Problem-Matched Basis Functions for Moment Method Analysis—An Application to Reflection Gratings

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***Abstract*—In the evaluation of the frequency response of a scattering object by the integral equation technique, generally a large linear system of equations has to be solved for each frequency point. This paper deals with a technique that drastically reduces the size of the linear system without loss of accuracy. The key point is the definition of a set of problem-matched basis functions. These basis functions are extremely efficient in the representation of the unknown in the parameter range of interest. In this way, the central processing unit (CPU) time required in the response evaluation is drastically reduced. Examples of application concerning reflection gratings are reported.**

***Index Terms*—Gratings, method of moments.**

I. Introduction

THE method of moments (MoM) can be considered as one of the most popular numerical techniques for solving either open or closed scattering problems. A crucial point is usually the choice of a suitable basis function set to represent the unknown of the problem. For instance, in the case of the scattering from a conducting body, the unknown is the current induced on the surface. As is well known, if N is the number of basis functions, the unknown is determined through the solution of a linear system of size $N \times N$. When one is interested in the frequency response of the structure, the MoM has to be applied over and over again for each value of the variable parameter.

Concerning the choice of the basis functions, a small number of them should be able to provide a good representation of the induced current. If the structure is very complicated, "general-purpose" type functions are used such as those introduced in [1], which allow the maximum flexibility. If the geometry of the structure is more or less canonical, entire domain functions are used and the efficiency of the representation can be increased by selecting functions that satisfy the edge condition [2]. Even so, when the current to be expanded is a two-dimensional (2-D) vector field, the required number of basis functions may still be too high for response investigations or optimization-based design activities.

In this paper, we present a new technique to generate a set of problem-matched basis functions, which is based on the MoM solution of the scattering problem for a selected set of values of the variable parameter. The expansion coefficients of the unknown function are arranged columnwise in a matrix, which is then subjected to a singular value decomposition. In this way, the orthogonalized problem-matched basis function set is defined. Moreover, by inspection of the dynamic range of the singular values, it is possible to control the accuracy level of the procedure.

A related method has been presented in the past to reduce the computational effort involved in the frequency analysis of printed antennas [3]. It was based on the use of the resonant modes of the antenna patch as basis functions for the representation of the current in the MoM discretization of the electric field integral equation (EFIE). The resonant modes were determined by applying the singular value decomposition to the part of the total impedance matrix that refers to the radiating element. A different technique, called "marching on in frequency" has been proposed in the area of transient electromagnetic scattering [4]. It is based on the the iterative solution (according to the conjugate gradient scheme) of the frequency domain discretized EFIE. The fact that the unknown function depends smoothly on frequency is taken into account in the setting of the iterative scheme.

The new method that we propose is presented by discussing an application to a structure consisting of a frequency selective surface (FSS) of metal patches printed on a dielectric support and backed by a solid plate. Generally, the determination of the response to an incident wave coming from different directions does not require the recomputation of the MoM system matrix, but only the relatively fast back-substitution process. However, in the case of periodic structures such as FSS, the Green's function is represented in terms of Floquet modes, which depend on the incidence direction. This implies that if one is interested in the incidence direction response of the structure, the MoM system matrix has to be recomputed for each value of the variable parameter. This new method can be applied with advantage also in this case. The geometry of the structure presented in the result section has been defined with the goal of reducing the specular reflection for a given frequency band and for a range of incident directions. The example shows that this method allows the drastic reduction of the number of basis functions without loss of accuracy and saving a considerable amount of computation time.

II. Theory

Even though the concept presented below can be applied to any kind of scattering problem, a specific case is presented to

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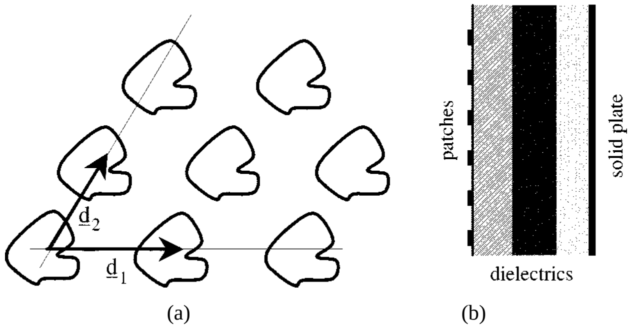

Fig. 1. Geometry of a FSS backed by a solid plate. (a) Top view. (b) Side view.

illustrate the idea. Consider the scattering from a FSS consisting of a periodic array of patches printed on a stratified dielectric support of thickness s and permittivity ϵ_r backed by a metal plate as shown in Fig. 1.The conducting patches are arranged in the plane $z = 0$ on the generally skew 2-D lattice defined by the basis vectors $\underline{d}_1, \underline{d}_2$. The problem is formulated as an EFIE, where the current induced on the patches is represented on the basis function set $\{\underline{\psi}_k(\rho)\}$ as

$$\underline{J}(\underline{\rho}) = \sum_{k=1}^{N} x_k \underline{\psi}_k(\underline{\rho}). \tag{1}$$

The application of the method of moments leads to a linear system that, following the formulation presented in [5] and [6], can be written as

$$(\underline{\underline{T}}^H \overset{\leftrightarrow}{\underline{\underline{Z}}} \, \underline{\underline{T}})\underline{X} = (\underline{\underline{T}}^H \underline{\underline{\Gamma}})\underline{V}^i \tag{2}$$

where $\underline{\underline{T}}$ is an abstract matrix of size $M \times N$, with M indicating the number of points of the reciprocal lattice, N the number of basis functions, and the superscript H indicates the hermitian adjoint. The element T_{mn} contains the 2-D Fourier transform $\underline{\Psi}_n(\underline{\sigma})$ of the vector basis functions $\underline{\psi}_n(\underline{\rho})$, evaluated in the mth point of the reciprocal lattice $\underline{\sigma}_m$. In particular, if the primary incident plane wave has a transverse wave vector $\underline{k}_t = k_0 \sin\theta^i(\cos\phi^i \hat{x} + \sin\phi^i \hat{y})$, the reciprocal lattice points are defined by

$$\underline{\sigma}_m = \underline{k}_t + p\underline{\sigma}_1 + q\underline{\sigma}_2 \tag{3}$$

where the couple of integers (p, q) denotes the mth point of the reciprocal lattice. The basis vectors $\underline{\sigma}_1$, $\underline{\sigma}_2$ are related to the direct lattice basis vectors $\underline{d}_1, \underline{d}_2$ via

$$\underline{\sigma}_1 = 2\pi \frac{\underline{d}_2 \times \hat{z}}{|\underline{d}_1 \times \underline{d}_2 \cdot \hat{z}|}; \quad \underline{\sigma}_2 = 2\pi \frac{\hat{z} \times \underline{d}_1}{|\underline{d}_1 \times \underline{d}_2 \cdot \hat{z}|} \tag{4}$$

where the carets denote unit vectors. Moreover, it is well known that $\underline{\sigma}_m$ are the transverse wave vectors of the Floquet modes excited by the discontinuity. The $M \times M$ diagonal abstract matrix $\overset{\leftrightarrow}{\underline{\underline{Z}}}$ appearing in (2) is the spectral representation of the Green's function evaluated in the points of the reciprocal lattice $\underline{\sigma}_m$. The (m, m)th element of this matrix has the dyadic expression

$$\begin{aligned}\overset{\leftrightarrow}{\underline{\underline{Z}}}(\underline{\sigma}_m) = &Z_{\mathrm{TM}}(\underline{\sigma}_m)\hat{\sigma}_m\hat{\sigma}_m \\ &+ Z_{\mathrm{TE}}(\underline{\sigma}_m \times \hat{z})(\hat{\sigma}_m \times \hat{z})\end{aligned} \tag{5}$$

where Z_{TM} and Z_{TE} are the total load impedances of the modal current generator that represents the unknown current distribution in circuit terms. In this case, one has

$$\begin{aligned}Z_{\mathrm{TM}}(\underline{\sigma}_m) &= \left[\frac{\omega\epsilon_0}{k_z} - j\frac{\omega\epsilon_0\epsilon_r}{k_{z\epsilon}}\cot(k_{z\epsilon}s)\right]^{-1}; \\ Z_{\mathrm{TE}}(\underline{\sigma}_m) &= \left[\frac{k_z}{\omega\mu_0} - j\frac{k_{z\epsilon}}{\omega\mu_0}\cot(k_{z\epsilon}s)\right]^{-1}\end{aligned} \tag{6}$$

where

$$k_z = \sqrt{k_0^2 - \underline{\sigma}_m \cdot \underline{\sigma}_m}; \quad k_{z\epsilon} = \sqrt{k_0^2\epsilon_r - \underline{\sigma}_m \cdot \underline{\sigma}_m}. \tag{7}$$

Analogously, $\underline{\underline{\Gamma}}$ is an $M \times M$ abstract diagonal matrix, describing the reflection of the structure in absence of the metallic patches. Its (m, m)th element has the dyadic expression

$$\begin{aligned}\underline{\underline{\Gamma}}(\underline{\sigma}_m) = &\Gamma_{\mathrm{TM}}(\underline{\sigma}_m)\,\hat{\sigma}_m\,\hat{\sigma}_m \\ &+ \Gamma_{\mathrm{TE}}(\underline{\sigma}_m)(\hat{\sigma}_m \times \hat{z})\,(\hat{\sigma}_m \times \hat{z})\end{aligned} \tag{8}$$

where

$$\begin{aligned}\Gamma_{\mathrm{TM}} &= \frac{1 + j\cot(k_{z\epsilon}s)\dfrac{k_z\epsilon_r}{k_{z\epsilon}}}{1 - j\cot(k_{z\epsilon}s)\dfrac{k_z\epsilon_r}{k_{z\epsilon}}} \\ \Gamma_{\mathrm{TE}} &= \frac{1 + j\cot(k_{z\epsilon}s)\dfrac{k_{z\epsilon}}{k_z}}{1 - j\cot(k_{z\epsilon}s)\dfrac{k_{z\epsilon}}{k_z}}.\end{aligned} \tag{9}$$

The incident electric field is described by the abstract column vector $\underline{V}^i$, whose elements are the transverse electric field of the incident plane waves with transverse wave vector $\underline{\sigma}_m$. In particular, in the case of a single incident plane wave from the direction θ^i, ϕ^i, only the $m = 0$ element is different from zero. Finally, the column vector $\underline{X}$ contains the unknown expansion coefficients $\{x_k\}$ of (1).

What is of interest is the multimodal reflection coefficient $\underline{\underline{R}}$ of the structure and this can be computed by the following operator expressed in the Floquet basis:

$$\underline{\underline{R}} = \underline{\underline{\Gamma}} + \overset{\leftrightarrow}{\underline{\underline{Z}}}\,\underline{\underline{T}}[\underline{\underline{T}}^H \overset{\leftrightarrow}{\underline{\underline{Z}}}\,\underline{\underline{T}}]^{-1}\underline{\underline{T}}^H\underline{\underline{\Gamma}}. \tag{10}$$

When the frequency or the incidence direction changes, the reciprocal lattice moves in the spectral plane and the Fourier transforms of the basis functions are sampled in different points, unless these spectra are rigidly connected with the lattice. This can be obtained by defining new basis functions which contain the phase shift imposed by the incident plane wave

$$\tilde{\underline{\psi}}_n(\underline{\rho}) = \underline{\psi}_n(\underline{\rho})\exp(-j\underline{k}_t \cdot \underline{\rho}). \tag{11}$$

In this way, the transform of $\tilde{\underline{\psi}}_n(\rho)$ is $\underline{\Psi}_n(\underline{\sigma} - \underline{k}_t)$ and the relevant $\underline{\underline{T}}$ matrix does not depend on $\underline{k}_t$, i.e., on frequency and incidence direction and can be computed once for all, which entails a computer time saving that can be significant if the transforms have to be evaluated numerically.

In order to reduce drastically the computer time, only a few basis functions must be used. Obviously, these functions must fit closely the actual current distribution.

Consider the problem where the frequency response of the structure is required. A possible set of "problem-matched basis functions" is generated as follows. Let $\{\underline{\psi}_p^{(\omega)}(\underline{\rho})\}$ with $p = 1, \cdots, P$ be the current distributions at P frequency points, evenly spaced in the band of interest. They are obtained by the MoM as described above and have the following representation in the basis $\{\underline{\tilde{\psi}}_k(\underline{\rho})\}$:

$$\underline{\psi}_p^{(\omega)}(\underline{\rho}) = \sum_{k=1}^{N} X_{kp}\underline{\tilde{\psi}}_k(\underline{\rho}) \tag{12}$$

where $X_{kp} = x_k(\omega_p)$ with $k = 1, \cdots, N$ and $p = 1, \cdots, P$. These P new basis functions are certainly problem matched, but one has to decide how many of them are necessary. If too few are chosen, there is an unacceptable loss of accuracy in the evaluation of the response curve. If too many are selected, even if $P \leq N$, one has a redundant description and there may be the risk that some basis functions are scarcely linearly independent from the others.

The solution to the above problem can be found by applying the singular value decomposition to the $N \times P$ matrix $\underline{\underline{X}}$ with elements X_{kp} introduced in (12). It is well known [7] that this decomposition has the form

$$\underline{\underline{X}} = \underline{\underline{U}}\,\underline{\underline{S}}\,\underline{\underline{V}}^H \tag{13}$$

where $\underline{\underline{U}}$ is a $N \times N$ unitary matrix, $\underline{\underline{S}}$ is a $N \times P$ diagonal matrix with positive elements (singular values), and $\underline{\underline{V}}$ is a $P \times P$ unitary matrix. The columns of $\underline{\underline{U}}$ are the singular vectors $\{\underline{u}_p\}$ and those corresponding to nonzero singular values $\{S_p\}$ form an orthonormal basis in the subspace spanned by the current vectors $\underline{X}(\omega_p)$. The significance of the various singular vectors $\{\underline{u}_p\}$ in the description of the current distribution is measured by the amplitude of the corresponding singular values S_p. Since these range over several orders of magnitude, not all the singular vectors are needed to obtain accurate results in the response curve. Also, inspection of the dynamic range of the singular value set allows one to ascertain whether the frequency sampling is adequate. A small dynamic range means, in fact, that the corresponding singular vectors do not have sufficient span for an acceptable representation of the current distribution.

Let then $Q \leq P$ be the number of singular vectors assumed to be adequate to represent the unknown. Hence, Q is the "equivalent dimension" of the subspace that contains the current representation, at least in the band of interest. Define a new set of basis functions via

$$\underline{\psi}_q^{(s)}(\underline{\rho}) = \sum_{k=1}^{N} \underline{\tilde{\psi}}_k(\underline{\rho})u_{kq} \quad \text{with} \quad q = 1, \cdots, Q \tag{14}$$

which can be interpreted as "orthogonalized problem-matched basis functions." They lead to the definition of a new projection matrix $\underline{\underline{T}}^{(s)}$ given by

$$\underline{\underline{T}}^{(s)} = \underline{\underline{T}}\,\underline{\underline{U}}^{(Q)} \tag{15}$$

where the $N \times Q$ matrix $\underline{\underline{U}}^{(Q)}$ consists of the first Q columns of the matrix $\underline{\underline{U}}$. The $M \times Q$ projection matrix $\underline{\underline{T}}^{(s)}$ obviously does not depend on frequency and can be computed at the beginning of the response computation and stored. The linear system to be solved for each frequency value is still expressed by (2) with $\underline{\underline{T}}$ substituted with $\underline{\underline{T}}^{(s)}$ and has size $Q \times Q$ where Q may be much smaller than N.

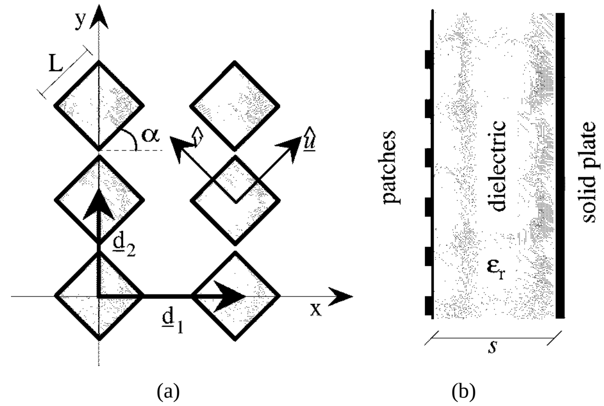

Fig. 2. Geometry of the reflection grating $L = 14$ mm, $\alpha = 45°$, $d_1 = 29$ mm, $d_2 = 20.5$ mm, $\epsilon_r = 2.56$, $s = 4.8$ mm. (a) Top view. (b) Side view.

III. Results

To illustrate the above idea, the scattering properties of a reflection grating are analyzed. The structure consists of an array of conducting patches printed on a dielectric substrate and backed by a ground plane, as shown in Fig. 2. Such structures may have the property of eliminating the specular reflection with a proper choice of the grating geometry [8], [9].

The grating under consideration is printed on a dielectric with $\epsilon_r = 2.56$ and thickness $s = 4.8$ mm. The lattice geometry is rectangular with $d_1 = 29$ mm and $d_2 = 20.5$ mm so that the first higher order Floquet modes can propagate at X band. The conducting patches are square with a rotation angle of $\alpha = 45°$ so that in conjunction with the choice of the aspect ratio of the lattice, the structure is less sensitive to the direction of the incident wave in the XZ plane.

The structure has been designed so that the specular reflection is eliminated. The current induced on the patches is represented by weighted Chebyshev polynomials [10] in order to satisfy the edge conditions. In particular, the divergent and convergent behaviors on a generic length w along the x-axis are represented as follows:

$$\begin{aligned} D_m(x,w) &= \frac{T_m\left(\frac{2x}{w}\right)}{\sqrt{1-\left(\frac{2x}{w}\right)^2}} \\ C_m(x,w) &= U_m\left(\frac{2x}{w}\right)\sqrt{1-\left(\frac{2x}{w}\right)^2} \\ &\qquad m = 0, 1, 2, 3 \cdots \end{aligned} \tag{16}$$

where T_m and U_m are the Chebyshev polynomials of the first and second kind, respectively. The Fourier transforms of D_m and C_m have analytical expressions, which, apart from factors that can be included in the definition of the current coefficients

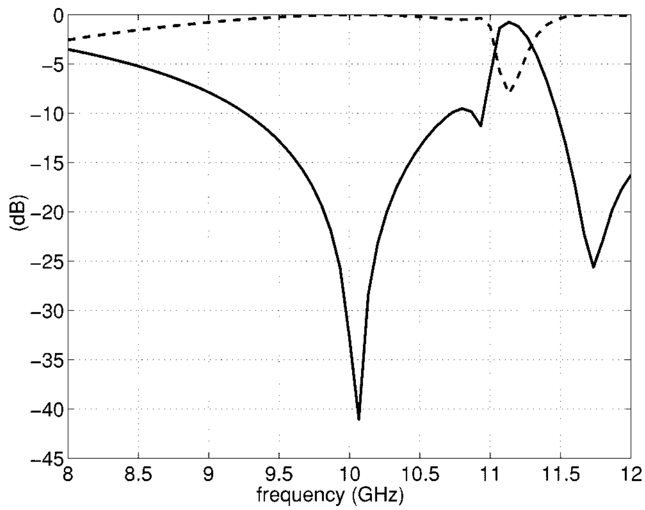

Fig. 3. TE reflection coefficient of the structure of Fig. 2 versus frequency for the incidence direction $\theta = 30°$, $\phi = 0°$: $TE^{(s)}_{00}/TE^{(i)}_{00}$ (solid line); $TE^{(s)}_{-10}/TE^{(i)}_{00}$ (dashed line).

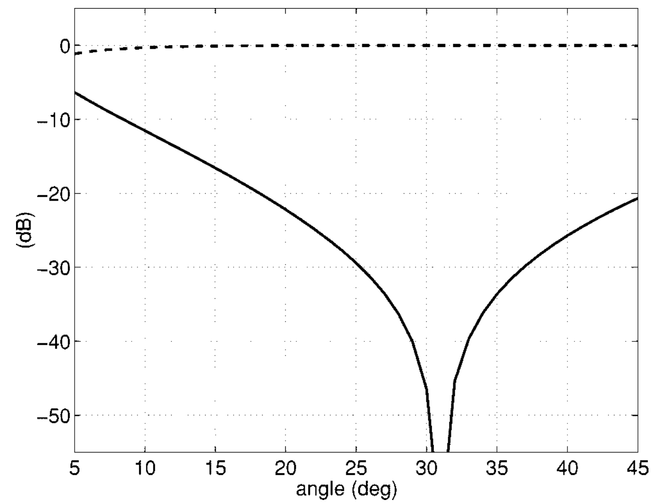

Fig. 4. TE reflection coefficient of the structure of Fig. 2 versus angle of incidence at the frequency 10.05 GHz and $\phi = 0°$: $TE^{(s)}_{00}/TE^{(i)}_{00}$ (solid line); $TE^{(s)}_{-10}/TE^{(i)}_{00}$ (dashed line).

x_k, can be represented in terms of Bessel functions of first kind J_ν as

$$D_m(\xi, w) = J_m\left(\frac{\xi w}{2}\right)$$
$$C_m(\xi, w) = \frac{1}{\xi} J_{m+1}\left(\frac{\xi w}{2}\right) \qquad m = 0, 1, 2, 3 \cdots \tag{17}$$

where ξ is the spectral variable conjugate to x. Combining the expressions (17), the basis functions for a square patch of side L assume the following forms in the spectral domain:

$$\underline{\Psi}_k(\underline{\sigma}) = \begin{cases} \dfrac{\hat{u}}{\underline{\sigma} \cdot \hat{u}} J_{r+1}\left(\dfrac{L}{2}\underline{\sigma} \cdot \hat{u}\right) J_s\left(\dfrac{L}{2}\underline{\sigma} \cdot \hat{v}\right) \\ \quad \text{for } \hat{u} \text{ directed current} \\ \dfrac{\hat{v}}{\underline{\sigma} \cdot \hat{v}} J_{r+1}\left(\dfrac{L}{2}\underline{\sigma} \cdot \hat{v}\right) J_s\left(\dfrac{L}{2}\underline{\sigma} \cdot \hat{u}\right) \\ \quad \text{for } \hat{v} \text{ directed current} \end{cases} \tag{18}$$

with

$$r = [0, 1 \cdots R]; \qquad s = [0, 1 \cdots S]$$

and the unit vectors $\hat{u}$ and $\hat{v}$ are the symmetry axes of the patch (see Fig. 2) and are given by

$$\hat{u} = \hat{x}\cos(\alpha) + \hat{y}\sin(\alpha)$$
$$\hat{v} = -\hat{x}\sin(\alpha) + \hat{y}\cos(\alpha).$$

In this way, the total number of basis functions N is $2(R+1)(S+1)$. Fig. 3 shows the plots of the specular TE reflection coefficient relative to an incident plane wave with $\theta^i = 30°$, $\phi^i = 0°$. As one can see, around 10 GHz there is a strong reduction of the specular reflection. This property is maintained also while varying the angle of incidence as shown in Fig. 4. The incident wave in this case is converted mainly into the scattered TE_{-10} Floquet mode that for $\theta^i = 30°$ at 10 GHz has the direction $\theta = -32.31°$. Figs. 3 and 4 also report the TE_{-10}/TE_{00} reflection coefficients. The slope discontinuity in the specular

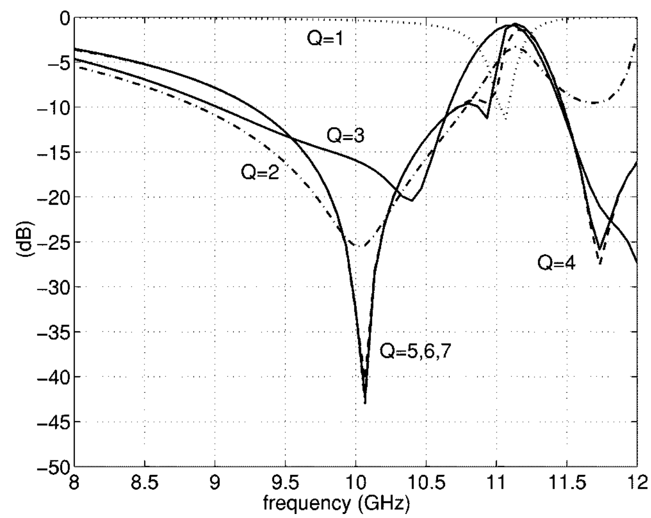

Fig. 5. $TE^{(s)}_{00}/TE^{(i)}_{00}$ reflection coefficient versus frequency, computed using Q problem-matched basis functions; $Q = 1, \cdots, 7$.

reflection coefficient near 11 GHz is a resonance-type Wood anomaly. In this frequency range, 16 Floquet modes are above cutoff in the dielectric.

The convergence of the results presented above has been reached with 72 basis functions of the type (18) with the spectral summations extended over 1750 points of the reciprocal lattice. This means that the projection matrix $\underline{\underline{T}}$ of (2) has size 1750×72 and a 72×72 linear system must be solved for each frequency and incident direction.

The problem-matched basis function concept has been applied to this problem. In the case of frequency response computation, the current coefficients for TE incidence are evaluated at $P = 7$ frequency points, evenly spaced in the range of 8–12 GHz. The singular values of the 72×7 $\underline{\underline{X}}$ matrix containing the current coefficients X_{kp} were 17.673, 8.4713, 3.0933, 1.7738, 0.4401, 0.0125, and 0.0022. The range of the singular values confirms that the frequency sample rate is sufficient to represent the frequency variations of the induced currents. In fact, the sequence of singular values shows a clear exponential decay

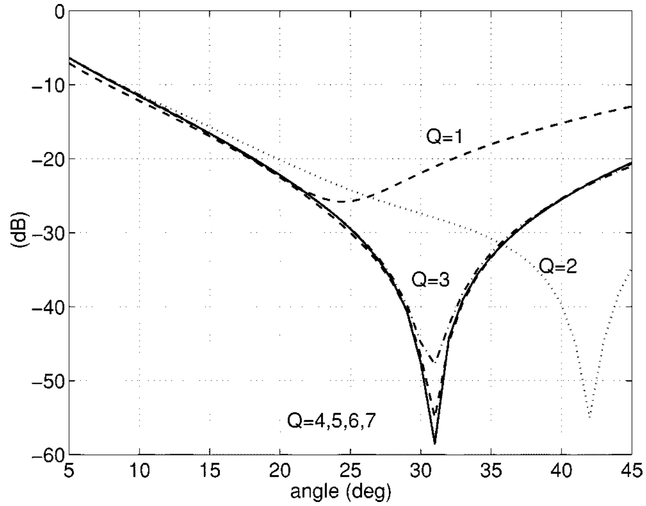

Fig. 6. $\mathrm{TE}_{00}^{(s)}/\mathrm{TE}_{00}^{(i)}$ reflection coefficient versus angle of incidence at frequency 10.05 GHz, computed using Q problem-matched basis functions; $Q = 1, \cdots, 7$.

starting at the fifth one. Fig. 5 shows the same specular reflection coefficient curve of Fig. 3, evaluated using Q problem-matched basis functions obtained by (14). It can be observed that with just $Q = 5$, the frequency response is reproduced. It has to be remarked that in this case the system to be solved is reduced to 5×5 from 72×72, giving a remarkable reduction in computation time without loss of accuracy.

In the MoM application, most of the CPU time is devoted to the computation of the system matrix rather than to its inversion. In this example the use of problem-matched basis functions led to the computation of just 25 matrix elements instead of 5184.

This method also produces good results when the variable parameter is the angle of incidence. In the case of Fig. 6, the current coefficients are evaluated at seven angles evenly spaced in the range from 5° to 45°. As can be observed, also in this case, just five problem-matched basis functions are sufficient to reproduce the angular response with excellent accuracy.

It has to be noticed that the structure used to illustrate the problem-matched basis function concept is analyzed in a frequency range where higher order Floquet modes are propagating in the dielectric support and even in free-space. Moreover, the presence of the metallic ground plane causes a multimodal standing wave in the dielectric support that modifies significantly the current distribution on the patches when the frequency changes. This proves the broad range of applicability of this technique.

As a final comment, one can observe that if the sampling rate is adequate, the sequence of singular values presents a knee, after which they decrease exponentially. As a general rule, the singular vectors corresponding to singular values at least two orders of magnitude below the dominant one can be safely neglected.

IV. Conclusion

The concept of problem-matched basis functions has been applied to moment-method analysis. This idea is very useful when, due to the complexity of the geometry, a large number of basis functions are required to represent the unknown. The method has been applied to study the specular reflection from a planar array of patches backed by a solid plate. However, this concept can be successfully applied to a wide class of problems such as the scattering from very complex objects or waveguide discontinuity problems. Consider for example the design of multiplexers or of dual-mode elliptical waveguide filters where the projection matrix has to be computed by numerical integration [11]. Often, the geometrical configuration is obtained by an optimization process, where several full wave analyzes are required. In this case, the application of this method produces a considerable reduction in computation time. Finally, the same concept can be applied to the mode-matching technique in the waveguide discontinuity problem. In fact, as demonstrated in [12], this technique can be seen as the MoM, where the basis functions are the waveguide eigenmodes.

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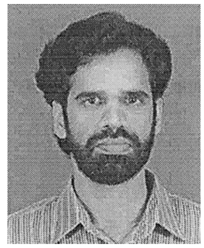

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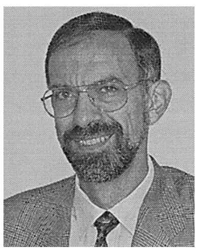

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