Multilevel Tabulated Interaction Method Applied to UHF Propagation over Irregular Terrain

Conor Brennan and Peter J. Cullen

***Abstract*—A multilevel tabulated interaction method (TIM) is described for application to the analysis of exceptionally large two-dimensional (2-D) UHF terrain propagation problems. While standard TIM schemes offer extremely rapid solutions to terrain propagation problems on a medium to large scale (∼20 km), the computation times increase quadratically with problem size. For problems of extremely large size (∼100 km), we show how the adoption of the multilevel scheme proves beneficial, reducing computation times and restoring the TIM's computational advantage. Numerical results are provided illustrating the concepts introduced.**

***Index Terms*—Electromagnetic propagation, UHF propagation, UHF radio propagation terrain factors.**

I. Introduction

The efficient and accurate calculation of UHF propagation loss over irregular terrain occupies a position of fundamental importance in the planning of wireless communications systems. While previously the lack of computational resources forced planners to use simplistic models such as single or multiple knife edge diffraction [1], the advent of the digital computer rendered feasible the usage of more sophisticated models such as geometric theory of diffraction (GTD) [2] or the parabolic equation [3], [4]. The potentially exact full wave solutions offered by an integral equation (IE) formulation are a tantalizing prospect, but until recently have remained elusive due to the inherent computational complexity of standard IE solutions. This computational burden has, in the past, relegated them to the position of reference solutions to the faster approximate solutions mentioned above. The last decade, however, has seen a great deal of activity in this important area. Hviid *et. al* [5] demonstrated the accuracy of the IE approach—the adoption of a forward scattering approximation eliminating the need for explicit storage of the impedance matrix associated with a moment method solution of the IE. They also illustrated the largest drawback of standard IE solutions, namely the extremely large computation times necessary. The natural basis set (NBS) [6]–[8] was shown to offer huge computational savings for propagation problems involving grazing incidence over undulating terrain, the chosen basis set accurately capturing much of the structure of the unknown surface current leaving an accordingly lower order matrix equation to numerically solve. Backscattering could be explicitly accounted for and the whole computation took a matter of seconds on a modest computational resource such as an HP workstation. The fast far-field algorithm (FAFFA) [9], which is related to the well-documented fast multiple method [10], was combined with the Green's function perturbation method (GFPM) [11], [12] and offers large computational savings over a wider range of problems than those amenable to rapid NBS solution. Though not as fast as the NBS, the FAFFA was further specialized (via the incorporation of certain assumptions, described in Section II, about the problem geometry and incident field) into an extremely rapid scheme namely the tabulated interaction method (TIM) [13] which, for terrain problems, combined the robustness of the FAFFA with the speed of the NBS. However, the admirable performance of the TIM when applied to medium to large sized problems, providing very accurate predictions in a matter of seconds, is not replicated when applied to very large problems (of the order of 100 km at typical radio frequencies). This paper is intended to set out a simple computational technique designed to improve the efficiency of the TIM in this regard. In the next section, we briefly review the workings of the standard TIM while Section III motivates and describes a multilevel implementation suitable for usage in the analysis of extremely large scale problems. We close with some numerical examples illustrating the advantages of the new method.

II. Basic TIM Scheme

The propagation model adopted by ourselves, that of a TM^z (horizontal polarization) linesource irradiating, with wavenumber β, a one-dimensional terrain profile (consisting of a series of perfectly conducting linear segments) allows us to use the two-dimensional (2-D) electric field integral equation (EFIE) [14]. Although the assumption of perfect reflection is introduced to make the analysis easier, it is by no means central to the ideas presented. In fact, it is partially justified by the fact that the propagation is grazing and comparison with experimental results, (see, for instance [5]) indicate that its usage is reasonable. Present work by ourselves focuses on reformulating (in the context of coupled boundary integral equations) the ideas herein for application to nonperfectly reflecting cases, a topic we leave for future discussion.

Standard solution of the integral equation involves describing the unknown surface current in terms of N known basis functions with unknown coefficients. The problem, thus discretized, reduces to one of linear algebra, specifically a large dense matrix equation that must be solved to yield the basis coefficients. Solution of this equation, even assuming forward

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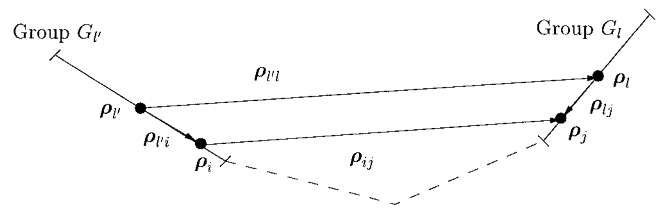

Fig. 1. Geometry for the TIM. Assumption that incident field is locally planar in conjunction with the assumption that $\rho_{ij} \simeq \rho_{l'l} - \boldsymbol{\rho}_{l'i} \cdot \hat{\boldsymbol{\rho}}_{l'l} - \boldsymbol{\rho}_{lj} \cdot \hat{\boldsymbol{\rho}}_{ll'}$ for all point-pairs $(\boldsymbol{\rho}_i, \boldsymbol{\rho}_j)$ allows the current residing on group G_l to be described in terms of reference currents.

scattering that allows the matrix to be approximated as lower triangular, is exceptionally time-consuming. The TIM replaces the basis function to basis function interaction of conventional moment methods with interactions between the smooth linear segments that are used to describe the terrain profile. These segment-to-segment interactions can be equivalently viewed as the interactions between large groups of pulse basis functions, having their domains on the segments in questions. It should be noted that the addition of random roughness to these linear segments leads to a more difficult problem not amenable to a rigorous TIM solution. However, these roughness effects are not so manifest in the grazing incidence forward scattering case and our smooth terrain model is a reasonable one for these problems.

Specifically, we note that the terrain profile is made up of a series of linear segments; G_l for $l = 1 \cdots D$. Using the standard pulse basis function and Dirac-delta testing function approach the discretized EFIE for this problem can be written as

$$\sum_{i=1}^{N} Z_{ji} J_i = V_j \tag{1}$$

for $j = 1 \cdots N$. V_j is the incident field at point $\boldsymbol{\rho}_j$ and Z_{ji} is a typical entry in the impedance matrix. Referring to Fig. 1, we can rewrite (1) in a manner that emphasizes the interaction between linear segments. Specifically for $\boldsymbol{\rho}_j \in G_l$, we can rearrange (1) to yield

$$\sum_{\boldsymbol{\rho}_i \in G_l} Z_{ji} J_i = V_j - \sum_{l' \neq l} \sum_{\boldsymbol{\rho}_i \in G_{l'}} Z_{ji} J_i. \tag{2}$$

Note that we can impose the forward scattering condition by restricting the l' summation to all groups prior to G_l. Now, consider Fig. 1 which depicts the interaction between two segments G_l and $G_{l'}$, the central points of which are $\boldsymbol{\rho}_l$ and $\boldsymbol{\rho}_{l'}$, respectively. The central approximation of the TIM is that for each point-pair, $\boldsymbol{\rho}_i \in G_{l'}$, $\boldsymbol{\rho}_j \in G_l$

$$\rho_{ij} \simeq \rho_{l'l} - \boldsymbol{\rho}_{l'i} \cdot \hat{\boldsymbol{\rho}}_{l'l} - \boldsymbol{\rho}_{lj} \cdot \hat{\boldsymbol{\rho}}_{ll'} \tag{3}$$

where ρ_{ij} is the magnitude of the vector $\boldsymbol{\rho}_{ij}$ from $\boldsymbol{\rho}_i$ to $\boldsymbol{\rho}_j$, other quantities being defined similarly. The symbol $\hat{\rho}$ is used to indicate a unit vector. Applying this approximation to all group pairs $(G_{l'}, G_l)$ for $l' = 1 \cdots l-1$, using the far-field form of the Hankel function and assuming the incident field to be locally plane wave in form over each segment allows us to rewrite (2) as

$$\sum_{\boldsymbol{\rho}_i \in G_l} Z_{ji} J_i \simeq V_l \exp(\jmath\beta \boldsymbol{\rho}_{lj} \cdot \hat{\boldsymbol{\rho}}_{la}) - \sum_{l'=1}^{l-1} Z_{ll'} \cdot \exp(\jmath\beta \boldsymbol{\rho}_{lj} \cdot \hat{\boldsymbol{\rho}}_{ll'}) F_{l'l} \tag{4}$$

for each $\boldsymbol{\rho}_j \in G_l$, $l = 1 \cdots D$. $\boldsymbol{\rho}_{la}$ is the vector from the group center $\boldsymbol{\rho}_l$ to the source point $\boldsymbol{\rho}_a$. Note that we have explicitly introduced the forward scattering approximation (this is not central to the method but does lessen the computational burden). The interactions between each segment pair $(G_{l'}, G_l)$ is given in terms of $F_{l'l}$

$$F_{l'l} \equiv \sum_{\boldsymbol{\rho}_i \in G_{l'}} J_i \exp(\jmath\beta \boldsymbol{\rho}_{l'i} \cdot \hat{\boldsymbol{\rho}}_{l'l}) \tag{5}$$

which are in turn given by interpolating between group $G_{l'}$'s far-field pattern $F_{l'}^m$, describing scatter in the K discrete directions $\hat{\boldsymbol{e}}^m$ for $m = 1 \cdots K$

$$F_{l'}^m \equiv \sum_{\boldsymbol{\rho}_i \in G_{l'}} J_i \exp(\jmath\beta \boldsymbol{\rho}_{l'i} \cdot \hat{\boldsymbol{e}}^m) \tag{6}$$

$\hat{\boldsymbol{e}}^m$ is a unit directional vector given by $\hat{\boldsymbol{e}}^m = \hat{\boldsymbol{x}} \cos\theta_m + \hat{\boldsymbol{y}} \sin\theta_m$ and $\theta_m = 2\pi m/K$ for $m = 1 \cdots K$. Upon calculation, the far-field pattern of a group can be used to approximately calculate via interpolation, the field scattered from that group to the center of another. The accurate and quick calculation of each group's far-field pattern thus holds the key to efficient solution of the problem and is the core aim of the TIM. This can be efficiently done upon noting that the form of (4), where the excitation (right-hand side) on a given segment is a weighted sum of plane waves, suggests that the current residing on group G_l (and, hence, *any* group as they are deemed physically identical) can be approximately expressed as a weighted sum of K known reference currents. As described in [13] we thus tabulate K reference currents, namely the currents excited on a typical perfectly electrically conducting (PEC) segment when illuminated by plane waves impinging from K discrete angles from 0 to 2π. Also tabulated are each reference current's far-field pattern. The essence of the TIM is that, because the current residing on any group G_l can be written in terms of the reference currents the far-fields of that group can now be written in terms of the tabulated reference far fields. The calculation of far-field patterns reduces to trivial bookkeeping (of incoming plane wave directions and amplitudes). These tabulated far-field patterns and reference currents are, in addition to being used to calculate the current on the scatterer surface, used in the final calculation of scattered field above the terrain surface. In short, the rigorous numerical analysis of the fields scattered from a typical terrain profile component provides us with the framework to build solutions to terrain scattering problems with a minimum of computational effort. These reference solutions once obtained are, of course, portable from radial propagation problem to radial propagation problem facilitating the rapid construction of area coverage maps. While the development above suggests that the tabulations occur for all discrete directions $0 < \theta_m < 2\pi$, it should be noted that due to the slowly undulating nature of typical terrain profiles, the tabulating of far fields in a small angular sector around $\theta_m = 0$ suffices for most propagation problems, thereby limiting the storage requirements. The TIM, thus described, has been used with success by ourselves in the analysis of propagation over many terrain profiles. While as with most numerical approaches it is difficult to quantify precisely the region of validity of its application, it should be noted that like most numerical schemes, the algorithm is inherently adaptive. If suitable reference currents and far fields are available, group sizes can be reduced to ensure the accuracy of the underlying geometrical approximation (3) in areas where it is deemed necessary (for example sharp ridges) or, indeed, the scheme can be easily hybridized with the FAFFA in these areas. While application to more complicated scattering environments (such as propagation around right angled wedges) would be nontrivial, we have yet to encounter a rural propagation problem that was not amenable to accurate analysis with the TIM.

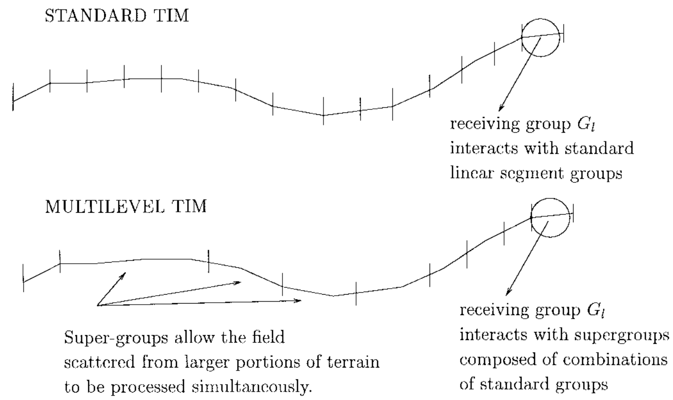

Fig. 2. Schematic of standard versus multilevel TIM. In standard TIM, each linear segment group interacts with each other linear segment group leading to quadratic growth in computation time with problem size. In multilevel case, we endeavor to allow as large portions of terrain interact with each other as is possible via the use of super groups.

III. MULTILEVEL TIM

While the basic TIM scheme, as described above, performs impressively when applied to propagation problems of modest to large size, it scales quadratically in computational complexity as the number of groups (linear segments) present in the problem increases. This is in contrast to the performance of the parabolic equation method whose computational complexity scales linearly with problem size. For propagation problems of a very large scale (~100 km) it is obvious that some modification to the basic TIM scheme is necessary to render it computationally competitive. The motivation for the multilevel TIM lies in the realization that for fixed group sizes the approximation of (3) becomes more accurate as the group separation distance $\rho_{l'l} \rightarrow \infty$. The multilevel TIM aims to exploit this by using larger group sizes, thus processing more scattering information in tandem wherever possible.

A multilevel TIM postulates, in addition to the standard TIM linear-segment groups, "super-groups" comprising of groups of standard TIM groups. Specifically, we introduce, in addition to the standard groups $G_1 \ldots G_D$, the following super groups.

- "two groups" (comprising two standard linear segments) $G_1^{(2)} \cdots G_{D/2}^{(2)}$ where $G_m^{(2)} = G_{2m-1} \cup G_{2m}$;
- "four groups" (comprising two "two groups") $G_1^{(4)} \cdots G_{D/4}^{(4)}$ where $G_m^{(4)} = G_{2m-1}^{(2)} \cup G_{2m}^{(2)}$

and so on with "eight groups" and higher. Associated with each super group is a central point easily defined in terms of the centers of the constituent standard groups.

Upon adopting such an approach, we can write (4) as

$$\sum_{\boldsymbol{\rho}_i \in G_l} Z_{ji} J_i = V_l \exp(\jmath\beta \boldsymbol{\rho}_{lj} \cdot \hat{\boldsymbol{\rho}}_{la}) - \sum_{l' \in \mathcal{G}_l} Z_{ll'} \exp(\jmath\beta \boldsymbol{\rho}_{lj} \cdot \hat{\boldsymbol{\rho}}_{ll'}) F_{l'l}. \tag{7}$$

Note that rather than stepping through the standard groups 1 to $l-1$ as in (4), the l' index is confined to groups deemed to be in $\mathcal{G}_l$. $\mathcal{G}_l$ is a set of standard **and** super groups such that each of the standard TIM groups $G_{l'}$, $l' = 1 \cdots l-1$ is represented once and only once. We will discuss the composition of $\mathcal{G}_l$ as well as the calculation of super group far fields later. However, for now we note that the larger the super groups used in each $\mathcal{G}_l$, the fewer the computations necessary to calculate the reference currents residing on each G_l and, hence, the faster the TIM scheme will be.

To illustrate these ideas, consider Fig. 2, which schematically depicts both the standard TIM algorithm and the proposed multilevel version. Note that in the standard algorithm we are constrained to using the same group size in calculating the interactions between different portions of terrain. The multilevel scheme offers much more flexibility, allowing us to choose larger or smaller contributing groupings as the terrain geometry allows and, hence, fewer computations.

Two issues naturally arise in the implementation of such a scheme: how to decide on the composition of $\mathcal{G}_l$ and how to calculate the far-field pattern of a super group. As stated above we wish to use as large super groups as is possible in calculating the fields scattered to the receiving group of interest G_l. A simple check can be incorporated into the code to address the first issue. Consider Fig. 3, which depicts a typical situation. We would like to know whether the "four group" $G_s^{(4)}$ consisting of four standard groups G_a, G_b, G_c, G_d can be used to calculate the field scattered to the target point $\boldsymbol{\rho}_t$ (the center point of some standard group or a point above the surface where we wish to know the total field perhaps) or whether smaller groupings would be more appropriate. $\boldsymbol{\rho}_s$ is the center point of $G_s^{(4)}$ while $\boldsymbol{\rho}_a \cdots \boldsymbol{\rho}_d$ are the centers of its constituent standard groups. This can be easily checked by ensuring that the fast far-field approximation is a reasonable one for each central point in the super group. In this case, we check that

$$\beta|\rho_{it} - \rho_{st} + \boldsymbol{\rho}_{si} \cdot \hat{\rho}_{st}| < \epsilon \tag{8}$$

for $i = a, b, c, d$ where ϵ is some prechosen threshold constant. If the threshold quantity is exceeded we can retry using "two groups" or even resort to using standard groups if necessary. The process of composing $\mathcal{G}_l$ thus involves stepping through the groups $G_1 \cdots G_{l-1}$ and trying to amalgamate them

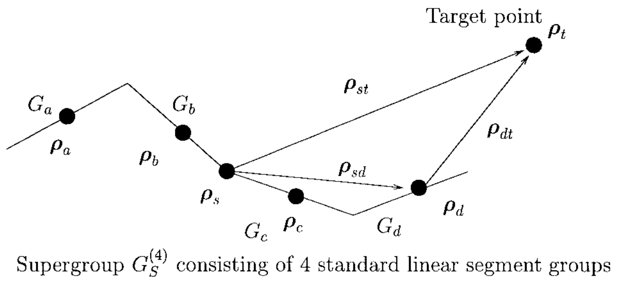

Fig. 3. Threshold procedure to decide whether to use super group of size four. Require $\beta|\rho_{it} - \rho_{st} + \boldsymbol{\rho}_{si} \cdot \hat{\rho}_{st}| < \epsilon$ for $i = a, b, c, d$.

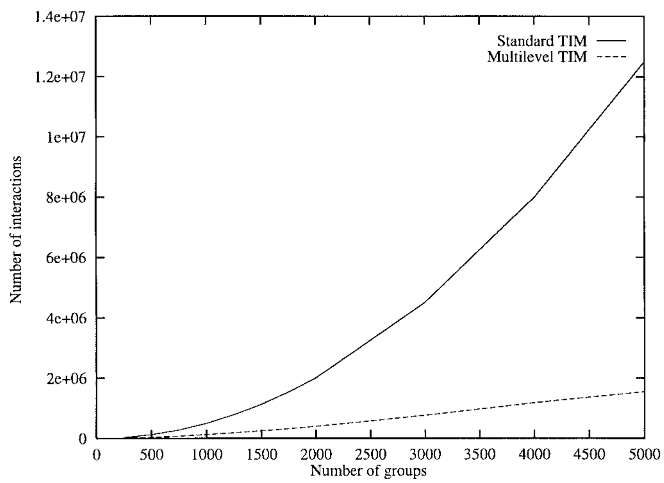

Fig. 4. Number of computations as a function of problem size for standard and multilevel TIM.

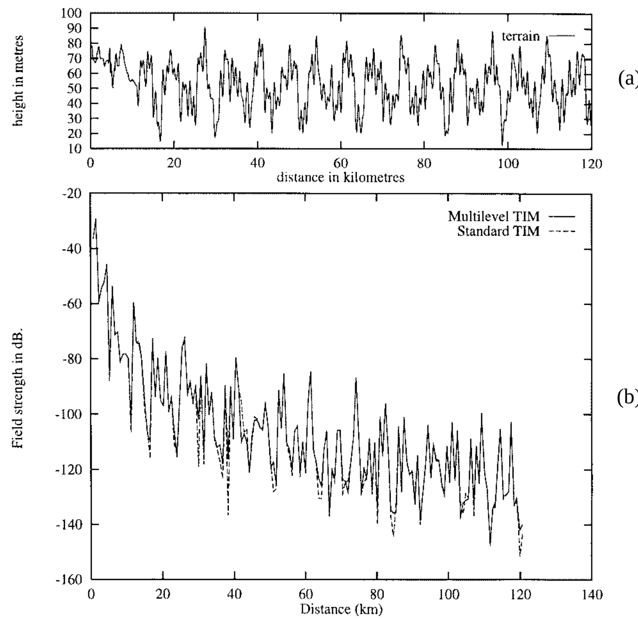

Fig. 5. Multilevel TIM versus standard TIM for large-scale propagation problem. Source radiating at 970 MHz, located over leftmost point. (a) Terrain surface. (b) Fields calculated 2.4 m over terrain surface. Multilevel used super groups as big as 1.6 km in length and threshold $\epsilon = 0.001$. K, the number of discrete plane wave directions, was equal to 200.

into as large super groups as is possible according to the threshold procedure above.

Having decided on what super groups are appropriate for a particular receiving group we are faced with the task of calculating their far-field scatter pattern. For each super group this is easily calculated in terms of a summation of shifted values of the far-field scatter patterns of the standard TIM groups of which it is ultimately comprised. For the example above we can write for the far-field scatter in the lth direction

$$F_s^{(4)l} = \sum_{i=a,\,b,\,c,\,d} F_i^l \exp(j\beta \boldsymbol{\rho}_{si} \cdot \hat{e}^l). \quad (9)$$

Thus, using the multilevel TIM we can process the fields scattered between larger portions of scatterer than is possible with a standard one-level TIM with a corresponding increase in efficiency. To illustrate this we conducted the following experiment. Pieces of terrain of varying lengths were generated and fields were calculated using the standard TIM and a multilevel TIM. The linear segments describing the terrain were 50 m in length and the multilevel scheme used super groups up to and including "32 groups," of length 1.6 km. A threshold level ϵ was set at 0.001 and super groups used according to the thresholding procedure described above. Fig. 4 plots the number of group-to-group interactions that occur in the standard TIM solution against those occurring in the multilevel scheme for the various problem sizes. We note that the multilevel complexity, scales much more gently with problem size than that of the standard TIM. We note that the efficiencies are not so manifest for smaller problem sizes and indeed the multilevel algorithm can, in practice, be actually slower in these circumstances due to the increased amount of bookwork necessary in defining super groups and checking threshold levels as per (8), etc. Fig. 5 shows a randomly generated terrain profile (top) and the fields predicted by both the standard TIM and the multilevel for the problem of size 2500 groups. The average difference in field strength predicted was 1.27 dB with a standard deviation 5.04 dB. The multilevel TIM took 63 s to run as opposed to the 130 s needed by the standard TIM. The computational savings garnered are not as high as predicted by Fig. 4 because of the inefficiencies introduced by the increased amount of "bookkeeping" needed but still represent a significant saving.

IV. Conclusions

We have described a multilevel tabulated interaction method, which improves the performance of the TIM in calculating UHF path loss over extremely large terrain profiles. Standard TIM groups are amalgamated into super groups and, wherever possible, it is these large groups that interact producing a correspondingly quicker method. Numerical results illustrated the potential of this new formulation.

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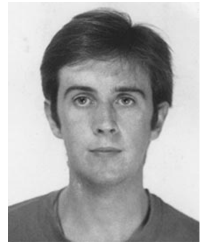

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