Interaction Effects in Two-Dimensional Bianisotropic Arrays

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***Abstract*—Electromagnetic excitation of two-dimensional (2-D) arrays of bianisotropic particles by plane waves is considered. Arrays (grids) are assumed to be infinite and the particles to be small compared to the wavelength, so that the dipole approximation is possible. The elecromagnetic interaction between all the particles changes the properties of these particles (in particular, chiral and omega particles are studied). The analytical model under consideration allows to express the electric and magnetic moments induced in each particle through the incident wave fields in terms of collective polarizability dyadics (CPD's). The proposed method to evaluate these dyadics combines numerical and analytical parts. The results of the calculations of the induced electric and magnetic moments by plane waves are presented for a planar arrangement of omega particles.**

***Index Terms*—Bianisotropic particles, dipole model, grid plane, interaction dyadics, particle interaction.**

I. Introduction

The modern electromagnetics literature contains many papers devoted to artificial bianisotropic media (see, e.g., [1]). At microwaves, we usually deal with composites of conductive inclusions embedded into dielectric matrices. These inclusions often have the shape similar to that of helices (chiral particles) or to the shape of capital Greek letter Ω (omega particles) [2]. Such particles have four polarizabilities: electric, magnetic, electromagnetic, and magnetoelectric. The polarizabilities are dyadic functions of frequency and geometrical parameters [1], [2]

$$\begin{aligned} \boldsymbol{p} &= \overline{\overline{a}}_{ee}\cdot\boldsymbol{E}+\overline{\overline{a}}_{em}\cdot\boldsymbol{H} \\ \boldsymbol{m} &= \overline{\overline{a}}_{me}\cdot\boldsymbol{E}+\overline{\overline{a}}_{mm}\cdot\boldsymbol{H}. \end{aligned} \tag{1}$$

Here, $\boldsymbol{E}$ and $\boldsymbol{H}$ are the local electric and magnetic fields amplitudes at the point where the particle is located. It is important to note that the magnetoelectric properties of such composites are essential only for frequencies near the particle resonance when the full length of the particle wire is about one half of the wavelength (see, e.g., [1]–[4]). It means that the stretched particle length $2l$ is small compared to the wavelength λ (about $\lambda/8$) and the dipole approximation of a particle is available. Often, studies of the polarizability dyadics of individual omega and chiral particles are carried out with

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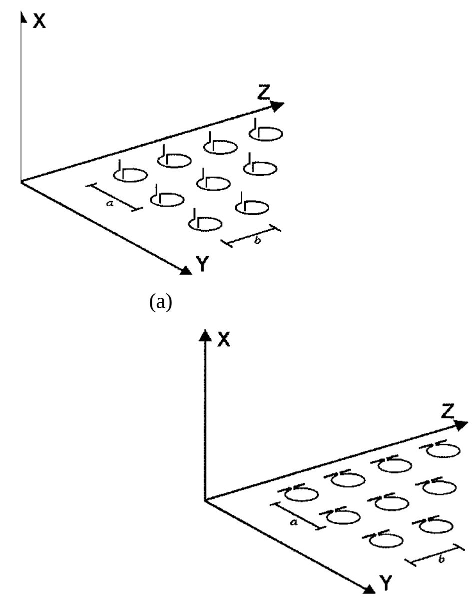


(a) (b)

Fig. 1. (a) Chiral particles and (b) omega particles planar arrangement.

the use of the so-called wire-and-loop model [3], [4]. It is an analytical model for single bianisotropic particles (BAP). Following [3], [4] we can write for a chiral particle

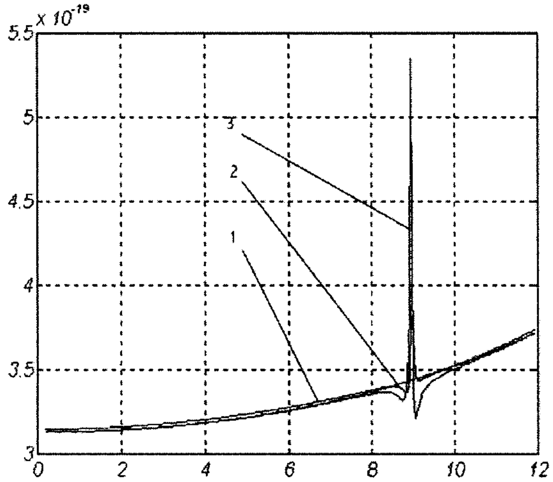
$$\begin{aligned} \overline{\overline{a}}_{ee} &= a_{ee}^{xx}x_0x_0+a_{ee}^{xz}x_0z_0+a_{ee}^{xz}z_0x_0+a_{ee}^{zz}z_0z_0+a_{ee}^{yy}y_0y_0 \\ \overline{\overline{a}}_{em} &= a_{em}^{xx}x_0x_0+a_{em}^{xz}z_0x_0 \\ \overline{\overline{a}}_{me} &= -a_{em}^{xx}x_0x_0-a_{em}^{xz}x_0z_0 \\ \overline{\overline{a}}_{mm} &= a_{mm}x_0x_0. \end{aligned} \tag{2a}$$

Here the axis x is directed alond the particle axis (see Fig. 1). For an omega particle with the stems oriented along the z axis [see Fig. 1(b)] we can write

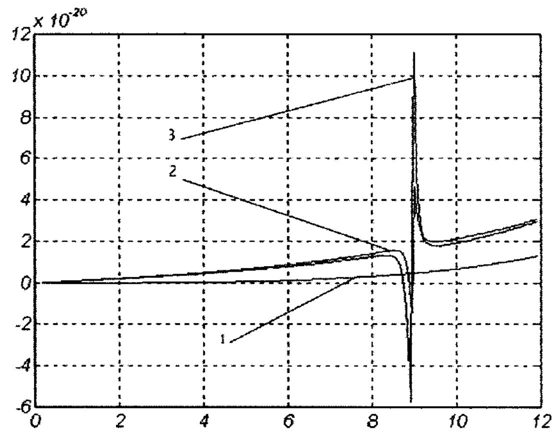
$$\begin{aligned} \overline{\overline{a}}_{ee} &= a_e^y y_0y_0+a_e^z z_0z_0 \\ \overline{\overline{a}}_{em} &= -a_{me}z_0x_0 \\ \overline{\overline{a}}_{me} &= a_{me}x_0z_0 \\ \overline{\overline{a}}_{mm} &= a_{mm}x_0x_0. \end{aligned} \tag{2b}$$

The scalar coefficients in these relations are given in [4].

It is clear that the question of electromagnetic coupling of BAP's in composite structures is important. But there is no complete theory that would take into account the electromagnetic interaction effects for such particles. The existing analytical [5] and numerical [6] are applicable for three-dimensional (3-D) (volume) mixtures and cannot be directly



(a)



(b)

Fig. 2. (a) $a = 10$ mm, $\mathrm{Re}(F_{ee}^{yy})$ and $\mathrm{Re}(a_{ee}^{yy})$. (b) $\mathrm{Im}(F_{ee}^{yy})$ and $\mathrm{Im}(a_{ee}^{yy})$. Horizontal axis—frequency in gigahertz.

used for planar two-dimensional (2-D) arrangements of particles. Maxwell Garnett approach [5] is available only for dilute volume mixtures of particles with 3-D distributions of inclusions. Papers [6] and [7] present an approach to solving the problem of mutual coupling for chiral and some planar particles. But they give no analytical solutions being based on the numerical modeling of scatterers within the frame of the multiple scattering method.

II. Formulation of the Problem

In this paper, a 2-D array (planar grid) of chiral and omega BAP's with a rectangular cell $a \times b$ is studied. In this case, the electromagnetic coupling is an important factor. We are interested in the cases when the particle size $2l$ is two or more times smaller compared to the space periods of the grid a, b. Strong electromagnetic interaction is expected for the case when $2l \approx \lambda/8, a \approx \lambda/4 \cdots \lambda/2$ (resonant particles and rather dense grids). So, it is possible to use the dipole approximation of BAP's in our analytical study. Of course, it is a serious problem to take into account the practical sizes and shapes of the particles in studying mutual coupling of adjacent particles. This question is considered in Appendix A.

Grids of BAP's under consideration (such as in Figs. 1 and 2) are placed in free-space. Let us choose among the BAP's an arbitrary particle named below *reference particle*. Let us place the origin of the Cartesian coordinate system at its center and denote the amplitude of the incident electric field at the point $x = y = z = 0$ as $\boldsymbol{E}_0$ and the electric and magnetic dipole moments of the reference particle as $p, m,$, respectively. For the field of the incident wave with the wave vector k, we have for the points belonging to the grid plane (time dependence is in the form $\exp(-i\omega t)$)

$$E_{\rm inc} = \boldsymbol{E}_0 \, \exp i(k_y y + k_z z). \tag{3}$$

Let us denote the amplitudes of the electric and magnetic fields produced by all the BAP's of the grid except the reference-particle at the point $x = y = z = 0$ as $\boldsymbol{E}', \boldsymbol{H}'$. The local field (that exciting the reference particle) equals

$$\begin{aligned} \boldsymbol{E} &= \boldsymbol{E}' + \boldsymbol{E}_0 \\ \boldsymbol{H} &= \boldsymbol{H}' + \boldsymbol{H}_0. \end{aligned} \tag{4}$$

Periodicity of our problem allows us to assume that for an arbitrary element of the array (with the numbers m, n of the cell corresponding to the OY, OZ axes, respectively) the following relations hold:

$$\begin{aligned}\boldsymbol{p}_{mn} &= p\ \exp i(k_y ma + k_z nb)\\ \boldsymbol{m}_{mn} &= m\ \exp i(k_y ma + k_z nb).\end{aligned} \tag{5}$$

Assumption (5), which means that the phases of the induced dipoles vary accordingly to the incident wave phase, probably cannot be strictly proven for bianisotropic particles. However, in most cases (5) is obviously correct, so we make this assumption

$$\begin{aligned}\boldsymbol{E}' &= \overline{\overline{A}}_{ee}\cdot\boldsymbol{p} + \overline{\overline{A}}_{em}\cdot\boldsymbol{m}\\ \boldsymbol{H}' &= \overline{\overline{A}}_{me}\cdot\boldsymbol{p} + \overline{\overline{A}}_{mm}\cdot\boldsymbol{m}.\end{aligned} \tag{6}$$

To find the dyadics $\overline{\overline{A}}_{ee}, \overline{\overline{A}}_{em}, \overline{\overline{A}}_{me}, \overline{\overline{A}}_{mm}$, we have to calculate the sums of the electric and magnetic fields produced by the electric and magnetic dipoles with the unit amplitudes and the phases given by (5), which are placed at the centers of the grid cells. These sums do not include contributions from the reference-particle dipoles. These fields are calculated at the point $x = y = z = 0$. We will call these dyadics *interaction dyadics*.

Let us denote by $\overline{\overline{F}}_{ee}, \overline{\overline{F}}_{em}, \overline{\overline{F}}_{me}, \overline{\overline{F}}_{mm}$ the dyadics that relate the electric and magnetic moments $\boldsymbol{p}$ and $\boldsymbol{m}$ of the reference particle with the incident wave fields amplitudes $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$, and call them *collective polarizability dyadics* (CPD's)

$$\begin{aligned}\boldsymbol{p} &= \overline{\overline{F}}_{ee}\cdot\boldsymbol{E}_0 + \overline{\overline{F}}_{em}\cdot\boldsymbol{H}_0\\ \boldsymbol{m} &= \overline{\overline{F}}_{me}\cdot\boldsymbol{E}_0 + \overline{\overline{F}}_{mm}\cdot\boldsymbol{H}_0.\end{aligned} \tag{7}$$

After some dyadic algebra we find from (1), (6), and (7)

$$\begin{aligned}\overline{\overline{F}}_{ee} &= (\overline{\overline{W}}_e - \overline{\overline{V}}_m\cdot\overline{\overline{W}}_m^{-1}\cdot\overline{\overline{V}}_e)^{-1}\\ &\quad\cdot(\overline{\overline{a}}_{ee} + \overline{\overline{V}}_m\cdot\overline{\overline{W}}_m^{-1}\cdot\overline{\overline{a}}_{me})\\ \overline{\overline{F}}_{em} &= (\overline{\overline{W}}_e - \overline{\overline{V}}_m\cdot\overline{\overline{W}}_m^{-1}\cdot\overline{\overline{V}}_e)^{-1}\\ &\quad\cdot(\overline{\overline{a}}_{em} + \overline{\overline{V}}_m\cdot\overline{\overline{W}}_m^{-1}\cdot\overline{\overline{a}}_{mm})\\ \overline{\overline{F}}_{me} &= (\overline{\overline{W}}_m - \overline{\overline{V}}_e\cdot\overline{\overline{W}}_e^{-1}\cdot\overline{\overline{V}}_m)^{-1}\\ &\quad\cdot(\overline{\overline{a}}_{me} + \overline{\overline{V}}_e\cdot\overline{\overline{W}}_e^{-1}\cdot\overline{\overline{a}}_{ee})\\ \overline{\overline{F}}_{mm} &= (\overline{\overline{W}}_m - \overline{\overline{V}}_e\cdot\overline{\overline{W}}_e^{-1}\cdot\overline{\overline{V}}_m)^{-1}\\ &\quad\cdot(\overline{\overline{a}}_{mm} + \overline{\overline{V}}_e\cdot\overline{\overline{W}}_e^{-1}\cdot\overline{\overline{a}}_{em})\end{aligned} \tag{8}$$

where

$$\begin{aligned}\overline{\overline{W}}_e &= \overline{\overline{I}} - \overline{\overline{a}}_{ee}\cdot\overline{\overline{A}}_{ee} - \overline{\overline{a}}_{em}\cdot\overline{\overline{A}}_{me}\\ \overline{\overline{W}}_m &= \overline{\overline{I}} - \overline{\overline{a}}_{mm}\cdot\overline{\overline{A}}_{mm} - \overline{\overline{a}}_{me}\cdot\overline{\overline{A}}_{em}\\ \overline{\overline{V}}_e &= \overline{\overline{a}}_{mm}\cdot\overline{\overline{A}}_{me} + \overline{\overline{a}}_{me}\cdot\overline{\overline{A}}_{ee}\\ \overline{\overline{V}}_m &= \overline{\overline{a}}_{ee}\cdot\overline{\overline{A}}_{em} + \overline{\overline{a}}_{em}\cdot\overline{\overline{A}}_{mm}.\end{aligned} \tag{9}$$

The polarizability dyadics $\overline{\overline{a}}_{ij}$ for our BAP's are known (see above) and the interaction dyadics $\overline{\overline{A}}_{ij}$ of our grid of the general kind we calculate below. The problem is, hence, to find the CPD's for grids of chiral and omega BAP's as functions of the interaction dyadics and polarizability dyadics.

The dependence on the polarization of the incident wave is included in the structure of CPD's dyadics because each component of these is a Cartesian component of an electric or magnetic moment induced in the reference-BAP by a certain component of the incident electric or magnetic fields with unit amplitudes and phases given by (5).

III. Interaction Dyadics Evaluation—Series Representation

Electric and magnetic fields of electric and magnetic dipoles at a point displaced from these dipoles by distance R are given by the well-known formulas

$$\begin{aligned}\boldsymbol{E}_p &= \frac{k^2}{4\pi\epsilon\epsilon_0}\frac{[[\boldsymbol{R}\times\boldsymbol{p}]\times\boldsymbol{R}]}{R^3}\left(1+\frac{i}{kR}-\frac{1}{k^2R^2}\right)e^{ikR}\\ &\quad+\frac{k^2}{2\pi\epsilon\epsilon_0}\frac{\boldsymbol{R}(\boldsymbol{R}\cdot\boldsymbol{p})}{R^3}\left(-\frac{i}{kR}+\frac{1}{k^2R^2}\right)e^{ikR}\\ \boldsymbol{H}_p &= \frac{k^2}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\frac{[\boldsymbol{R}\times\boldsymbol{p}]}{R^2}\left(1+\frac{i}{kR}\right)e^{ikR}\end{aligned} \tag{10a}$$

$$\begin{aligned}\boldsymbol{E}_m &= \frac{k^2}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\frac{[\boldsymbol{R}\times\boldsymbol{m}]}{R^2}\left(1+\frac{i}{kR}\right)e^{ikR}\\ \boldsymbol{H}_m &= \frac{k^2}{4\pi\mu\mu_0}\frac{[[\boldsymbol{R}\times\boldsymbol{m}]\times\boldsymbol{R}]}{R^3}\left(1+\frac{i}{kR}-\frac{1}{k^2R^2}\right)e^{ikR}\\ &\quad+\frac{k^2}{2\pi\mu\mu_0}\frac{\boldsymbol{R}(\boldsymbol{R}\cdot\boldsymbol{m})}{R^3}\left(-\frac{i}{kR}+\frac{1}{k^2R^2}\right)e^{ikR}.\end{aligned} \tag{10b}$$

It is instructive to rewrite (10) using three dyadics $\overline{\overline{A}}(x,y,z)$, $\overline{\overline{B}}(x,y,z)$, and $\overline{\overline{C}}(x,y,z)$

$$\begin{aligned}\boldsymbol{E}_p &= \frac{k^2}{4\pi\epsilon\epsilon_0}\frac{e^{ikR}}{R^3}\left(\overline{\overline{A}}(x,y,z)\right.\\ &\quad\left.+\left(-\frac{i}{kR}+\frac{1}{k^2R^2}\right)\overline{\overline{C}}(x,y,z)\right)\cdot\boldsymbol{p}\\ \boldsymbol{H}_p &= \frac{k^2}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\frac{e^{ikR}}{R^2}\left(1+\frac{i}{kR}\right)\overline{\overline{B}}(x,y,z)\cdot\boldsymbol{p}\end{aligned} \tag{11a}$$

$$\begin{aligned}\boldsymbol{E}_m &= \frac{k^2}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\frac{e^{ikR}}{R^2}\left(1+\frac{i}{kR}\right)\overline{\overline{B}}(x,y,z)\cdot\boldsymbol{m}\\ \boldsymbol{H}_m &= \frac{k^2}{4\pi\mu\mu_0}\frac{e^{ikR}}{R^3}\left(\overline{\overline{A}}(x,y,z)\right.\\ &\quad\left.+\left(-\frac{i}{kR}+\frac{1}{k^2R^2}\right)\overline{\overline{C}}(x,y,z)\right)\cdot\boldsymbol{m}.\end{aligned} \tag{11b}$$

The matrix form is more convenient to analyze the following dyadics:

$$\overline{\overline{A}}(x,y,z) = \begin{pmatrix}-(y^2+z^2) & xy & xz\\ xy & -(x^2+z^2) & yz\\ xz & yz & -(x^2+y^2)\end{pmatrix} \tag{12a}$$

$$\overline{\overline{B}}(x,y,z) = \begin{pmatrix}0 & -z & y\\ z & 0 & -x\\ -y & x & 0\end{pmatrix} \tag{12b}$$

$$\overline{\overline{C}}(x,y,z) = \begin{pmatrix}-2x^2+y^2+z^2 & xy & xz\\ xy & 2y^2+x^2+z^2 & yz\\ xz & yz & -2z^2+x^2+y^2\end{pmatrix}. \tag{12c}$$

For generality, let us consider here the interaction dyadics of a general periodic translationary invariant structure of scatterers including those that are 3-D, where the periodicity condition (5) is assumed to hold. From (5) and (11) we have

$$\overline{\overline{A}}_{ee} = \frac{k^2}{4\pi\epsilon\epsilon_0}\sum_{j\neq 0}\left[\overline{\overline{A}}(x_j,y_j,z_j) + \left(-\frac{i}{kR_j}+\frac{1}{k^2R_j^2}\right)\cdot\overline{\overline{C}}(x_j,y_j,z_j)\right]\frac{e^{ikR_j+i\boldsymbol{k}\cdot\boldsymbol{R}_j}}{R_j^3} \quad (13a)$$

$$\overline{\overline{A}}_{em} = \frac{k^2}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\sum_{j\neq 0}\left(1+\frac{i}{kR_j}\right)\overline{\overline{B}}(x_j,y_j,z_j)\cdot\frac{e^{ikR_j+i\boldsymbol{k}\cdot\boldsymbol{R}_j}}{R_j^2} \quad (13b)$$

$$\overline{\overline{A}}_{me} = \frac{k^2}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\sum_{j\neq 0}\left(1+\frac{i}{kR_j}\right)\overline{\overline{B}}(x_j,y_j,z_j)\cdot\frac{e^{ikR_j+i\boldsymbol{k}\cdot\boldsymbol{R}_j}}{R_j^2} \quad (13c)$$

$$\overline{\overline{A}}_{mm} = \frac{k^2}{4\pi\mu\mu_0}\sum_{j\neq 0}\left[\overline{\overline{A}}(x_j,y_j,z_j) + \left(-\frac{i}{kR_j}+\frac{1}{k^2R_j^2}\right)\cdot\overline{\overline{C}}(x_j,y_j,z_j)\right]\frac{e^{ikR_j+i\boldsymbol{k}\cdot\boldsymbol{R}_j}}{R_j^3}. \quad (13d)$$

Here, j numbers particles in the 2-D array. It is evident that

$$\overline{\overline{A}}_{ee} = \frac{\mu\mu_0}{\epsilon\epsilon_0}\overline{\overline{A}}_{mm}, \quad A_{em} = \overline{\overline{A}}_{me}. \quad (14)$$

The dyadics $\overline{\overline{A}}_{ee}, \overline{\overline{A}}_{mm}$ are symmetric and $\overline{\overline{A}}_{em}, \overline{\overline{A}}_{me}$ are antisymmetric. We evaluate $\overline{\overline{A}}_{ee}$ and $\overline{\overline{A}}_{em}$ and use (14) to find the other interaction dyadics.

Let us separate two terms in relations (10). The first term describes the wave-zone field (wave-field) of these dipoles (it decreases as $1/R$) and the second one is the near-zone field (which includes $1/R^2$ and $1/R^3$ terms). We name this second term in (10a) and (10b) as *near-zone*. The representation of the interaction dyadics with separated wave terms and near-zone terms is

$$\overline{\overline{A}}_{ee,em,me,mm} = \overline{\overline{A}}^{\rm w}_{ee,em,me,mm} + \overline{\overline{A}}^{\rm nz}_{ee,em,me,mm}$$

where

$$\overline{\overline{A}}^{\rm w}_{ee} = \frac{k^2}{4\pi\epsilon\epsilon_0}\sum_{j\neq 0}\overline{\overline{A}}(x_j,y_j,z_j)\frac{e^{ikR_j+i\boldsymbol{k}\cdot\boldsymbol{R}_j}}{R_j^3} \quad (15a)$$

$$\overline{\overline{A}}^{\rm w}_{em} = \frac{k^2}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\sum_{j\neq 0}\overline{\overline{B}}(x_j,y_j,z_j)\frac{e^{ikR_j+i\boldsymbol{k}\cdot\boldsymbol{R}_j}}{R_j^2} \quad (15b)$$

and near-zone terms are

$$\overline{\overline{A}}^{\rm nz}_{ee} = \frac{1}{4\pi\epsilon\epsilon_0}\sum_{j\neq 0}\left(-\frac{ik}{R_j}+\frac{1}{R_j^2}\right)\overline{\overline{C}}(x_j,y_j,z_j)\cdot\frac{e^{ikR_j+i\boldsymbol{k}\cdot\boldsymbol{R}_j}}{R_j^3} \quad (16a)$$

$$\overline{\overline{A}}^{\rm nz}_{em} = \frac{ik}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\sum_{j\neq 0}\overline{\overline{B}}(x_j,y_j,z_j)\cdot\frac{e^{ikR_j+i\boldsymbol{k}\cdot\boldsymbol{R}_j}}{R_j^3}. \quad (16b)$$

All the series in (16) absolutely converges and can be easily calculated numerically (terms decrease as $1/R^2$ or faster). But the convergence of the series in (15) is so slow that they cannot be calculated without using special methods. Such methods are known in the theory of microwave scanning antenna arrays formed by horizontal electric or magnetic dipoles [9]. We have applied the method of imaginary screen [9] in numerical calculations of series (15) to validate our analytical approach given below. This method is convenient for this goal but it is not realistic for detailed numerical study, because it still requires too much calculation time which increases with the grid density and it is complicated as such.

IV. Interaction Dyadics. Integral Representation

Let us take into account that all the scatterers of our grid are located in the plane $x=0$. In this case, the matrices $\overline{\overline{A}}$, $\overline{\overline{B}}$, and $\overline{\overline{C}}$ have the form

$$\overline{\overline{A}}(x,y,z) = \begin{pmatrix} -(y^2+z^2) & 0 & 0 \\ 0 & -z^2 & yz \\ 0 & yz & -y^2 \end{pmatrix} \quad (17a)$$

$$\overline{\overline{B}}(x,y,z) = \begin{pmatrix} 0 & -z & y \\ z & 0 & 0 \\ -y & 0 & 0 \end{pmatrix} \quad (17b)$$

$$\overline{\overline{C}}(x,y,z) = \begin{pmatrix} y^2+z^2 & 0 & 0 \\ 0 & 2y^2+z^2 & yz \\ 0 & yz & -2z^2+y^2 \end{pmatrix}. \quad (17c)$$

Let us introduce the notations presenting the interaction dyadics in the matrix form

$$\overline{\overline{A}}_{ee} = \begin{pmatrix} A & 0 & 0 \\ 0 & A_1 & A_2 \\ 0 & A_2 & A_3 \end{pmatrix}, \quad \overline{\overline{A}}_{em} = \begin{pmatrix} 0 & A' & A'' \\ -A' & 0 & 0 \\ -A' & 0 & 0 \end{pmatrix}. \quad (18)$$

In this section, we will take into account in all the components of dyadics (18) only the wave terms given by (15). Let us replace the sums in (15) by the corresponding integrals (validity conditions discussed in Appendix B)

$$A' = -\frac{k^2n}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\, v.p. \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{z}{y^2+z^2}\cdot e^{i(k\sqrt{z^2+y^2}+k_zz+k_yy)}\,dy\,dz \quad (19a)$$

$$A'' = \frac{k^2n}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\, v.p. \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{y}{y^2+z^2}\cdot e^{i(k\sqrt{z^2+y^2}+k_zz+k_yy)}\,dy\,dz \quad (19b)$$

$$A = -\frac{k^2n}{4\pi\epsilon\epsilon_0}\, v.p. \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{y^2+z^2}}\cdot e^{i(k\sqrt{z^2+y^2}+k_zz+k_yy)}\,dy\,dz \quad (19c)$$

$$A_1 = -\frac{k^2n}{4\pi\epsilon\epsilon_0}\, v.p. \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{z^2}{\sqrt{y^2+z^2}^3}\cdot e^{i(k\sqrt{z^2+y^2}+k_zz+k_yy)}\,dy\,dz \quad (19d)$$

$$A_2 = \frac{k^2 n}{4\pi\epsilon\epsilon_0} \, v.p. \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{yz}{\sqrt{y^2+z^2}^3} \cdot e^{i(k\sqrt{z^2+y^2}+k_z z+k_y y)} \, dy \, dz \tag{19e}$$

$$A_3 = -\frac{k^2 n}{4\pi\epsilon\epsilon_0} \, v.p. \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{y^2}{\sqrt{y^2+z^2}^3} \cdot e^{i(k\sqrt{z^2+y^2}+k_z z+k_y y)} \, dy \, dz. \tag{19f}$$

Here, $n = 1/ab$ is the surface concentration of particles and the meaning of symbol $v.p.$ is that we exclude from the plane OXY an area of the reference-particle cell (principal value of the integral).

Evaluation of all these integrals by the method of residues is given in Appendix B. Finally, we obtain for the wave terms of the interaction dyadics

$$\overline{\overline{A}}^{\mathrm{w}}_{ee} = \frac{ik^2 n}{2k_x\epsilon\epsilon_0}\left\{\boldsymbol{x}_0\boldsymbol{x}_0 + \frac{kk_x+k_x^2+k_z^2}{(k+k_x)^2}\boldsymbol{y}_0\boldsymbol{y}_0 + \frac{kk_x+k_x^2+k_y^2}{(k+k_x)^2}\boldsymbol{z}_0\boldsymbol{z}_0 - \frac{k_y k_z}{(k+k_x)^2} \cdot (y_0 z_0 + z_0 y_0)\right\} \tag{20a}$$

$$\overline{\overline{A}}^{\mathrm{w}}_{em} = \frac{ik^2 n}{2k_x\sqrt{\epsilon\epsilon_0\mu\mu_0}}\left\{\frac{k_z}{k+k_x}(\boldsymbol{y}_0\boldsymbol{x}_0 - \boldsymbol{x}_0\boldsymbol{y}_0) + \frac{k_y}{k+k_x}(\boldsymbol{x}_0\boldsymbol{z}_0 - \boldsymbol{z}_0\boldsymbol{x}_0)\right\}. \tag{20b}$$

We can see that using this method we can calculate only the imaginary parts of of all the wave terms of the interaction dyadics. Numerical validation shows that this limitation has little importance for dense grids $(ka, kb < 2)$, where the real part of each component of the interaction dyadics is practically that of its near-zone term.

V. An Example of CPD's for Arrays of Omega Particles

Consider the omega particles orientation presented in Fig. 2. The polarizability dyadics are given by (2b). After some dyadic algebra the CPD's are found in the form

$$\begin{aligned}
\overline{\overline{F}}_{ee} &= a_e^y \frac{K}{\Delta}\boldsymbol{y}_0\boldsymbol{y}_0 - a_e^y\frac{M}{\Delta}\boldsymbol{y}_0\boldsymbol{z}_0 - a_e^y\frac{M}{\Delta}\boldsymbol{z}_0\boldsymbol{y}_0 + a_e^z\frac{N}{\Delta}\boldsymbol{z}_0\boldsymbol{z}_0\\
\overline{\overline{F}}_{em} &= -a_e^y\frac{a_{mm}}{a_{me}}\frac{M}{\Delta}\boldsymbol{y}_0\boldsymbol{x}_0 - a_{me}\frac{N}{\Delta}\boldsymbol{z}_0\boldsymbol{x}_0\\
\overline{\overline{F}}_{me} &= a_e^y\frac{a_{mm}}{a_{me}}\frac{M}{\Delta}\boldsymbol{x}_0\boldsymbol{y}_0 + a_{me}\frac{N}{\Delta}\boldsymbol{x}_0\boldsymbol{z}_0\\
\overline{\overline{F}}_{mm} &= a_{mm}\frac{N}{\Delta}\boldsymbol{x}_0\boldsymbol{x}_0,
\end{aligned} \tag{21}$$

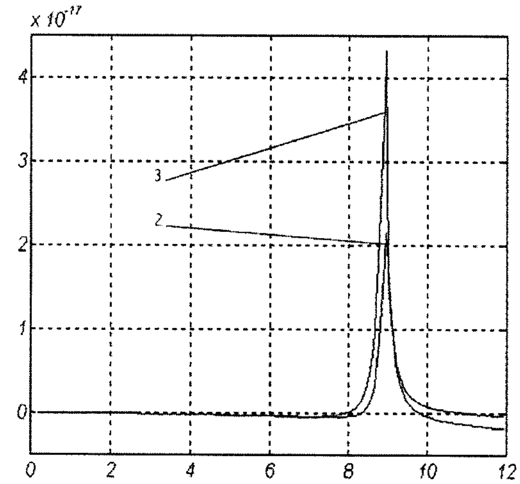
where $K = 1 - a_e^z A_3 + 2a_{me}A'' - a_{mm}A_m, M = a_{me}A' - a_e^z A_2, N = 1 - a_e^y A_1, \Delta = KN + (a_e^y/a_e^z)M^2$. This shows the role of the interaction terms in the polarizability dyadics. Due to this interaction the new components appears in the magnetoelectric, electromagnetic and electric polarizabilities (term M is zero for noninteracting particles as well as $a_{ee}^{yz,zy}, a_{me}^{xy}, a_{em}^{yx}$). Also the terms describing the interactions *electric dipole–electric dipole* and *magnetic dipole–magnetic dipole*, enter K and N with the minus (it corresponds to decreasing of electric and magnetic polarizabilities as it is discussed below), whereas the interaction *electric dipole–magnetic dipole* leads to the term entering K with the plus sign, which is responsible for the new frequency dependence of yy component of the electric polarizability (also see the next section).

VI. Numerical Study of a Planar Omega Grid

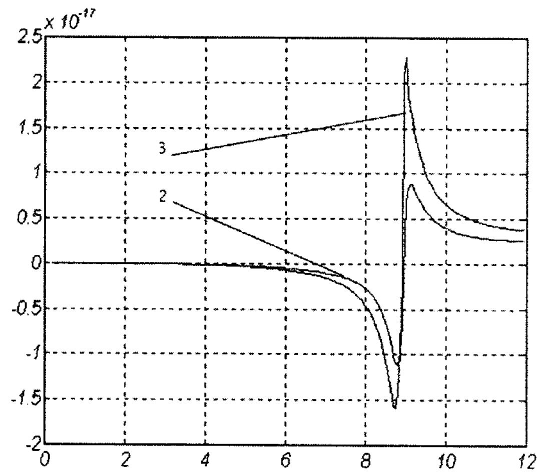
Among all the data, we choose to discuss the results for a planar omega grid. This structure is probably the easiest and cheapest to manufacture variant of bianisotropic arrays. It is shown in several investigations (for example, [8]) that planar omega layers with finite depth (which can be obtained with a few planar 2-D omega arrays) possess very interesting properties. We consider here a grid with square cells $(a = b)$ where the incidence plane is bisectorial with respect to octant $OXYZ$ and $k_x = k_y = k_z$. We show only the results where the numerical results for the interaction dyadics obtained as described above practically coincide with our analytical calculations.

Figs. 2–5 present the frequency dependence of the components of the excitation and polarizability dyadics for the grid with the cell size $a = 10$ mm. The polarizability dyadics components are marked as 1, the CPD's components are marked as 2 if the near-zone terms are included into the interaction dyadics, and as 3 if they are neglected. Components F_{ee}^{yz} and F_{me}^{xy} have no analogues in the particle's polarizability dyadic being results of the electromagnetic interaction of particles. As it is seen, considerable cross-polarizing effect is observed in the reflected and transmitted waves. In this example, perfectly conducting omega particles have the following geometrical parameters: $l = 1.5$ mm, $R_0 = 1.5$ mm, $r_0 = 0.05$ mm, where l is the length of one stem of the particle (one half of the straight portion length), R_0 is the loop radius, and r_0 is the wire radius. The frequency band of the analysis includes the individual particle resonance. Note that resonances that correspond to various periods in the grid cannot be seen in the case of small distances between the particles $(a < \lambda/2)$, which is studied here. Also, the reflection coefficient at that resonances becomes practically zero (since the resonance interaction, which has been studied separately corresponds to the sharp mutual depression of the dipoles).

The CPD's components frequency behavior is considered in comparison with the corresponding components of the polarizability dyadics. When $a < 6$ mm and smaller, it turns out that the results become obviously incorrect since the negative values of the imaginary parts of $\overline{\overline{F}}_{ee}, \overline{\overline{F}}_{mm}$, and of the real part of $\overline{\overline{F}}_{em}$ become large. In the same cases the correction terms obtained in Appendix B for the dipole moments of the reference particle become essential, too. It can be understood as that the dipole model of interaction is not available for such small distances.



(a)



(b)

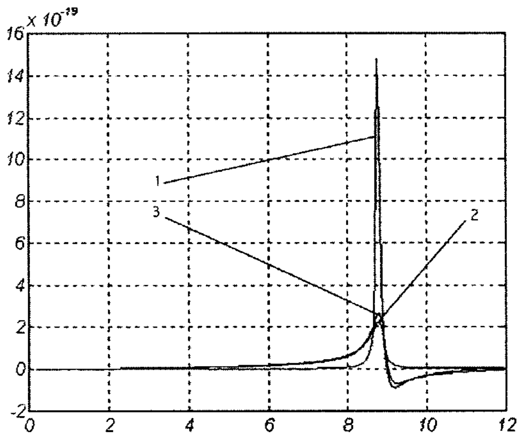
Fig. 3. (a) $a = 10$ mm, $\mathrm{Re}(F_{ee}^{yz})$. (b) $\mathrm{Im}(F_{ee}^{yz})$. Horizontal axis—frequency in gigahertz.

The general rule is that the electromagnetic interaction decreases the resonant values of all the components of CPD's compared to their analogues in the polarizability dyadics, but increase the components having no such analogues. That decreasing especially concerns the imaginary parts of $\overline{\overline{F}}_{mm}$, $\overline{\overline{F}}_{ee}$, and the real part of $\overline{\overline{F}}_{me}$. Their analogues for noninteracting particles are responsible for particle radiation losses in the theory of bianisotropic bulk composites. Small shift of the resonance frequency increases with the grid density and the incidence angle, but it does not exceed 5–7%.

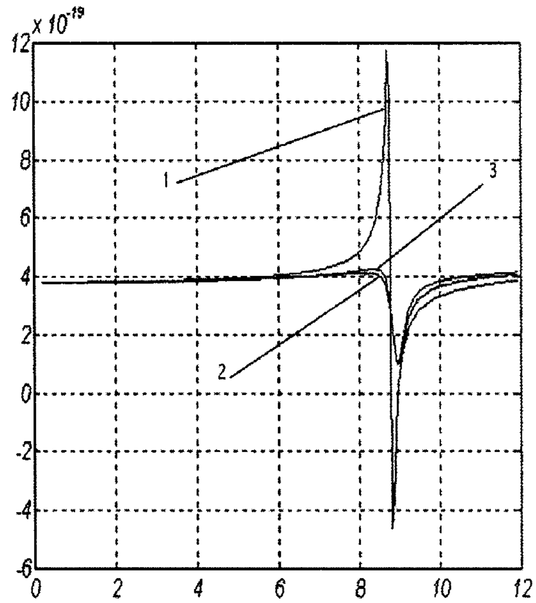
We have studied separately the influence of the near-zone fields produced by our grid elements on the excitation of the zero element. When we neglect these fields in the interaction dyadics, all the resonance peaks of the CPD's become more pronounced. The electromagnetic and especialy near-zone particles interaction decreases the resonance properties, radiation losses, and bianisotropy of particles.

In Fig. 2, the polarizability component describes excitation of electric dipole moment in the loop (along y) in response to electric field along the same axis. For noninteracting particles, the particle as a whole is not excited by y-directed electric vector [4]. Therefore, the resonance peak is absent and the excitation model is quasi-static. The interaction leads to that y-directed incident electric field produces (for oblique incidence only) the magnetic dipoles which are induced by y-directed electric ones. These magnetic dipoles via the term with A'' entering K in (21) are responsible for the resonance peak that appears.

Fig. 3 demonstrates another interesting phenomenon. For an individual particle, z-directed electric field does not excite a y-directed electric dipole moment. However, interaction leads to nonzero interaction, which again has a resonance at the resonant frequency of the interacting scatterer. Fig. 4 shows the effect of mutual depression mentioned above. It especially refers to the imeginary part of this main component of the polarizability (the real part also decreases but not that sharply). That demonstrates transition to a continuous surface, where no scattering loss exist. Similar phenomena



(a)



(b)

Fig. 4. (a) $a = 10$ mm, $\mathrm{Re}(F_{ee}^{zz})$, and $\mathrm{Re}(a_{ee}^{zz})$. (b) $\mathrm{Im}(F_{ee}^{zz})$ and $\mathrm{Im}(a_{ee}^{zz})$. Horizontal axis—frequency in gigahertz.

can be observed in analysing the magnetic components of CPD's Fig. 5 corresponds to that magnetoelectric component of CPD's which appears also as the result of the interaction (y-polarized external electric field excites the magnetic dipoles).

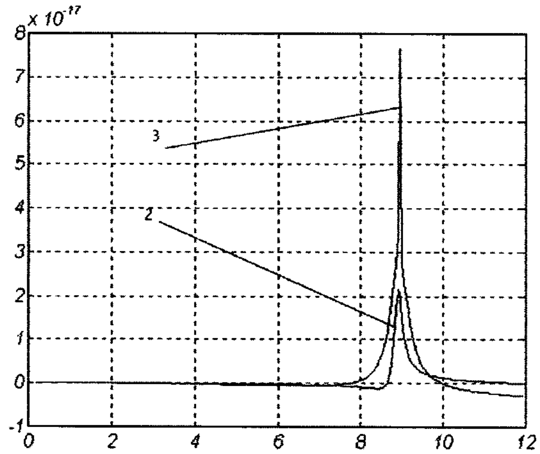
VII. Conclusion

The analytical model of excitation of planar arrays of bianisotropic particles has been developed. All the results are in agreement with the reciprocity conditions and they are physically sound in the limiting cases. The following conclusions regarding the electromagnetic interaction of omega particles in 2-D arrays can be made.

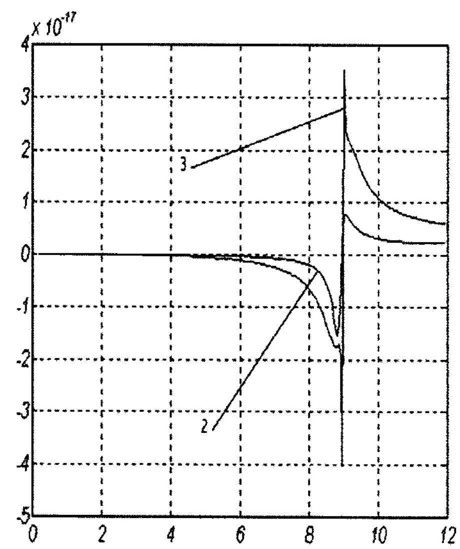
1) The resonance frequency shift increases with the incidence angle and the grid density, but it is rather small.
2) Bianisotropy and magnetic susceptibility of particles decrease, which especially refers to the components that are responsible for the radiation losses in bulk composites.
3) Comparing to noninteracting particles, collective polarizability dyadics contain more nonzero components whose amplitude increases with the grid density.
4) The dipole approximation applied here is valid when the cell size is at least twice the particle size.
5) Interaction leads to new resonances of polarizability components which can be described by the quasi-static model in the noninteracting case.

Appendix A

Let us consider a planar omega grid. At first, we should note that since the stretched size of the particle is about $\lambda/8$ and the grid period under consideration is about $\lambda/6 \cdots \lambda/3$, the dipole model is not available to describe the electromagnetic interaction only for adjacent particles.



(a)



(b)

Fig. 5. (a) $a = 10$ mm, $\mathrm{Re}(F_{me}^{xy})$. (b) $\mathrm{Im}(F_{me}^{xy})$. Horizontal axis—frequency in gigahertz.

As to the particles located around the reference one, the distance from the reference particle to them has the same order that their sizes. So, the field produced by these particles is nonuniform over the reference particle. Contribution of that field to the electric and magnetic dipole moments induced in the reference particle can be described in terms of the so-called additional impedance. This impedance (referred to the particle center) produced by an arbitrary particle with the number 1 in a particle with the number 0 can be presented as [11], [12]

$$Z_{\mathrm{add}} = Z_{01} I_1 / I_0$$

where Z_{01} is the mutual impedance of particles 0 and 1 and I_0, I_1 are the current amplitudes at the centers of the 0 and 1 particles, respectively.

To calculate the mutual impedances, we use the well-known method of induced electromotive forces (EMF) by Brillouin [12]. The periodicity condition (5) allows

$$\frac{I_1}{I_0} = e^{-j\boldsymbol{k}\cdot\boldsymbol{d}} \tag{22}$$

where d is the radius-vector of the 1 particle's center (where the origin is the 0 particle's center). The mutual impedance Z_{01} can be presented as

$$Z_{01} = Z_{\mathrm{ww}} + Z_{\mathrm{wl}} + Z_{\mathrm{ll}} \tag{23}$$

where Z_{ww} is the mutual impedance of two straight wire portions of interacting particles, Z_{wl} is one of the wire and the loop portions, and Z_{ll} is one of their loops. The last term in (23) can be estimated as

$$Z_{\mathrm{ll}} = j\omega M$$

where M is the well-known (see [10, eq. (5.41)]) mutual inductance of coplanar loops, which is very small compared to the self-intuctance L when $d \geq 3R_0$ (where R_0 is the loop radius).

The mutual impedance of the loop and the straight wire antennas has been studied in [11]. It was shown that at the frequencies which are about that of the main resonance (or less) $Z_{\rm wl}$ is also small compared to the loop and the wire proper impedances $Z_{\rm w}$ and $Z_{\rm l}$. The term $Z_{\rm ww}$ has the same order that $Z_{\rm w}$ since the stems of omega particles are parallel.

Practically, $Z_{\rm ww}$ is the only term to be taken into account in (26) For $Z_{\rm ww}$ we have from [12]

$$Z_{\rm ww} = R_{\rm ww} + jX_{\rm ww}$$

where

$$\begin{aligned}\frac{R_{\rm ww}}{\eta/2\pi} =& 2{\rm ci}(kd) - {\rm ci}(kd')c - {\rm ci}(kd'') \\ &+ 0.5\ \sin 2kl(2{\rm si}(-kd'') - 2{\rm si}(kd') \\ &+ {\rm si}(kd_1) + {\rm si}(kd_2)) + 0.5\ \cos 2kl(2{\rm ci}(kd) \\ &- 2{\rm ci}(kd') - 2{\rm ci}(kd'') + {\rm ci}(kd_1) + {\rm ci}(kd_2)) \\ \frac{X_{\rm ww}}{\eta/2\pi} =& -2{\rm si}(kd) + {\rm si}(kd')c + {\rm si}(kd'') \\ &+ 0.5\ \sin 2kl(2{\rm ci}(kd'') - 2{\rm ci}(kd') + {\rm ci}(kd_1) \\ &+ {\rm ci}(kd_2)) + 0.5\ \cos 2kl(-2{\rm si}(kd) + 2{\rm si}(kd') \\ &- 2{\rm si}(kd'') - {\rm si}(kd_1) + {\rm ci}(kd_2))\end{aligned}$$

and

$$\begin{aligned}d' &= l + \sqrt{l^2 + d^2} \\ d'' &= l - \sqrt{l^2 + d^2} \\ d_1 &= 2l + \sqrt{4l^2 + d^2} \\ d_2 &= 2l - \sqrt{4l^2 + d^2}.\end{aligned}$$

Here si and ci are the integral sine and cosine functions, respectively.

There are eight adjacent particles around the reference particle (which is named below 0 particle since its array numbers are $m = n = 0$) and we have to determine only three mutual impedances of the reference particle and of one) the particle with numbers $m = 0, n = 1$ (denoted below as Z_{001}); 2) the particle with numbers $m = 1, n = 0$ (Z_{010}); and 3) the particle with numbers $m = 1, n = 1$ (Z_{011}). The other five mutual impedances are equivalent to these as results from the problem geometry.

Let us present the EMF induced in the zero-particle as the sum of EMF's induced by the adjacent particles ($E_{\rm adj}$) and that induced by the far sources (the distant particles and the incident wave) ($E_{\rm far}$). We have in these terms: $E_{\rm far} + E_{\rm adj} = I_0 Z_0$ where I_0 is the current in the reference particle center and Z_0 is its proper impedance evaluated in [11]. Also, from (22) we yield

$$E_{\rm adj} = I_0 \sum_{m,n=-1}^{1}{}' e^{-j(k_x ma + k_y nb)} Z_{0mn}. \tag{24}$$

Here the prime means that the term $m = n = 0$ is excluded. Let us present the current I_0 as a similar sum $I_0 = I_{\rm far} + I_{\rm adj}$, where $I_{\rm far} = E_{\rm far}/Z_0$, $I_{\rm adj} = E_{\rm adj}/Z_0$. From (24) the relation between these terms results

$$I_{\rm adj} = I_{\rm far} G$$

where

$$G = \frac{\displaystyle\sum_{m,n=-1}^{1}{}' e^{-j(k_x ma + k_y nb)} Z_{0mn}/Z_0}{1 - \displaystyle\sum_{m,n=-1}^{1}{}' e^{-j(k_x ma + k_y nb)} Z_{0mn}/Z_0}.$$

With these equations we can express $p_z^{\rm adj}$ via $p_z^{\rm far}$

$$p_z^{\rm adj} = p_z^{\rm far} G \tag{25a}$$

$$p_z = p_z^{\rm adj} + p_z^{\rm far}. \tag{25b}$$

To find $p_z^{\rm far}$ we have to substract the terms with numbers $m, n = 0, \pm 1$ from the series (13) describing the interaction dyadics. These reduced interaction dyadics being substituted into (8) and (7) give the vector $p^{\rm far}$ with the z component $p_z^{\rm far}$. Substituting $p_z^{\rm far}$ into (25), we obtain an appreciate value of p_z where the real wire-and-loop shape of the omega particle is taken into account within the frame of the impedance model.

This result can be compared with that of the dipole approximation given by (7), (8), and (13). Such a numerical comparison being made for an array described in Section VI shows that the dipole model is available if the distance between particles is greather than $4l$ (that is, the double stretched length of one particle). This result is the same as that of [13] in which the interaction of two chiral particles is considered.

Appendix B

To evaluate all the integrals in (19), we rewrite them in polar coordinates and obtain a tabulated integral over the polar radius. The estimations have been made for the contribution of a small domain with the dimension $a \times b$ around the origin. In these estimations, this rectangular area has been replaced by a circle with the radius $(a+b)/2$. In the case when the dipole approximation is valid and sums (15) can be replaced by integrals (19) ($a, b \approx \lambda/3 \cdots \lambda/6$), the contribution of this domain does not exceed the error given by such an approximation itself. Therefore, this contribution can be neglected and the mark $v.p.$ can be dropped.

Integrating along the polar radius from 0 to ∞ we obtain

$$A' = \frac{k^2 n}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\, I_z i$$

$$I_z = \int_0^{2\pi} \frac{\cos\varphi d\varphi}{k + k_z\cos\varphi + k_y\sin\varphi} \tag{26a}$$

$$A'' = -\frac{k^2 n}{4\pi\sqrt{\epsilon\epsilon_0\mu\mu_0}}\, I_y i$$

$$I_y = \int_0^{2\pi} \frac{\sin\varphi d\varphi}{k + k_z\cos\varphi + k_y\sin\varphi} \tag{26b}$$

$$A = \frac{k^2 n}{4\pi\epsilon\epsilon_0}\, Ii$$

$$I = \int_0^{2\pi} \frac{d\varphi}{k + k_z\cos\varphi + k_y\sin\varphi} \tag{26c}$$

$$A_1 = \frac{k^2 n}{4\pi\epsilon\epsilon_0} I_{z^2} i$$

$$I_{z^2} = \int_0^{2\pi} \frac{\cos^2\varphi d\varphi}{k + k_z\cos\varphi + k_y\sin\varphi} \tag{26d}$$

$$A_2 = -\frac{k^2 n}{4\pi\epsilon\epsilon_0} I_{yz} i$$

$$I_{yz} = \int_0^{2\pi} \frac{\sin\varphi\cos\varphi d\varphi}{k + k_z\cos\varphi + k_y\sin\varphi} \tag{26e}$$

$$A_3 = \frac{k^2 n}{4\pi\epsilon\epsilon_0} I_{y^2} i$$

$$I_{y^2} = \int_0^{2\pi} \frac{\sin^2\varphi d\varphi}{k + k_z\cos\varphi + k_y\sin\varphi}. \tag{26f}$$

Using the well-known relation

$$\int_0^{2\pi} F(\cos\varphi, \sin\varphi)\, d\varphi = \int_0^{2\pi} F(\sin\varphi, \cos\varphi)\, d\varphi$$

we find from (23)

$$\begin{aligned} I_y(k, k_z, k_y) &= I_z(k, k_y, k_z) \\ I_{y^2}(k, k_z, k_y) &= I_{z^2}(k, k_y, k_z) \\ I &= I_{z^2} + I_{y^2}. \end{aligned} \tag{27}$$

We can evaluate I_z, I_{z^2}, I_{yz}. The other integrals we calculate using (27). Substituting $t = \tan\varphi/2$ we obtain

$$\begin{aligned} I_z &= \int_{-\infty}^{+\infty} \frac{4t dt}{((k-k_y)t^2 + 2k_z t + (k+k_y))(1+t^2)} \\ I_{z^2} &= \int_{-\infty}^{+\infty} \frac{8t^2 dt}{((k-k_y)t^2 + 2k_z t + (k+k_y))(1+t^2)^2} \\ I_{yz} &= \int_{-\infty}^{+\infty} \frac{8t(1-t^2) dt}{((k-k_y)t^2 + 2k_z t + (k+k_y))(1+t^2)^2}. \end{aligned} \tag{28}$$

All the integrands analytically expanded in the complex plane have poles of the first or second kind at

$$p_{1,2} = \frac{-k_z \pm k_x i}{k - k_y}, \quad p_{3,4} = \pm i$$

and the required behavior in the complex infinity. We can therefore evaluate the integrals in (28) as

$$\begin{aligned} I_z &= \frac{2\pi k_z (k_x - k)}{k_x (k_y^2 + k_z^2)} \\ I_{z^2} &= \frac{2\pi (k - k_x)(k_y^2 k_x + k_z^2 k)}{k_x (k_y^2 + k_z^2)^2} \\ I_{yz} &= \frac{2\pi k_y k_z (k - k_x)^2}{k_x (k_y^2 + k_z^2)^2}. \end{aligned}$$

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