)}(x, y, 0) - H_y^{(2)}(x, y, 0) = \begin{cases} -J_x(x, y), & \text{on the strip} \\ 0, & \text{elsewhere} \end{cases} \quad (12)$$

$$E_y^{(1)}(x, y, 0) = \begin{cases} M_x(x, y), & \text{on the slot} \\ 0, & \text{elsewhere} \end{cases} \quad (13)$$

$$E_x^{(1)}(x, y, 0) = 0, \qquad \text{on the strip} \quad (14)$$

$$H_x^{(1)}(x, y, 0) = H_x^{(2)}(x, y, 0), \qquad \text{the slot} \quad (15)$$

where $J_x(x, y)$ and $M_x(x, y)$ are the unknown strip electric and slot magnetic currents, respectively, and can be expanded into a product of a series of cosine functions and a function satisfying the edge condition as

$$J_x(x, y) = Y(y)X_0(x)\sqrt{1-(x/w)^2} \cdot \sum_{\nu=0}^{\infty}(j)^{\nu}A_{\nu}\cos w_{\nu}(x+w), \qquad |x| \leqslant w \tag{16}$$

$$M_x(x, y) = Y(y)X_0(x)\sqrt{1-[(x-a_c)/a]^2} \cdot \sum_{l=0}^{\infty}(j)^{l}B_l\cos a_l(x-a_c+a), \ |x-a_c| \leqslant a \tag{17}$$

where $w_\nu = \nu\pi/2w$, $a_l = l\pi/2a$, $a_c = p/2$, and A_ν and B_l are unknown coefficients.

Boundary conditions (10)–(13), field equations (1)–(8), current expressions (16) and (17), and the orthogonality property of the space harmonics lead to following relations between the unknown complex amplitudes of the Floquet mode and the coefficients of the electric and magnetic currents as

$$R_n = -E_y^i\delta_{n,0} + \frac{1}{p}\sum_{l=0}^{\infty}B_lF(n, l, a)e^{-j2n\pi a_c/p} \tag{18}$$

$$P_n = \frac{1}{1-e^{j2k_{1zn}h}}\left(R_n + E_y^i\delta_{n,0}\right) \tag{19}$$

$$S_n = \Gamma_H(k_{x0})H_y^i\delta_{n,0} - \frac{1}{2p}\{1-\Gamma_H(k_{xn})\} \cdot \sum_{\nu=0}^{\infty}A_\nu F(n, \nu, w) + \frac{k_yk_{xn}}{\eta_0k_0}C^{eh}(k_{xn})\frac{1}{p} \cdot \sum_{l=0}^{\infty}B_lF(n, l, a)e^{-j2n\pi a_c/p} \tag{20}$$

$$Q_n = \frac{1}{1+e^{j2k_{1zn}h}} \cdot \left\{H_y^i\delta_{n,0} + S_n + \frac{1}{p}\sum_{\nu=0}^{\infty}A_\nu F(n, \nu, w)\right\} \tag{21}$$

where

$$\Gamma_H(k_{xn}) = \frac{\varkappa_1^2k_{zn} - \varkappa_0^2\mu_rk_{1zn}j\tan k_{1zn}h}{\varkappa_1^2k_{zn} + \varkappa_0^2\mu_rk_{1zn}j\tan k_{1zn}h}$$

$$C^{eh}(k_{xn}) = \frac{\varkappa_1^2 - \varkappa_0^2}{\varkappa_1^2k_{zn} + \varkappa_0^2\mu_rk_{1zn}j\tan k_{1zn}h}$$

$$F(n, \nu, w) = \frac{w\pi}{2}\left\{\frac{J_1(2n\pi w/p + \nu\pi/2)}{2n\pi w/p + \nu\pi/2} + (-1)^\nu\frac{J_1(2n\pi w/p - \nu\pi/2)}{2n\pi w/p - \nu\pi/2}\right\} \tag{22}$$

$\delta_{n,0}$ is Kronecker delta, i.e., $\delta_{n,0} = 1$ for $n = 0$, $\delta_{n,0} = 0$ for $n \neq 0$, and $J_1(\cdot)$ denotes Bessel function. Inserting the field (3) into the tangential electric field condition on the strip (14) and multiplying both sides by $X_0(x)^*(-j)^\mu \sqrt{1-(x/w)^2}\cos w_\mu(x+w)$ and integrating over the strip region, we have

$$\sum_{n=-\infty}^{\infty}(k_yk_{xn}R_n - \eta_0k_0k_{zn}S_n)F(n, \mu, w) = \varkappa_o^2E_x^iF(0, \mu, w). \tag{23}$$

Inserting (4) and (5) into the remaining boundary condition (15) and multiplying both sides by $X_0(x)^*(-j)^m \sqrt{1-[(x-a_c)/a]^2}\cos a_m(x-a_c+a)$ and integrating over the slot region, we obtain the following:

$$\sum_{n=-\infty}^{\infty}\left\{\frac{k_0/\eta_0k_{zn}R_n + k_yk_{xn}S_n}{\varkappa_0^2} - \frac{\epsilon_rk_0/\eta_0k_{1zn}P_n + k_yk_{xn}Q_n}{\varkappa_1^2}\left(1+e^{j2k_{1zn}h}\right)\right\} \cdot F(n, m, a)e^{j2n\pi a_c/p} = H_x^iF(0, m, a). \tag{24}$$

After substituting (18)–(21) into (23) and (24) and some algebraic manipulations, the following coupled linear equations for the unknown coefficients of the strip electric and slot magnetic currents are obtained:

$$\sum_{\nu=0}^{\infty}Z_{\mu\nu}A_\nu + \sum_{l=0}^{\infty}T_{\mu l}^{-}B_l = V_\mu \tag{25}$$

$$-\sum_{v=0}^{\infty}T_{m\nu}^{+}A_\nu + \sum_{l=0}^{\infty}Y_{ml}B_l = I_m \tag{26}$$

where

$$\begin{aligned}
Z_{\mu\nu} &= \frac{\eta_0k_0}{2}\sum_{n=-\infty}^{\infty}k_{zn}\{1-\Gamma_H(k_{xn})\} \cdot F(n, \nu, w)F(n, \mu, w)\\
T_{\mu l}^{\pm} &= \sum_{n=-\infty}^{\infty}k_yk_{xn}\{1-k_{zn}C^{eh}(k_{xn})\} \cdot F(n, l, a)F(n, \mu, w)e^{\pm j2n\pi a_c/p}\\
Y_{ml} &= \sum_{n=-\infty}^{\infty}\frac{k_0}{\eta_0\varkappa_1^2}\{D_\Gamma^E(k_{xn}) + (\epsilon_r\mu_r - 1) \cdot (k_yk_{xn})^2C^{eh}(k_{xn})\}F(n, l, a)F(n, m, a)\\
V_\mu &= -p\eta_0k_0k_{z0}\{1-\Gamma_H(k_{x0})\}H_y^iF(0, \mu, w)\\
I_m &= 2pk_{z0}\left\{2\frac{k_0}{\eta_0}E_y^i - k_yk_{x0}C^{eh}(k_{x0})H_y^i\right\} \cdot F(0, m, a)\\
D_\Gamma^E(k_{xn}) &= \varkappa_1^2k_{zn} - \varkappa_0^2\epsilon_rk_{1zn}j\cot k_{1zn}h.
\end{aligned} \tag{27}$$

Once the unknown coefficients A_ν and B_l are known, the complex amplitudes R_n, P_n, S_n, and Q_n are determined by using (18)–(21). Then the fields in both regions are computed by (1)–(8).

B. E and H Polarization

One finds that $\phi = 0°$, $\gamma = 0°$, and $\phi = 0°$, $\gamma = 90°$ correspond to E polarization ($H_y = 0$) and E polarization ($E_y = 0$) cases, respectively. For these cases, the coupled linear equations (25) and (26) are no longer coupled, and can be expressed as the following equations by setting $k_y = 0$,

$H_y^i = 0$ for E polarization and $k_y = 0$, $E_y^i = 0$ for H polarization.

E Polarization:

$$\sum_{l=0}^{\infty} B_l \sum_{n=-\infty}^{\infty} \left\{ k_{zn} - \frac{k_{1zn}}{\mu_r} j \cot k_{1zn} h \right\} \cdot F(n, l, a) F(n, m, a) = 2pk_{z0} E_y^i F(0, m, a). \tag{28}$$

H Polarization:

$$\sum_{\nu=0}^{\infty} A_\nu \sum_{n=-\infty}^{\infty} \frac{k_{zn}}{2p} \{1 - \Gamma_H(k_{xn})\} F(n, \nu, w) F(n, \mu, w) = -k_{z0}\{1 - \Gamma_H(k_{x0})\} H_y^i F(0, \mu, w). \tag{29}$$

C. Approximate Solution—Single Edge-Mode Solution

When the period of the structure is much smaller than the free-space wavelength, i.e., $p \ll \lambda$, it is sufficient to take one expansion function for both the strip electric and slot magnetic currents (single edge-mode expansion). From the coupled linear equations and field equations, the following explicit expressions for reflection coefficients are derived:

$$R_0 = -E_y^i + \frac{\pi a}{2p} B_0 \tag{30}$$

$$S_0 = \Gamma_H(k_{x0}) H_y^i - \frac{\pi w}{4p} \{1 - \Gamma_H(k_{x0})\} A_0 + C^{eh}(k_{x0}) \frac{\pi a}{2p} \frac{k_y k_{x0}}{\eta_0 k_0} B_0 \tag{31}$$

where

$$A_0 = \frac{V_0 Y_{00} - I_0 T_{00}^-}{Z_{00} Y_{00} + T_{00}^+ T_{00}^-}$$

$$B_0 = \frac{I_0 Z_{00} + V_0 T_{00}^+}{Z_{00} Y_{00} + T_{00}^+ T_{00}^-}$$

$$Z_{00} = \frac{p^2}{8} \sum_{n=-\infty}^{\infty} \eta_0 k_0 k_{zn} \{1 - \Gamma_H(k_{xn})\} \frac{\{J_1(2n\pi w/p)\}^2}{n^2}$$

$$Y_{00} = \frac{p^2}{4} \sum_{n=-\infty}^{\infty} \frac{k_0}{\eta_0 \varkappa_1^2} \{D_\Gamma^E(k_{xn}) + (\epsilon_r \mu_r - 1)(k_y k_{xn})^2 C^{eh}(k_{xn})\} \frac{\{J_1(2n\pi a/p)\}^2}{n^2}$$

$$T_{00}^{\pm} = \frac{p^2}{4} \sum_{n=-\infty}^{\infty} k_y k_{xn} \{1 - k_{zn} C^{eh}(k_{xn})\} \cdot \frac{J_1(2n\pi w/p) J_1(2n\pi a/p)}{n^2} e^{\pm j2n\pi a_c/p}$$

$$V_0 = -p\eta_0 k_0 k_{z0} \{1 - \Gamma_H(k_{x0})\} \frac{\pi w}{2} H_y^i$$

$$I_0 = pk_{z0} \left\{ \frac{k_0}{\eta_0} E_y^i - k_y k_{x0} C^{eh}(k_{x0}) H_y^i \right\} \pi a. \tag{32}$$

These expressions (30), (31) are very useful when the period is smaller than a half free-space wavelength. It will be shown in the next section that these expressions are valid for small periods ($p < 0.5\lambda$).

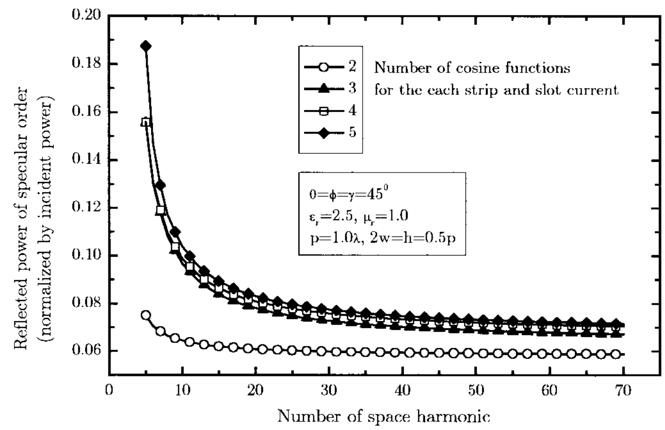

Fig. 2. Convergence curve for the reflected power of the specular order mode.

The procedure described in this section can be applied to the analysis of electromagnetic scattering by a periodic strip grating on a dielectric/magnetic slab.

III. Numerical Results and Discussion

To test the convergence of our numerical method a structure having $\theta = \phi = \gamma = 45°$, $\epsilon_r = 2.5$, $\mu_r = 1.0$, $p = 1.0\lambda$, and $2w = h = 0.5p$ is analyzed for an increasing number of space harmonics and cosine functions. Fig. 2 shows the reflected power density of the specular-order mode ($n = 0$) normalized by the incident power density versus the number of space harmonics for different numbers of cosine functions a parameter. It is seen that 30 space harmonics for any number of cosine functions gives results with excellent convergence. However, the converged values for the two and three cosine functions are not same as those for different number of cosine functions. Because the period of the structure is $p = 1.0\lambda$, two or three cosine functions are not sufficient to model the strip electric and slot magnetic current distributions. As shown in Fig. 2, converged values are obtained when the number of space harmonics and cosine functions are greater than 30 and 3, respectively. This means that an 8 × 8 matrix is enough to analyze the structure so the analysis presented here requires very small computational memory size.

To be certain that the reflected power density of the specular-order mode converges to the correct value, we calculate the tangential electric and magnetic fields at the interface ($z = 0$) for the same structure and plot in Fig. 3 when 60 space harmonics and five cosine functions are employed. The tangential electric fields are identical on both sides of the interface and vanish on the metal strip surface. The tangential magnetic fields are continuous over the slot region and are discontinuous over the strip region due to the strip surface current density, as shown in the figure. Fig. 3 shows that the boundary conditions at the interface are satisfied and the problem is solved in the correct way.

Fig. 4 shows the phases of the $n = 0$ order reflection coefficients R_0 and S_0 versus normalized period for a structure

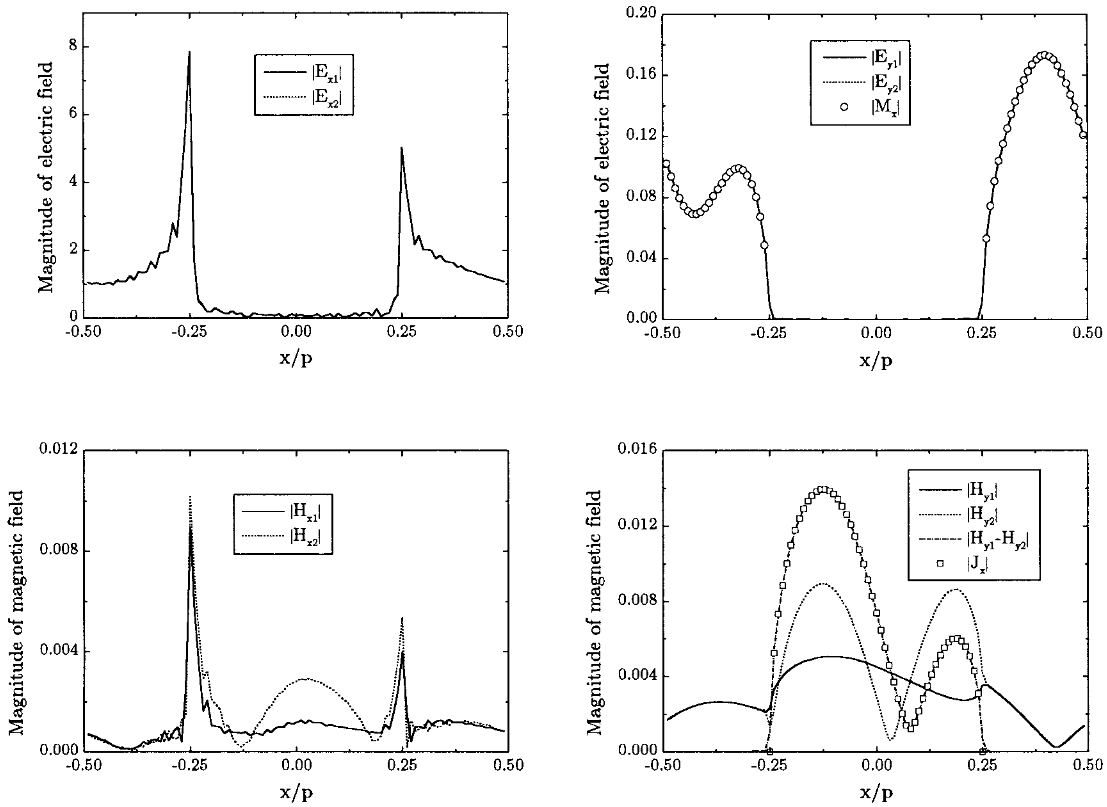

Fig. 3. Field amplitudes at the discontinuity ($z = 0$).

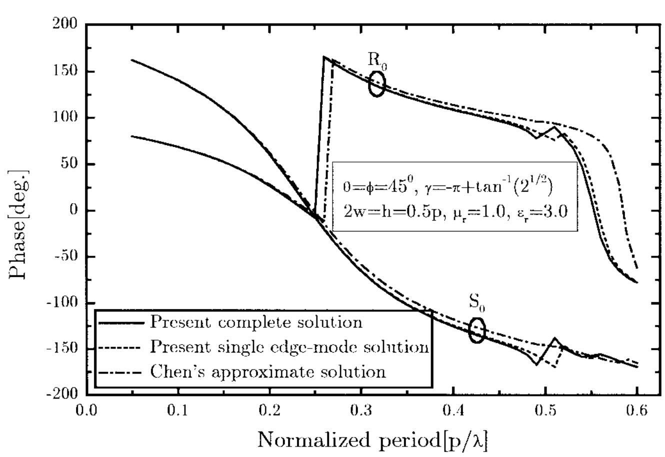

Fig. 4. Phases of R_0 and S_0 for a normalized period (p/λ).

having $\theta = \phi = 45^\circ$, $\gamma = -\pi + \tan^{-1}\sqrt{2}$, $2w = h = 0.5p$, $\mu_r = 1.0$, and $\epsilon_r = 3.0$. It is found that the result obtained by the present complete solution is almost same as that of the point-matching method plotted in [5]. The present approximate solution is very well matched with the complete solution up to $p = 0.5\lambda$; however, the approximate expression [5] is closely matched with our complete solution only up to a normalized period of 0.3λ and not matched for a normalized period more than 0.3λ.

Using the coordinate transformation and field equations (1)–(8), we obtain the following electric field components in spherical coordinates as:

$$E_\theta^r = \cos\theta\cos\phi E_x^r + \cos\theta\sin\phi E_y^r - \sin\theta E_z^r = -\frac{\cos\theta\cos\phi}{1-\sin^2\theta\sin^2\phi}(\tan\phi R_0 + \sec\theta\eta_0 S_0)e^{j\underline{k}\cdot\underline{r}} \quad (33)$$

$$E_\phi^r = -\sin\phi E_x^r + \cos\phi E_y^r = \frac{\sin\phi}{1-\sin^2\theta\sin^2\phi}(-\cot\phi R_0 + \cos\theta\eta_0 S_0)e^{j\underline{k}\cdot\underline{r}} \quad (34)$$

where $\underline{k} = k_0(\sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} - \cos\theta\hat{z})$ and $E^r_{(\theta,\phi,x,y,z)}$ is the reflected (θ, ϕ, x, y, z) component of the electric field in region (1). In the derivation of (33) and (34), we assumed that the period is smaller than $\lambda/2$; that is, only one propagating mode ($n = 0$) exists. If the incident field is assumed to be $E_\theta^i = 1$ and $E_\phi^i = 0$, the polarization suppression ratio (E_θ^r/E_θ^i) is given by (33). The polarization suppression ratio given in (33) is similar to that of [1] but is not the same. From (33), we can find two conditions for perfect conversion of E_θ^i to E_ϕ^r

$$\frac{R_0}{\eta_0 S_0} = \text{real} \quad (35)$$

$$\frac{R_0}{\eta_0 S_0} = -\frac{\sec\theta}{\tan\phi}. \quad (36)$$

The polarization suppression ratio is calculated and plotted in Fig. 5 for the structure having $\gamma = 90^\circ$ and $p = 0.228\lambda$,

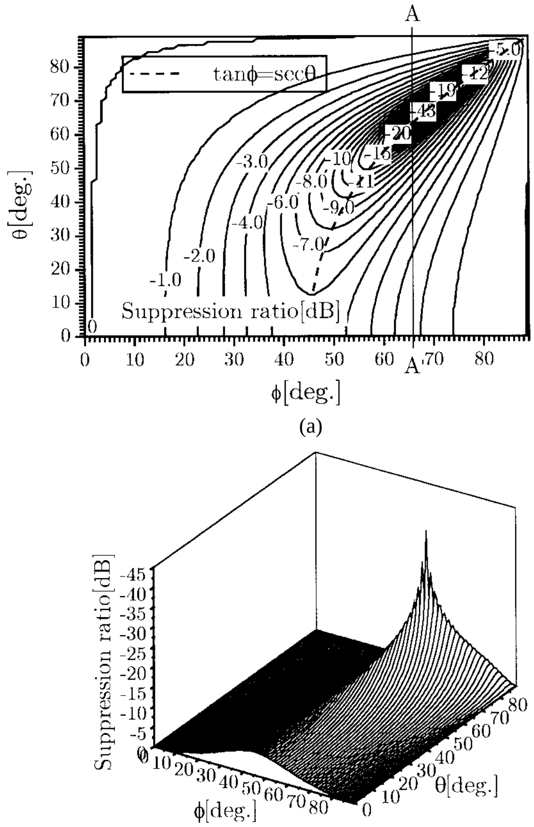

Fig. 5. (a) Contour and (b) surface graphs of the suppression ratio for incident angles.

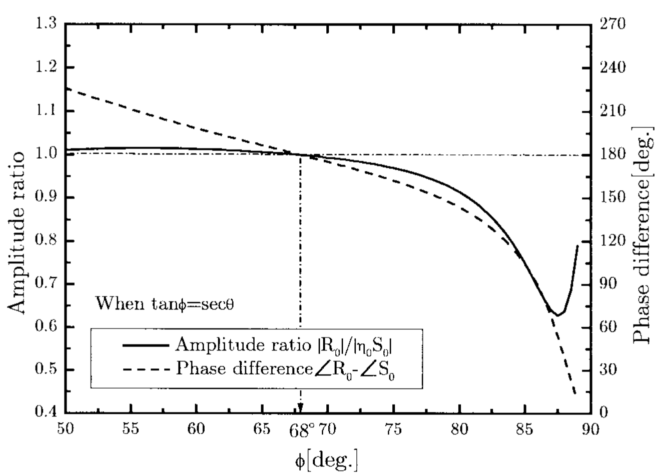

Fig. 6. Amplitude ratio and phase difference for incident angles, when $\tan\phi = \sec\theta$.

$2w = 0.5p$, $h = 0.164\lambda$, and $\epsilon_r = 2.57$. In this calculation, 50 space harmonics and two cosine functions for each electric and magnetic current are employed. In the contour graph, the maximum suppression ratio is obtained around $\tan\phi = \sec\theta$ for a specified incident angle (ϕ), which is similar to the result found in [1]. In this figure, the maximum suppression ratio is obtained when the incident angles are $\phi \simeq 68°$ and $\theta \simeq 66°$ and the value is almost −43.5 dB. For the incident angles

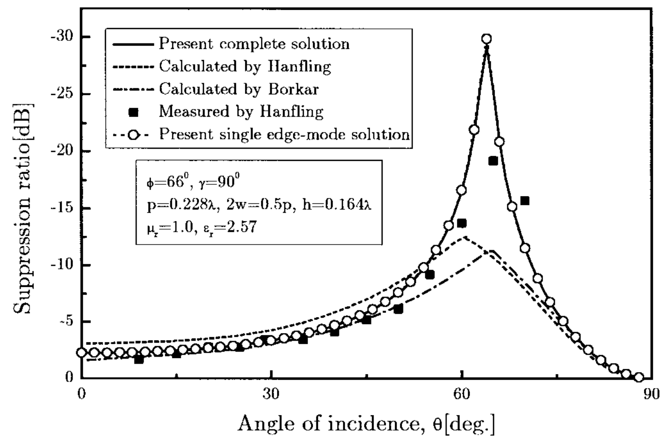

Fig. 7. Variation of suppression ratio for an angle of incidence.

which satisfying the condition $\tan\phi = \sec\theta$, the amplitude ratio ($|R_0|/|\eta_0 S_0|$) and phase difference ($\angle R_0 - \angle\eta_0 S_0$) are calculated and plotted in Fig. 6. When the incident angles satisfy $\tan\phi = \sec\theta$, from (36) the condition for perfect conversion of E_θ^i to E_ϕ^r is $R_0/\eta_0 S_0 = -1$, which means that the amplitude ratio and phase difference are 1 and 180°, respectively. In Fig. 6, it is seen that these conditions are satisfied when the incident angle is $\phi \simeq 68°$, thus, the maximum suppression ratio is obtained at this angle.

Fig. 7 shows the cross-sectional view of suppression ratio along A–A′ in Fig. 5. In Fig. 7, it is seen that the calculated result of [1] widely deviates from the measured one in [1] and that of [2] is closely matched with the measured one up to an angle of incidence of 50° but not matched for an angle of incidence more than 50°. The theoretical prediction of the present complete solution, however, is closely matched with the measured one for all ranges of the angle of incidence. So this method has excellent accuracy. The polarization suppression ratio obtained by using (30) and (31) is also plotted in Fig. 7. The result of the approximate solution completely matches that of the complete integral equation solution. So this approximate solution is very useful for the small period case.

In Fig. 7, the peak can be predicted by the use of the above two conditions (35), (36). The first condition is satisfied when the phase difference ($\angle R_0 - \angle S_0$) is 0° or 180° (in or out of phase). As seen in Fig. 5, the maximum suppression is occurred around $\tan\phi = \sec\theta$, thus, the incident angle is $\theta \approx 63.56°$ for $\phi = 66°$. Therefore, the second condition is satisfied when the amplitude of $R_0/\eta_0 S_0$ is unity and the phase difference ($\angle R_0 - \angle S_0$) is 180°. Fig. 8 shows the amplitude ratio ($|R_0|/|\eta_0 S_0|$) and phase difference for various angles of incidence. It is seen that $R_0/\eta_0 S_0 = -1$ is satisfied when the incident angle is $\theta \approx 64°$. Therefore, $R_0/\eta_0 S_0 = -1$ and $\tan\phi = \sec\theta$ conditions are satisfied for almost the same angle. For this incident angle, the peak occurs as shown in Fig. 7.

IV. Conclusion

The analysis of electromagnetic scattering by a periodic strip grating on a grounded dielectric slab for an oblique incident plane waves with an arbitrary polarization is considered. The

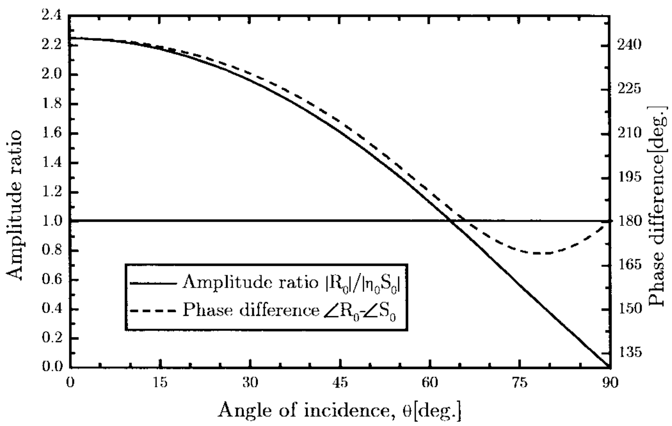

Fig. 8. Amplitude ratio and phase difference for an angle of incidence.

coupled linear equations for the unknown strip electric and slot magnetic currents are obtained in functional forms. When the incident wave direction is in the plane perpendicular to the grating axis and the polarization angle is E and H polarization, the linear equations that are no longer coupled are obtained as a special case. In this analysis, the electric and magnetic currents that flow in the direction perpendicular to the grating axis are expanded into a product of a series of cosine functions and a function satisfying the edge condition. From the coupled linear equations, the explicit form expressions for the reflection coefficients of the specular order mode are obtained under the assumption that the period is much smaller than the free-space wavelength. It is found that the numerical results exhibit a good convergence. The validity of the presented method is examined by examining numerically calculated boundary conditions. The results obtained by the closed-form expressions for the reflection coefficients are compared with those of the complete solution and it is found that the simple expressions are very efficient when the period of the structure satisfies $p < 0.5\lambda$. The comparison between the theoretical prediction and experimental results shows an excellent accuracy of the present method. The rigorous and complete analysis method without constraints that are described in the introduction is proposed in this paper. This method can be applied to the analysis of the structure that is used as a twist reflector or soft and hard surfaces.

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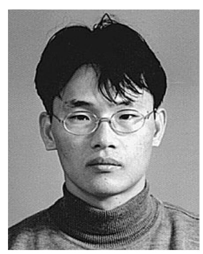

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