Phase Synthesis of Antennas for a Given Radiation Pattern in One Plane Using Piecewise Linear Aperture Phase Distribution

Felix K. Povolotsky and Tatiana P. Sydorova

***Abstract*— A phase-only method for the synthesis of planar aperture antennas for a given complex radiation pattern in one plane is reported. The problem is reduced to determining an appropriate aperture phase distribution in the form of a ruled function and solved for apertures with rectangular shape and an arbitrary amplitude distribution and for apertures of arbitrary shape and amplitude distribution. This method can be used for controlling the pattern of phased-array antennas. Results of computer modeling are presented.**

***Index Terms*—Aperture antenna, phase synthesis.**

I. Introduction

IN our previous paper [1], we proposed a method for the phase synthesis and phase control of radiation patterns of aperture antennas in one plane based on works [2]–[4]. The phase distribution was searched for in the class of two-dimensional ruled functions. Only uniform amplitude distributions on rectangular apertures were considered. In this method, the aperture is represented as a combination of parallel strips with linear phase distributions specific for each strip. Each strip corresponds to a particular term of the Woodward expansion of the pattern with the amplitude proportional to the strip width.

In this paper, we extend this method to apertures of arbitrary shape and arbitrary amplitude distribution. As a result, the strip boundaries can become curvilinear.

II. The Rectangular Aperture with a Nonuniform Amplitude Distribution

Consider a $2a \times 2b$ rectangular aperture with an amplitude distribution $F(x,y)$. It is necessary to find a phase distribution providing a given complex pattern $G(\theta,\varphi)$ in the plane $\varphi = 0$, where θ and φ are the polar and azimuth angles of the spherical coordinate system.

Let us represent the unknown amplitude-phase distribution (APD) $I(x)$ on the equivalent linear antenna parallel to the x axis (ELAx) providing the given pattern $G(\theta, 0)$ with the help of the Woodward method [3] as

$$G(\theta,0) = \sum_{n=-N}^{N} g(n\pi)[\sin(u-n\pi)]/(u-n\pi),$$
$$I(x) = \sum_{n=-N}^{N} g(n\pi)e^{jn\pi x/a} \quad (1)$$

where $u = (2\pi a/\lambda)\sin\theta$, $g(n\pi)$ is the amplitude of the nth partial pattern, and λ is the wavelength.

The APD on the ELAx due to the initial distribution $F(x,y)$ is

$$I_{\text{init}}(x) = \int_{-b}^{b} F(x,y)\,dy = F_1(x,b) - F_1(x,-b) \quad (2)$$

where $F_1(x,y)$ is the integral of $F(x,y)$ with respect to y.

The ratio of the required APD $I(x)$ on the ELAx to the initial one can be represented as

$$f(x) = I(x)/I_{\text{init}}(x) = \sum_{n=-N}^{N} g(n\pi)e^{jn\pi x/a}/[F_1(x,b) - F_1(x,-b)]. \quad (3)$$

By expanding $f(x)$ in the Fourier series we obtain

$$f(x) = \sum_{n=-N}^{N} g_1(n\pi)e^{jn\pi x/a} \quad (4)$$

where $g_1(n\pi)$ are the Fourier coefficients.

Equation (4) shows that the required APD on the ELAx can be implemented by representing the antenna aperture as a combination of $2N+1$ strips with linear phase distributions. The amplitude distribution on the strips is identical to the initial aperture distribution. The width $h(x,n)$ of the nth $(n = -N, \cdots, N)$ strip is determined from the coefficients $g_1(n\pi)$ by solving the following integral equations for its lower and upper boundaries y_n and y_{n+1}

$$g_1(n\pi) = \int_{y_n}^{y_{n+1}} F(x,y)\,dy. \quad (5)$$

$$h(n,x) = g_1(n\pi)(F_1(x,b) - F_1(x,-b)) \Big/ \sum_{m=-N}^{N} g_1(m\pi). \quad (6)$$

This can be done as follows.

1) Determine the function $F_1(x,y)$ from the amplitude distribution $F(x,y)$ (Fig. 1).
2) For each x, divide the interval $[F_1(x,-b), F_1(x,b)]$ into $2N+1$ segments with lengths determined by formula (6), i.e., the upper boundary of the lowest strip is $F_1(x,-b) + h(-N,x)$, the upper boundary of the next strip is $F_1(x,b) + h(-N,x) + h(-N+1,x)$, etc.

Manuscript received April 8, 1997; revised December 1, 1998.
The authors are with NPO Altair, Moscow 111024 Russia.
Publisher Item Identifier S 0018-926X(99)07078-7.

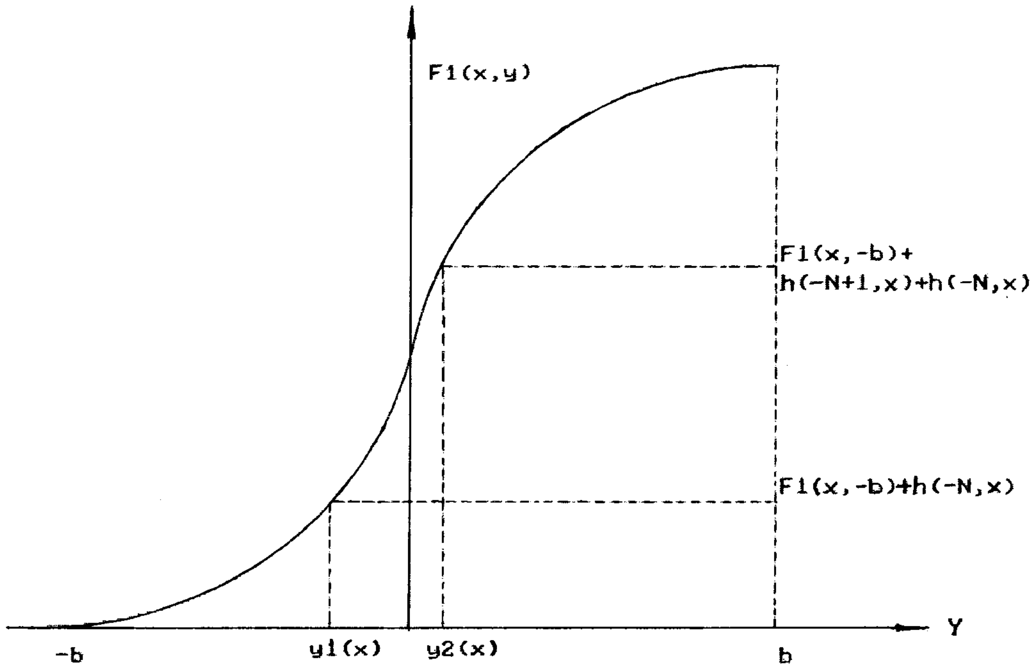

Fig. 1. Determination of the strip boundaries with the help of the integral function.

3) Draw lines parallel to the y axis through the points of division. The corresponding y coordinates of points of intersection of these lines and the curve of the function $F_1(x,y)$ yield the required positions of the strip boundaries for a particular x. As a result, we determine the curvilinear boundaries of the regions of the antenna aperture with linear phase distributions $\varphi(x) = n\pi x/a$ [we designate the upper boundary of the nth region as $L_n(x)$]. This piecewise linear phase distribution provides the required pattern $G(\theta, 0)$ for the given aperture amplitude distribution $F(x,y)$.

Let us consider an example. Let $F(x,y) = \cos(\pi x/2a)\cos(\pi y/2b)$. It is necessary to synthesize an antenna with a less than -40-dB sidelobe level in one plane. In particular, such sidelobe level has an antenna with the following amplitude-phase distribution on the ELAx [5]

$$I(x) = \cos^3(\pi x/2a). \tag{7}$$

In this case, (3)–(6) for the strip boundaries can be solved explicitly. From (2) and (3), we obtain

$$\begin{aligned} F_1(x,y) &= (2b/\pi)\cos(\pi x/2a)\sin(\pi y/2b), \\ I_{\text{init}}(x) &= (4b/\pi)\cos(\pi x/2a), \\ f(x) &= (\pi/4b)\cos^2(\pi x/2a) \\ &= (\pi/16b)(e^{j\pi x/a} + 2 + e^{-j\pi x/a}). \end{aligned}$$

Therefore, the aperture can be represented as a combination of three strips with phase distributions $\pi x/a$, 0, and $-\pi x/a$, and relative widths $g_1(-\pi) = \pi/16b$, $g_1(0) = 2\pi/16b$, and $g_1(\pi) = \pi/16b$, respectively. The lower boundary is $F_1(x,-b) = (-2b/\pi)\cos(\pi x/2a)$, and the upper boundary is $F_1(x,b) = (2b/\pi)\cos(\pi x/2a)$.

The widths of the strips can be found from (6): $h(-1,x) = (b/\pi)\cos(\pi x/2a)$, $h(0,x) = 2(b/\pi)\cos(\pi x/2a)$, and $h(1,x) = (b/\pi)\cos(\pi x/2a)$. If the distance is measured from the lower boundary of the antenna aperture ($y = -b$), we obtain that the upper boundaries of the lower region is $y = y_1 = -b/\pi\cos(\pi x/2a)$ and the upper boundary of the central region is $y = y_2 = b/\pi\cos(\pi x/2a)$. Thus,

$$(2b/\pi)\cos(\pi x/2a)\sin(\pi y_{1,2}/2b) = \mp b/\pi\cos(\pi x/2a). \tag{8}$$

Therefore, $L_1(x) = y_1 = -b/3$ and $L_2(x) = y_2 = b/3$; that is, the widths of the regions are equal. This property remains valid for all x; therefore, $L(x)$ are the straight lines parallel to the x axis. They divide the aperture into three equal parts. Throughout the central region the phase is zero. In the outer regions, the phase distribution is described by the linear functions $\pi x/a$ and $-\pi x/a$.

Note that the order of the regions can be changed. For example, if we assume that the equiphase region with the width $(2b/\pi)\cos(\pi x/2a)$ starts at the lower aperture boundary, (8) takes the form

$$\begin{aligned} &(2b/\pi)\cos(\pi x/2a)\sin(\pi y_1/2b) = 0 \\ &(2b/\pi)\cos(\pi x/2a)\sin(\pi y_2/2b) = (b/\pi)\cos(\pi x/2a). \end{aligned} \tag{9}$$

From (9), we obtain that $y_1 = 0$ and $y_2 = b/3$. Therefore, the equiphase region occupies the lower half of the aperture, while the regions with the linear phase distributions occupy $1/6$ part of the aperture adjacent to its center line and $1/3$ part of the aperture adjacent to its upper aperture boundary. Both phase distributions provide the same given pattern.

III. Arbitrary Aperture with Nonuniform Amplitude Distribution

Consider an aperture of an arbitrary shape with overall dimensions $2a \times 2b$. Between the points $x = -a$ and a, the upper and lower boundaries of the aperture are described by the functions $Y_+(x)$ and $Y_-(x)$, respectively. The aperture amplitude distribution is given by the function $F(x, y)$. It is necessary to determine the phase distribution providing the given complex pattern $G(\theta, \varphi)$ in the plane $\varphi = 0$.

The required amplitude-phase distribution $I(x)$ on the ELAx is given by (1). The initial distribution on the ELAx is

$$I_{\text{init}}(x) = \int_{Y_-(x)}^{Y_+(x)} F(x, y)\, dy = F_1(x, y_+) - F_1(x, y_-). \quad (10)$$

The additional multiplier $f(x)$, which is necessary for obtaining the distribution $I(x)$ on the ELAx, is

$$\begin{aligned} f(x) &= I(x)/I_{\text{init}}(x) \\ &= \sum_{n=-N}^{N} g(n\pi) e^{jn\pi x/a} / (F_1(x, y_+) - F_1(x, y_-)). \end{aligned} \quad (11)$$

Expanding $f(x)$ in the Fourier series, we obtain

$$f(x) = \sum_{n=-N}^{N} g_1(n\pi) e^{jn\pi x/a}.$$

In order to divide the aperture into regions with linear phase distributions with account of the amplitude distribution, we use a procedure similar to the one used above for the rectangular aperture. As a result, we calculate the widths of the regions

$$h(n, x) = g_1(n\pi)(F_1(x, y_+) - F_1(x, y_-)) \Big/ \sum_{n=-N}^{N} g_1(n\pi) \quad (12)$$

where $n = -N, \ldots, N$, i.e., the upper boundary of the lowest strip is $F_1(x, y_-) + h(-N, x)$, the upper boundary of the next strip is $F_1(x, y_-) + h(-N, x) + h(-N + 1, x)$, etc.

IV. Possibility of Obtaining an Exact Required Amplitude-Phase Distribution on the ELAx

The method for synthesizing a given pattern $G(\theta, 0)$ (proposed above) includes two levels of approximation. The first level is the representation of the given pattern as a truncated series. The second level is the representation of the multiplier $f(x)$ in the form of the truncated series. In both cases, the exact solution is possible only for a limited class of functions $G(\theta, 0)$ and $f(x)$ that can be represented as a finite exponential series.

However, this method provides a possibility of obtaining an exact formula for $f(x)$ and, therefore, the exact field distribution on the ELAx.

Let us consider this possibility. Assume that

$$f(x) = \cos^N(\alpha(x)). \quad (13)$$

That is, in order to implement $f(x)$, we should divide the aperture into $N + 1$ strips whose phase distributions are proportional to $\alpha(x)$, where

$$\alpha(x) = \arccos \sqrt[N]{f(x)}. \quad (14)$$

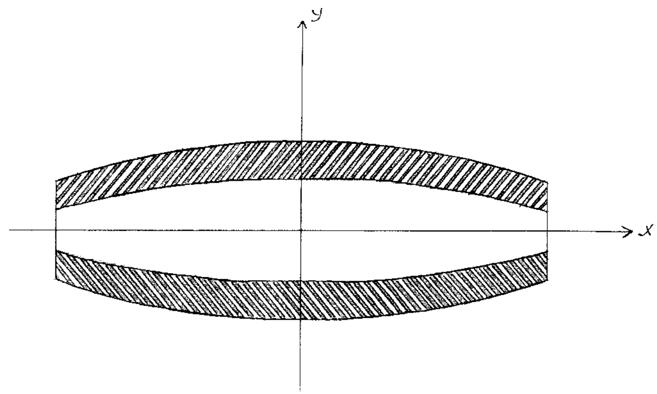

Fig. 2. Antenna aperture divided into curvilinear strips with the phase distributions $-2\alpha(x)$, 0, and $2\alpha(x)$ across the lower, middle, and upper strips, respectively.

The widths of these strips and the lines $L_n(x)$ are calculated by (10)–(12). As in (11), in order to find the phase distribution yielding the desired distribution $I(x)$ on the ELAx, we introduce the function $f(x) = I(x)/I_{\text{init}}(x)$.

In particular, at $N = 1$

$$f(x) = \cos(\alpha_1(x)) = 0.5\left(e^{j\alpha_1(x)} + e^{-j\alpha_1(x)}\right)$$

and, at $N = 2$

$$f(x) = \cos^2(\alpha_2(x)) = 0.25\left(e^{j2\alpha_2(x)} + 2 + e^{-j2\alpha_2(x)}\right).$$

That is, in the first case, $f(x)$ can be realized exactly by constructing the phase distributions $\alpha_1(x)$ on the upper strip and $-\alpha_1(x)$ on the lower strip. In the second case, one should provide distributions $2\alpha_2(x)$, 0, and $-2\alpha_2(x)$ on the upper, central, and lower strips, respectively.

Clearly, it is more convenient to divide the aperture into two strips $(N = 1)$ since when the aperture shape and amplitude distribution are symmetric with respect to the x axis, i.e.,

$$y_+(x) = -y_-(x) \quad \text{and} \quad F(x, y) = F(x, -y)$$

the upper half of the aperture plays the part of the upper strip and the lower half represents the lower one $[L(x) = 0]$. Therefore, there is no need in a rather complex searching for the lines $L(x)$.

Note that in this way we obtain identical patterns in the plane $\varphi = 0$ for different N, but in other directions, the patterns may differ substantially. Therefore, the shape of the pattern for angles different from $\varphi = 0$ can be used as a criterion for the best number of strips $N + 1$.

Consider an example. Let the antenna aperture (Fig. 2) be bounded by two arcs $y_+(x)$ and $y_-(x)$ of radius $3.2b$ (with centers symmetrically displaced from the origin by $R_0 = 2.2b$) and by two linear segments about $0.6b$ in length. The shape of this aperture is close to a $2a \times 2b$ ellipse with $a = 2b$

$$y_+(x) = \sqrt{R^2 - x^2} - R_0 \quad \text{and}$$
$$y_-(x) = -(\sqrt{R^2 - x^2} - R_0).$$

Assume that the amplitude distribution on the aperture is

$$F(x, y) = (1 - k_1 x^2/a^2)(1 - k_2 y^2/b^2). \quad (15)$$

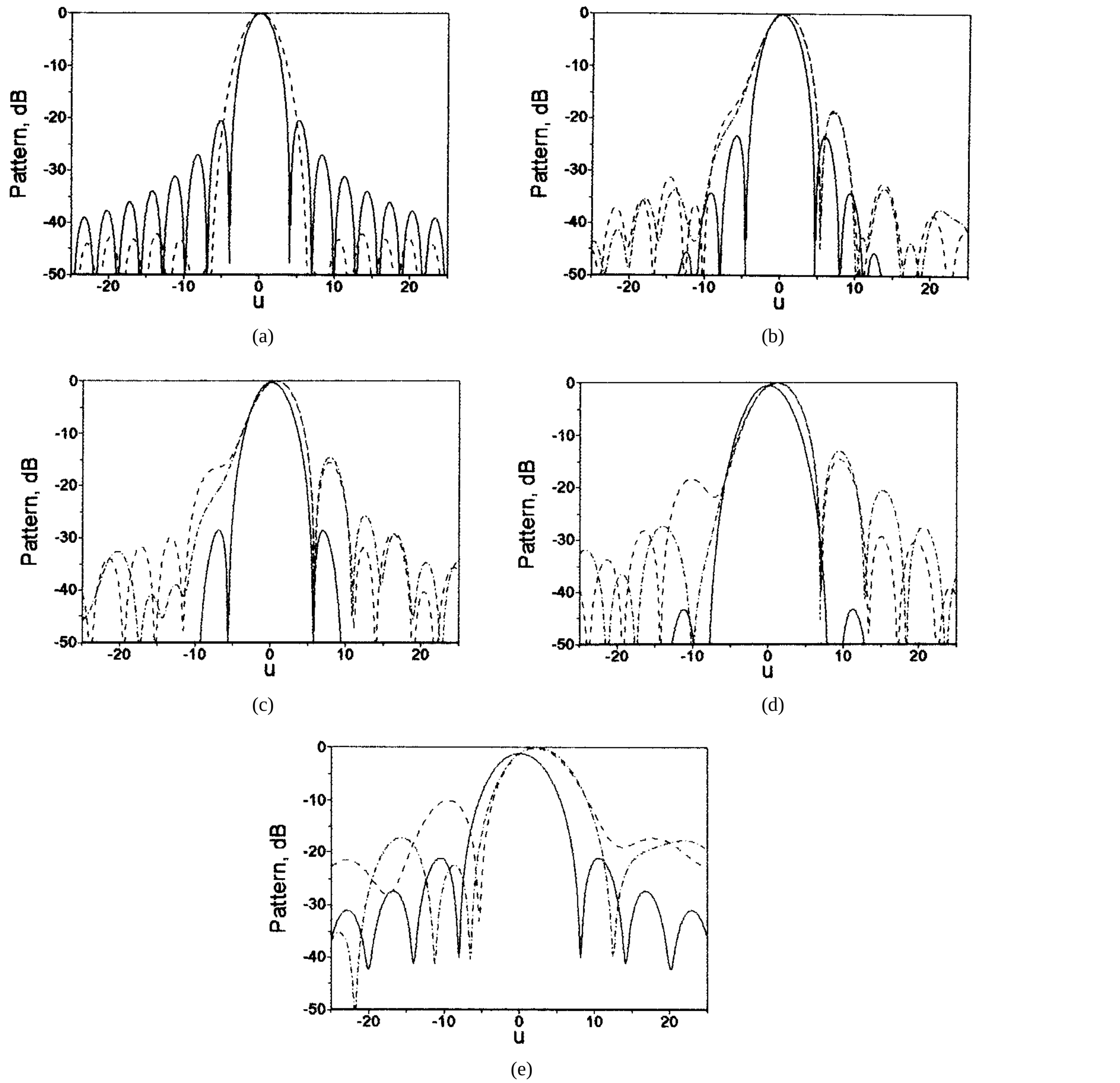

Fig. 3. Initial (solid line) and synthesized patterns for the aperture divided into two (dash line) and three (dotted lines) regions for $k_1 = k_2 = 0.8$. (a) $\varphi = 0°$. (b) $30°$. (c) $45°$. (d) $60°$. (e) $90°$, $u = 2\pi a \sin\vartheta/\lambda$.

It is necessary to provide the Hamming amplitude distribution [6]

$$I_H(x) = 0.54 + 0.46\cos(\pi x/a). \tag{16}$$

on the ELAx. Then,

$$\begin{aligned} f(x) &= I_H(x)/I_{\text{init}}(x) \\ &= (0.54 + 0.46\cos(\pi x/a))/(F_1(x, y_+) - F_1(x, y_-)). \end{aligned}$$

Consider two particular cases.

1) The aperture is divided into two strips, i.e., $f(x) = \cos(\alpha_1(x))$. In this case, $L(x) = y(0)$ and the phase distributions over the upper and lower halves of the aperture are $\alpha_1(x)$ and $-\alpha_1(x)$, respectively, where $\alpha_1(x) = \arccos[f(x)]$. Then

$$F_\alpha(x, y) = F(x, y)e^{j\alpha_1(x)\,\text{sign}(y)}. \tag{17}$$

2) The aperture is divided into three strips, i.e., $f(x) = \cos^2[\alpha_2(x)]$. In this case, the phase distributions $2\alpha_2(x)$, 0, and $-2\alpha_2(x)$ are imposed on the upper, middle, and lower strips, respectively, where $\alpha_2(x) = \arccos\sqrt{[f(x)]}$. The widths of the strips are determined from (10), (12), (15), and (16)

$$\begin{aligned} h(-1, x) &= (F_1(x, y_+) - F_1(x, y_-))/4 = h(+1, x) \\ h(0, x) &= 2h(1, x), \end{aligned} \tag{18}$$

$$F_1(x, y)|_{y_-}^{y_+} = \int_{y_-}^{y_+} (1 - k_1x^2/a^2)(1 - k_2y^2/b^2)\,dy \tag{19}$$

$$\begin{aligned} I_{\text{init}}(x) = 2(1 - k_1x^2/a^2)\{\sqrt{(R^2 - x^2)} - R_0 \\ - k_2[\sqrt{(R^2 - x^2)} - R_0]^3/3b^2\}. \end{aligned} \tag{20}$$

The upper boundary of the lowest strip is $F_1(x, y_-) + h(-1, x)$. The upper boundary of the middle strip is

$F_1(x, y_-) + h(-1, x) + h(0, x)$. Equations (18)–(20) yield

$$y_{1,2} - k_2 y_{1,2}^3/b^2 = \pm 0.5\{\sqrt{(R^2 - x^2)} - R_0 - k_2[\sqrt{(R^2 - x^2)} - R_0]^3/3b^2\}$$

where $y_{1,2} = L_{1,2}(x)$ are the equations of the strip boundaries (Fig. 2). Therefore

$$F_\alpha(x,y) = F(x,y)\begin{cases} e^{j2\alpha_2(x)} & \sqrt{(R_0^2 - x^2)} - R_0 > Y > L_2(x) \\ 1 & L_1(x) < Y < L_2(x) \\ e^{-j2\alpha_2(x)} & L_1(x) > Y > -[\sqrt{(R_0^2 - x^2)} - R_0] \end{cases} \quad (21)$$

We calculated patterns of this aperture with amplitude-phase distributions (17) and (21) by

$$G(\theta, \varphi) = \int_{-a}^{a} dx \int_{-[\sqrt{(R^2-x^2)}-R_0]}^{\sqrt{(R^2-x^2)}-R_0} F_\alpha(x,y) \times e^{j(2\pi/\lambda)\sin\theta(x\cos\varphi + y\sin\varphi)}\, dy.$$

Fig. 3(a)–(e) shows the calculated patterns of amplitude-phase distributions (17) (dashed lines) and (21) (dotted lines) and of the initial distribution (solid lines) for $k_1 = k_2 = 0.8$ and $\varphi = 0, 30, 45, 60$, and $90°$, respectively. Fig. 3(a) shows that the patterns in the plane $\varphi = 0$ are independent of the way in which the Hamming distribution on the ELAx is obtained.

At $\varphi \neq 0$, maximum sidelobe level and distortions of the pattern shape are smaller for the aperture divided into three parts. In particular, at $\varphi = 90°$, for distribution (17), the maximum sidelobe level is −8.6 dB, while for distribution (21) it is −13.5 dB. Note that the pattern maximum shifts from the broadside direction in the plane $\varphi = 90°$. This shift Δu is 2.05 and 2.1 for the apertures divided into two and three parts, respectively.

V. Conclusion

The method for the phase synthesis of antenna patterns in one plane proposed in [1] is extended to antennas with arbitrary apertures and amplitude distributions. On the basis of this method, a procedure of exact implementation of a given amplitude distribution along the equivalent linear antenna is developed.

This method provides a simple algorithm for controlling the shape of the phased array pattern.

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