

# Mutual Coupling Between Slots Printed at the Back of Elliptical Dielectric Lenses

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**Abstract**—This paper presents calculations of the mutual admittance between two slots on a ground plane that are the primary feed of an elliptical dielectric lens antenna. This admittance may also be regarded as the basic constituent of a Galerkin method of moments formulation for the analysis of more complex feeding arrangements of slot antennas and coplanar waveguide circuits. It is found that the contribution of the reflection mechanisms inside the lens significantly affects the mutual admittance especially for *H*-plane coupling.

**Index Terms**—Lens antennas, printed antennas.

## I. INTRODUCTION

**D**ILECTRIC lens antennas [1] offer interesting perspectives for millimeter- and submillimeter-wave applications [2], [3] owing to their capability to be integrated with electronic components such as detecting diodes, oscillators, and mixers. The elliptical (or the equivalent hyperhemispherical) shape of the lens provides useful focusing properties when the ellipse eccentricity  $e$  is designed to satisfy the relationship  $e = 1/\sqrt{\epsilon_r}$ , where  $\epsilon_r$  is the relative permittivity of the lens. Indeed, under this condition those outgoing rays, fed by a focal source that impinge on the lens surface above the plane of maximum waist [A-A' in Fig. 1(a)], are transmitted in free-space in broadside direction and provide a uniform phase aperture distribution.

Although many contributions in the recent literature have considered this topic, most of them do not pay attention to the effects that multiple reflections inside the lens may produce on the feeding structure. As pointed out in [4] and [5], the rays emanating from a source at the lower focus  $F_1$ , after reflecting at the interface, cross the upper focus  $F_2$  of the ellipse. Then, the rays are reflected once again at the lens interface and eventually are refocused at  $F_1$ . Therefore, a point caustic of doubly reflected rays occurs at the focal point where the primary source is located; thus, affecting its input impedance. We note that those rays that are launched within the solid angle  $\Omega$  [Fig. 1(b)] experience a second reflection on the ground plane and then undergo multiple incoherent reflections without focusing at  $F_1$ . Consequently, their contributions at  $F_1$  are negligible. Introducing a matching layer on the lens surface could reduce the reflections toward the feed. However, its manufacturing at millimeter waves may encounter practical difficulties in layer thickness control and appropriate dielectric

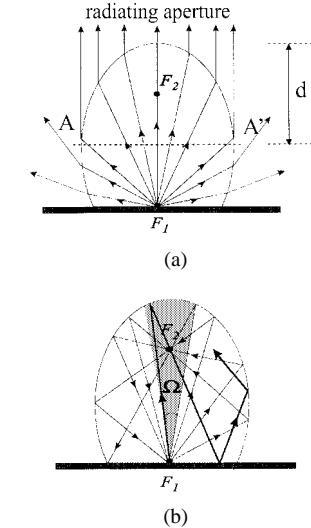


Fig. 1. Geometry of the lens antenna;  $F_1$  and  $F_2$  are the foci of the ellipsoid and  $d$  is the major semiaxis. (a) Ray construction of the aperture field. (b) Reflection mechanisms inside the lens: paths of the doubly reflected rays and shadow angle  $\Omega$ .

material supply. In most practical instances, the interaction between the various elements constituting the feeding structure need to be calculated, taking also into account these reflection mechanisms that are excited inside the lens. The effects of the internal reflections on the input impedance has been investigated in [5] by referring to two different types of focused primary feeds. In this paper, we present calculations of the coupling admittance between two nonfocused slots etched on the ground plane underneath an elliptical lens. This example was chosen taking into account that the most practical configurations of feeds consist of slots and coplanar waveguides, as demonstrated by recent papers [3], [6], [7]. This paper may be considered an extension of [6], which includes the internal reflections.

## II. FORMULATION

The geometrical configuration is shown in Fig. 2. An axially symmetric elliptical dielectric lens with permittivity  $\epsilon_{r1}$  is placed on an infinite ground plane where two resonant slots are etched. The lower focus of the lens is located on the ground plane; it is assumed that an infinite dielectric medium with permittivity  $\epsilon_{r2}$  is placed below the ground plane. Two configurations have been considered separately, which consist of collinear (*H*-plane coupled) and parallel (*E*-plane coupled) slots located on opposite sides with respect to the ellipse focus.

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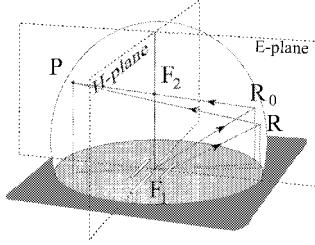


Fig. 2. Geometry of the slots printed at the back of the lens and ray-paths from one transmitting slot to  $P$  and from the lens focus  $F_1$  to  $P$ ; the reflection point  $R_0$  is used as initial guess for the ray-path minimization procedure.

We have focused our analysis to these configurations, because they are the most critical from the point of view of the lens influence.

By applying the equivalence principle, the slot  $i$  ( $i = 1, 2$ ) is replaced by two magnetic current distributions  $\pm \mathbf{M}_i$  ( $i = 1, 2$ ) juxtaposed above and below the ground plane. The continuity of the tangential electric field through the slots imposes equal amplitudes and opposite signs for these currents. Next, these magnetic currents are described by a normalized resonant-type sinusoidal basis function with its nulls at the end-points of the slot and with wavenumber  $k_s = k_0 \sqrt{(\epsilon_{r1} + \epsilon_{r2})/2}$ . The mutual admittance is expressed as

$$Y_{12} = Y_{12}^- + Y_{12}^+ + Y_{12}^l \\ = - \iint_{\text{slot}} \mathbf{M}_1 \cdot (\mathbf{H}_2^- + \mathbf{H}_2^+ + \mathbf{H}_2^l) dS \quad (1)$$

where  $\mathbf{H}_2^+$  is the magnetic field radiated by  $\mathbf{M}_2$  above the ground plane, when it is assumed that the upper half-space is filled by an unbounded medium with the same permittivity as that of the lens.  $\mathbf{H}_2^-$  is the magnetic field radiated by  $-\mathbf{M}_2$  in the medium below the ground plane.  $\mathbf{H}_2^l$  is the magnetic field that accounts for the discontinuity at the lens surface. It is calculated as the field radiated in a homogeneous, unbounded medium by the equivalent sources  $\mathbf{J}_2^{eq}$  and  $\mathbf{M}_2^{eq}$  associated to the actual field at the lens interface due to  $\mathbf{M}_2$ . In order to estimate  $\mathbf{J}_2^{eq}$  and  $\mathbf{M}_2^{eq}$ , a generalization of the method presented in [4] and [5] is introduced, which consists of asymptotically approximating the local field at the lens interface by geometrical optics (GO) ray tracing. In particular, the transmitting slot is represented as an offset point source of rays weighted by the far-field pattern of the slot. These rays are reflected two times at the lens surface; consequently the equivalent currents are subdivided into two contributions—those associated to the singly and those to the doubly GO rays, respectively. This latter contribution is the most significant for calculating of the mutual coupling, when the two slots are placed in close to the focal caustic.

In order to define the equivalent currents relevant to doubly reflected rays, a ray tracing procedure has been implemented from each point  $P$  on the lens surface to the transmitting slot (Fig. 2). This requires the minimization of a nonlinear function; however, its numerical implementation is greatly alleviated thanks to the particular geometry we are dealing with. Indeed, the procedure for the determining the reflection point  $R$  for any given  $P$  on the lens surface is performed by

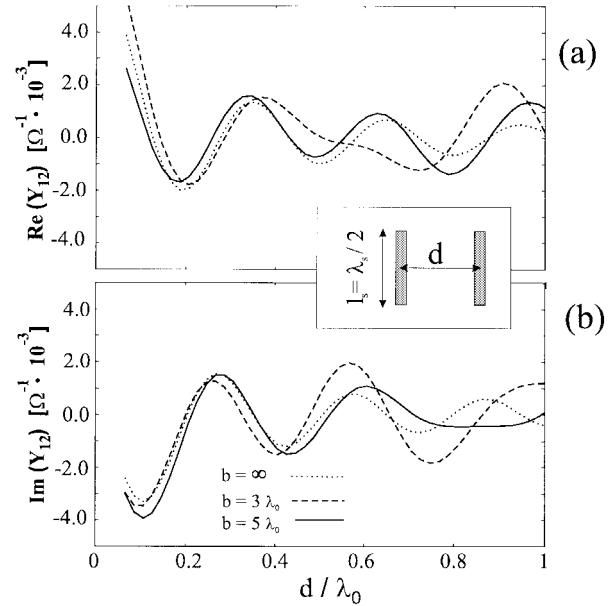


Fig. 3. Mutual admittance for two parallel slots ( $E$ -plane coupling) printed at the back of a silicon lens versus their distance normalized to the free-space wavelength  $\lambda_0$ ;  $b = 5 \lambda_0$  (continuous line),  $b = 3 \lambda_0$  (dashed line), coupling in homogeneous silicon (dotted line). (a) Real part. (b) Imaginary part.

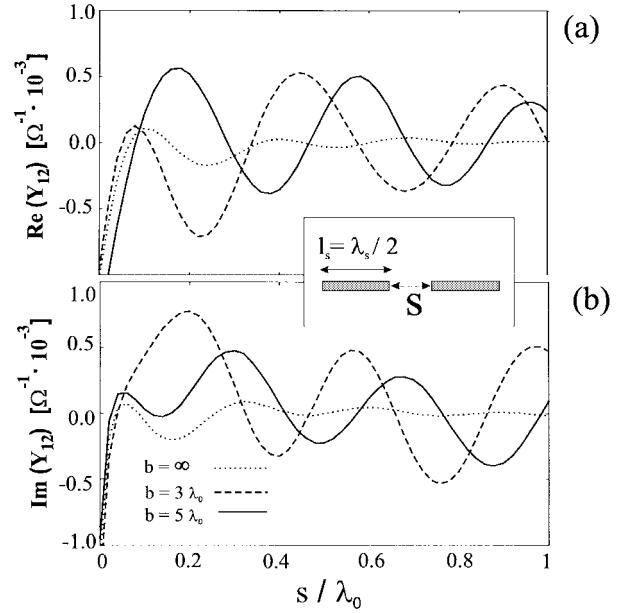


Fig. 4. Mutual admittance for two collinear slots ( $H$ -plane coupling) printed at the back of a silicon lens versus their distance normalized to the free-space wavelength  $\lambda_0$ ;  $b = 5 \lambda_0$  (continuous line),  $b = 3 \lambda_0$  (dashed line), coupling in homogeneous silicon (dotted line). (a) Real part. (b) Imaginary part.

using a suitable initial guess in the minimization process. This initial guess is provided by the position of the reflection point  $R_0$  which is obtained by tracing the ray from  $P$  on to the focus  $F_1$  (Fig. 2). First, since such a ray reaches  $F_1$  after crossing the upper focus  $F_2$  of the ellipse, the position of  $R_0$  is found in analytical form. Next, since  $R_0$  is generally close to the actual reflection point  $R$ , the minimization of the process requires a very few steps. When the transmitting slot is not so close to the focus, the search for a minimum may fall into a trap.

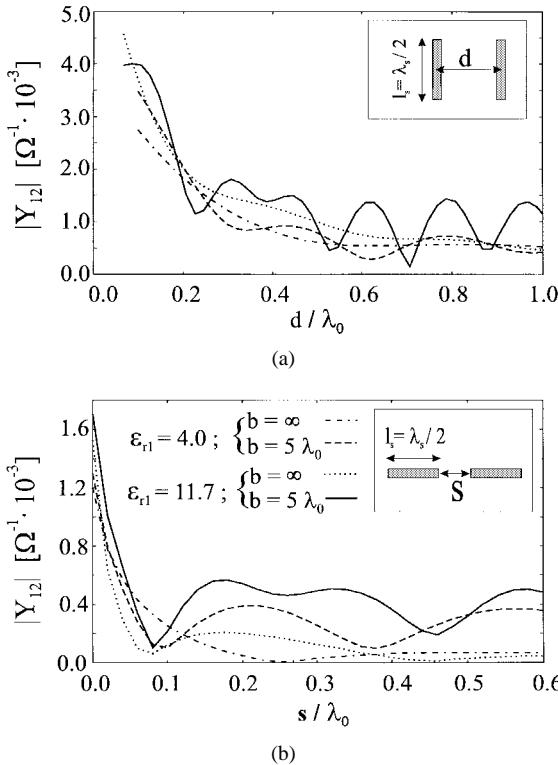


Fig. 5. Amplitude of the mutual admittance for a quartz lens ( $\epsilon_{r1} = 4$ ) and a silicon lens ( $\epsilon_{r1} = 11.7$ ). (a) Parallel slots. (b) Collinear slots. Silicon lens with  $b = 5\lambda_0$  (continuous line); quartz lens with  $b = 5\lambda_0$  (dashed line); homogeneous silicon (dotted line); homogeneous quartz (dash-dotted line).

In order to cure for this occurrence an appropriate check is included in the process, which consists of testing whether the Snell law is satisfied at the estimated point  $R$ . When a trap is found, a different initial guess  $R'_0$  is used by only changing the azimuthal angle of  $R_0$ .

### III. NUMERICAL RESULTS AND CONCLUSIONS

In Fig. 3, calculations of the mutual admittance between two  $E$ -plane coupled slots are presented in terms of its real [Fig. 3(a)] and imaginary [Fig. 3(b)] parts as a function of their distance normalized to the free-space wavelength. The lens dielectric is silicon ( $\epsilon_{r1} = 11.7$ ) and the medium below the ground plane is free-space; two different values of the major semiaxis  $b$  have been considered ( $b = 5\lambda$ , continuous line, and  $b = 3\lambda$ , dashed line, respectively), and, of course, the eccentricity of the ellipsoid is  $e = 1/\sqrt{\epsilon_{r1}}$ . The mutual admittances are compared with those calculated in a homogeneous silicon half-space (dotted line). It is rather apparent that the lens produces a noticeable influence on the coupling, especially for the smaller lens diameter. For collinear slots (Fig. 4), i.e., when the direct coupling is weak, the mutual admittance is entirely dominated by the lens effect. The amplitude of the mutual admittance between two slots tapped by either a quartz lens ( $\epsilon_{r1} = 4$ , dashed line) or a silicon lens ( $\epsilon_{r1} = 11.7$ , continuous line) are presented in Fig. 5, for parallel [Fig. 5(a)] and collinear [Fig. 5(b)] slot configurations, and compared with those obtained for the corresponding homogeneous dielectric half-space. As expected, although the

lens dimensions are the same, the effect of quartz lens is weaker than that of the silicon lens; such an effect is, however, still significant for  $H$ -plane coupling [Fig. 5(b)].

The results presented here demonstrate the importance of including reflections inside the lens in calculating the slot mutual admittance. The appropriate description of the lens effects via a doubly reflected ray tracing requires a moderate calculation efforts. The results presented here suggest to construct a hybrid technique that employs in a Galerkin method of moments formulation the present technique for calculating the admittance matrix. This may provide an effective alternative to a computationally expensive full-wave analysis of the overall structure.

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