Formulation of Spatial Correlation Statistics in Nakagami–Rice Fading Environments

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***Abstract*—Propagation environments in line-of-sight (LOS) communications, such as mobile-satellite communications and indoor radio communications, can be expressed by a Nakagami–Rice (or Rician) fading model. Since the Nakagami–Rice fading model also covers the Rayleigh fading model as an extreme case, it seems a very widely applicable model. This paper gives a theoretical formulation of spatial correlation characteristics in the Nakagami–Rice fading environment. Based on this theoretical model, spatial correlation characteristics of Nakagami–Rice fading are clarified comparing with those of Rayleigh fading. Moreover, the difference in spatial correlation characteristics between wide-band signals and narrow-band signals is discussed.**

***Index Terms*—Fading channels, spatial correlation.**

I. Introduction

A Nakagami–Rice fading model (also known as a Rician model) is a model expressing a fading environment comprising a stationary wave component and an incoherently fluctuating wave component. The stationary wave often appears in the radio wave environment of line-of-sight (LOS) communication systems corresponding to the direct wave, and the incoherent component comes from multipath waves. Typical examples of LOS mobile radio systems are indoor wireless local area networks (LAN's) and personal satellite communications. In the Nakagami–Rice fading model, if the strength of the stationary wave component that can be set arbitrarily is zero, the situation is the same as that of the Rayleigh fading model, which is used frequently in terrestrial mobile propagation modeling. Hence, the Nakagami–Rice model might be extremely universal.

A considerable amount of theoretical work has been carried out on spatial correlation characteristics in a Rayleigh fading environment, and its fundamental nature has now been made clear [1]–[4]. On the other hand, concerning spatial correlation characteristics in a Nakagami–Rice fading environment, few theoretical analyses have been performed, excluding one where we had derived a basic equation contributing to the analysis of correlation characteristics in the Nakagami–Rice fading environment [5], [6].

In this paper, a general formula is derived for spatial correlation characteristics in wideband signal transmission in a Nakagami–Rice fading environment. Using this theoretical model, we calculate the spatial correlation characteristics (more specifically, spatial correlation characteristics of in-band power fluctuations) for some typical fading environments usually appearing in mobile radio systems. This analysis allows clarification of the difference in spatial correlation characteristics between Rayleigh fading and Nakagami–Rice fading, and of that between wide-band signals and narrow-band signals.

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II. Formulation

Let the frequency of a carrier wave be f_c and the radio frequency centered on f_c be f_{RF}. We will assume that the frequency for an equivalent base-band system is $f\ (=f_{RF}-f_c)$. Let a transfer function of a propagation channel at an arbitrary frequency f and an arbitrary location x be $T(f, x)$. For reception at the base station, the transfer function itself changes with time due to the motion of user equipment. For reception by each user terminal, on the other hand, temporal variation of the transfer function comes from spatial change due to the motion of the terminal. In Nakagami–Rice fading, we can distinguish the coherent component (= direct wave component) and the incoherent component (= reflected-wave component), denoted as $T_D(f, x)$ for the former and $T_R(f, x)$ for the latter. Let the bandwidth of a transmitted signal be B. Fig. 1 shows an image of the transfer function and signal bandwidth in positions x_1 and $x_2\ (=x_1+\Delta x)$.

For T_D, the amplitude can be assumed constant with respect to both x and f in the limited range of consideration and we may therefore consider a normalized transfer function T_{R0}

$$T_{R0}(f, x) \equiv T_R(f, x)/T_D(f, x). \tag{1}$$

For simplification, we write

$$a_1 \equiv T_{R0}(f, x) \tag{2a}$$

$$a_2 \equiv T_{R0}(f, x+\Delta x). \tag{2b}$$

The amplitudes of normalized transfer functions $1+a_1$, $1+a_2$ are then

$$A_1(f) = |1+a_1| \tag{3a}$$

$$A_2(f) = |1+a_2|. \tag{3b}$$

Power characteristics $A_1^2(f)$, $A_2^2(f)$ of the channel are therefore given by

$$A_i^2(f) = 1 + a_i(f) + a_i^*(f) + a_i^*(f)a_i(f) \qquad (i=1, 2) \tag{4}$$

where $*$ implies complex conjugate.

The instantaneous values of the powers P_1, P_2 of wide-band signals may be found by in-band power integration of $A_1^2(f)$,

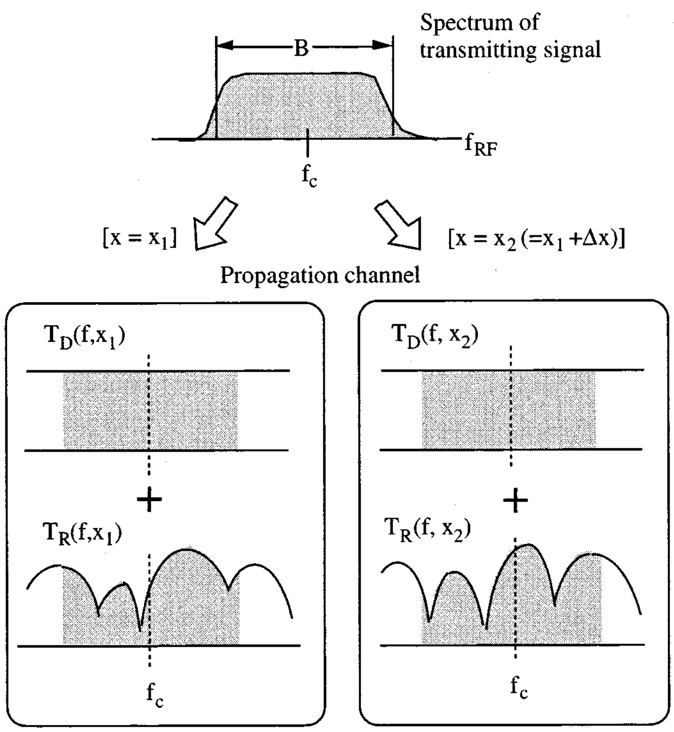

Fig. 1. Transfer function for the direct-wave component and multipath-wave component received at two different points.

$A_2^2(f)$ whose integration time range is much shorter than the fading period

$$P_i = \frac{1}{B}\int_{-(B/2)}^{B/2} A_i^2(f)\,df = 1 + \frac{1}{B}\int_{-(B/2)}^{B/2} \{a_i(f) + a_i^*(f) + a_i^*(f)a_i(f)\}\,df. \quad (5)$$

Here we will focus our attention on the correlation $\rho_{\delta P}$ of the power fluctuation, which is defined as a spatial correlation by (6) (this corresponds to $\rho_{\delta A^2}$ in [5] and [7], but we use $\rho_{\delta P}$ since A represents an in-band signal at a point frequency). This is due to the fact that for Nakagami–Rice fading, as in the case of Rayleigh fading, the correlation of amplitude fluctuation and power fluctuation is approximately the same and the analysis is easier if we consider power fluctuation [5]

$$\rho_{\delta P} = \frac{\langle P_1 P_2\rangle - \langle P_1\rangle\langle P_2\rangle}{\sqrt{\langle P_1^2\rangle - \langle P_1\rangle^2}\sqrt{\langle P_2^2\rangle - \langle P_2\rangle^2}} = \frac{\langle P_1 P_2\rangle - \langle P_1\rangle^2}{\langle P_1^2\rangle - \langle P_1\rangle^2}. \quad (6)$$

Herein, $\langle\,\rangle$ means the ensemble average, but when it appears as time-dependent fluctuations as in the case of base-station reception, it may be taken as time average in a sufficiently longer time than the fading period.

Writing

$$a \equiv \frac{1}{B}\int_{-(B/2)}^{B/2} a_i(f)\,df \quad (7a)$$

$$\beta_i^2 \equiv \frac{1}{B}\int_{-(B/2)}^{B/2} a_i^*(f)a_i(f)\,df \quad (7b)$$

with considering that $\langle a_i\rangle = 0$, $\langle a_i a_i\rangle = 0$, $\langle a_i a_j^* a_j\rangle = 0$, and $\langle\beta_1^2\rangle = \langle\beta_2^2\rangle$, the terms in (6) are

$$\langle P_1\rangle = \langle P_2\rangle = 1 + \langle\beta_1^2\rangle \quad (8a)$$

$$\begin{aligned}\langle P_1P_2\rangle = &\Big\langle\Big[1 + \frac{1}{B}\int_{-(B/2)}^{B/2}\{a_1(f) + a_1^*(f) + a_1^*(f)a_1(f)\}\,df\Big] \cdot \Big[1 + \frac{1}{B}\int_{-(B/2)}^{B/2}\{a_2(f) + a_2^*(f) + a_2^*(f)a_2(f)\}\,df\Big]\Big\rangle \\ = &1 + \frac{1}{B^2}\Big\langle\int_{-(B/2)}^{B/2} a_1^*(f)\,df\int_{-(B/2)}^{B/2} a_2(f)\,df\Big\rangle + \frac{1}{B^2}\Big\langle\int_{-B/2}^{B/2} a_1(f)\,df\Big\rangle \\ &+ \frac{1}{B}\Big\langle\int_{-(B/2)}^{B/2} a_1^*(f)a_1(f)\,df\Big\rangle + \frac{1}{B}\Big\langle\int_{-(B/2)}^{B/2} a_2^*(f)a_2(f)\,df\Big\rangle \\ &+ \frac{1}{B^2}\Big\langle\int_{-(B/2)}^{B/2} a_1^*(f)a_1(f)\,df \cdot \int_{-(B/2)}^{B/2} a_2^*(f)a_2(f)\,df\Big\rangle \\ = &1 + \langle\alpha_1^*\alpha_2\rangle + \langle\alpha_1\alpha_2^*\rangle + 2\langle\beta_1^2\rangle + \langle\beta_1^2\beta_2^2\rangle\end{aligned} \quad (8b)$$

$$\langle P_1^2\rangle = \langle P_2^2\rangle = 1 + 2\langle\alpha_1^*\alpha_1\rangle + 2\langle\beta_1^2\rangle + \langle\beta_1^4\rangle. \quad (8c)$$

Using the above relations, (6) can be written as

$$\rho_{\delta P}(\Delta x) = \frac{\langle\alpha_1\alpha_2^*\rangle + \langle\alpha_1^*\alpha_2\rangle + \langle\beta_1^2\beta_2^2\rangle - \langle\beta_1^2\rangle^2}{2\langle\alpha_1^*\alpha_1\rangle + \langle\beta_1^4\rangle - \langle\beta_1^2\rangle^2}. \quad (8d)$$

By defining a space–frequency correlation coefficient of a_1 and a_2 at the frequency f_{RF} (= correlation coefficient between two signals differing in frequency by Δf and in position by Δx) as

$$\begin{aligned}\rho_{a,fx}\{\Delta f, \Delta x; f_{RF}(\equiv f_c + f)\} &\equiv \frac{\langle T_{R0}^*(f,x)T_{R0}(f+\Delta f, x+\Delta x)\rangle}{\langle T_{R0}^*(f,x)T_{R0}(f,x)\rangle} \\ &= \frac{\langle a_1^*(f)a_2(f+\Delta f)\rangle}{\langle a_1^*(f)a_1(f)\rangle}\end{aligned} \quad (9)$$

the following relation can be obtained based on [7, eq. (52)])

$$\begin{aligned}&\langle a_1^*(f)a_1(f)a_2^*(f+\Delta f)a_2(f+\Delta f)\rangle \\ &= s^4\left(1 + |\rho_{a,fx}(\Delta f, \Delta x; f_c + f)|^2\right)\end{aligned} \quad (10)$$

where s^2 is average scattered wave power normalized by the direct-wave power [a key parameter of a Nakagami–Rice fading environment [8], which, in this paper, equals to $\langle \beta_1^2 \rangle \, (= \langle \beta_2^2 \rangle)$]. Using the above relations

$$\begin{aligned}\langle \alpha_1^* \alpha_2 \rangle &= \frac{1}{B^2} \int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)}^{B/2} \langle a_1^*(f_1) a_2(f_2) \rangle \, df_2 \, df_1 \\ &= \frac{1}{B^2} \int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)-f}^{(B/2)-f} \\ &\quad \cdot \langle a_1^*(f) a_2(f + \Delta f) \rangle \, d\Delta f \, df \\ &= \frac{s^2}{B^2} \int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)-f}^{(B/2)-f} \\ &\quad \cdot \rho_{a,\,fx}(\Delta f, \, \Delta x; \, f_c + f) \, d\Delta f \, df \end{aligned} \tag{11a}$$

$$\begin{aligned}\langle \alpha^* \alpha_1 \rangle &= \frac{s^2}{B^2} \int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)-f}^{(B/2)-f} \\ &\quad \cdot \rho_{a,\,fx}(\Delta f, \, 0; \, f_c + f) \, d\Delta f \, df \end{aligned} \tag{11b}$$

$$\begin{aligned}\langle \beta_1^2 \beta_2^2 \rangle &= \frac{1}{B^2} \int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)}^{B/2} \\ &\quad \cdot \left\langle a_1^*(f_1) a_1(f_1) a_2^*(f^2) a_2(f_2) \right\rangle \, df_2 \, df_1 \\ &= \frac{1}{B^2} \int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)-f}^{(b/2)-f} \\ &\quad \cdot \langle a_1^*(f) a_1(f) a_2^*(f + \Delta f) \, a_2(f + \Delta f) \rangle \, d\Delta f \, df \\ &= s^4 \left\{ 1 + \frac{1}{B^2} \int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)-f}^{(B/2)-f} \right. \\ &\quad \left. \cdot \left| \rho_{a,\,fx}(\Delta f, \, \Delta x; \, f_c + f) \right|^2 \, d\Delta f \, df \right\} \end{aligned} \tag{12a}$$

$$\begin{aligned}\langle \beta_1^4 \rangle &= s^4 \left\{ 1 + \frac{1}{B^2} \int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)-f}^{(B/2)-f} \right. \\ &\quad \left. \cdot \left| \rho_{a,\,fx}(\Delta f, \, 0; \, f_c + f) \right|^2 \, d\Delta f \, df \right\}. \end{aligned} \tag{12b}$$

Finally, (6) with (8d) can then be given as shown in (13) at the bottom of the page. Expression (13) is the general expression for spatial correlation characteristics in a wide-band Nakagami–Rice fading environment.

In available mobile radio systems, since relative bandwidth defined by the ratio between signal bandwidth (B) and carrier frequency (f_c) is much smaller than 10%, we can imagine easily that there is very little dependence of frequency on spatial correlation characteristics. The space–frequency correlation coefficient $\rho_{a,\,fx}(\Delta f, \, \Delta x; \, f_{RF})$ at the frequency f_{RF} may then be approximated by the following equation [9]:

$$\rho_{a,\,fx}(\Delta f, \, \Delta x; \, f_{RF}) \approx \rho_{a,\,f}(\Delta f) \, \rho_{a,\,x}(\Delta x; \, f_{RF}). \tag{14}$$

Herein, $\rho_{a,\,f}(\Delta f)$ is the frequency correlation coefficient, and $\rho_{a,\,x}(\Delta x; \, f_{RF})$ is the spatial correlation coefficient at the RF $f_{RF} \, (= f_c + f)$.

If the delay power profile of incident waves normalized by the direct wave power is $p(\tau)$ and the normalized angular power profile is $\Omega(\theta)$, writing the scattered wave components as $p_s(\tau)$ and $\Omega_s(\theta)$, the two profiles can be expressed by the following equations (angular relationships of spatial correlation characteristics are shown in Fig. 2):

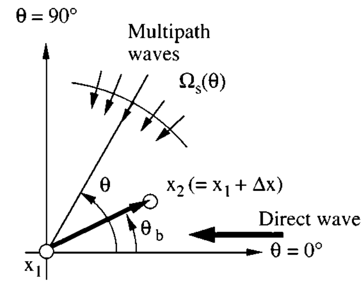

Fig. 2. Coordinate system for the analysis.

$$p(\tau) = \delta(\tau) + p_s(\tau) \tag{15}$$

$$\Omega(\theta) = \delta(\theta) + \Omega_s(\theta) \tag{16}$$

where δ is the delta function for expressing the delay and incident direction of the direct-wave path with $\tau = 0$ and $\theta = 0$.

The frequency correlation coefficient $\rho_{a,\,f}(\Delta f)$ in (14) is given by the following equation [7]:

$$\rho_{a,\,f}(\Delta f) = \frac{1}{s^2} \int_{-\infty}^{\infty} p_s(\tau) \exp(-j 2\pi \Delta f \tau) \, d\tau. \tag{17}$$

Similarly, the spatial correlation coefficient $\rho_{a,\,x}(\Delta x; \, f_{RF})$ at the frequency f_{RF} can be obtained by the following equation:

$$\begin{aligned}\rho_{a,\,x}(\Delta x; \, f_{RF}) &= \frac{\langle a_1^*(f_{RF}) a_2(f_{RF}) \rangle}{\langle a_1^*(f_{RF}) a_1(f_{RF}) \rangle} \\ &= \frac{1}{s^2} \exp\left(-\frac{j 2\pi f_{RF} \Delta x \cos\theta_b}{c} \right) \int_0^{2\pi} \Omega(\theta) \\ &\quad \cdot \exp\left\{ \frac{j 2\pi f_{RF} \Delta x \cos(\theta - \theta_b)}{c} \right\} d\theta \end{aligned} \tag{18}$$

where c is speed of light.

$$\rho_{\delta P}(\Delta x) = \frac{\int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)-f}^{(B/2)-f} \left\{ 2R_e \left[\rho_{a,\,fx}(\Delta f, \, \Delta x; \, f_c + f) \right] + s^2 \left| \rho_{a,\,fx}(\Delta f, \, \Delta x; \, f_c + f) \right|^2 \right\} d\Delta f \, df}{\int_{-(B/2)}^{B/2} \cdot \int_{-(B/2)-f}^{(B/2)-f} \left\{ 2R_e \left[\rho_{a,\,fx}(\Delta f, \, 0; \, f_c + f) \right] + s^2 \left| \rho_{a,\,fx}(\Delta f, \, 0; \, f_c + f) \right|^2 \right\} d\Delta f \, df} \tag{13}$$

The difference from the spatial correlation in Rayleigh fading is the exponential function outside the integral on the right-hand side of the above equation. This is due to the fact that "a" in (2), which was normalized by the phase of the direct wave, is used instead of the correlation characteristics of the scattered wave itself. To find the correlation characteristics for Nakagami–Rice fading, it is therefore necessary to consider this phase change which does not give a significant meaning for Rayleigh fading.

Thus, the spatial correlation characteristics of a wide-band signal given by (13) may be found by specifying the delay profile $p(\tau)$, angular profile $\Omega(\theta)$ of the incoming waves, carrier frequency f_c, and signal bandwidth B.

When a narrow-band signal is considered (i.e., $B \ll f_c$), (13) becomes

$$\rho_{\delta P}(\Delta x;\, f_c)|_{B\to 0} = \frac{2\mathrm{Re}\left[\rho_{a,x}(\Delta x;\, f_c)\right] + s^2 \left|\rho_{a,x}(\Delta x;\, f_c)\right|^2}{2+s^2} \quad \text{(Nakagami–Rice fading)} \qquad (19a)$$

$$\rho_{\delta P}(\Delta x;\, f_c)|_{\substack{B\to 0\\ s^2 \gg 1}} = \left|\rho_{a,x}(\Delta x;\, f_c)\right|^2 \quad \text{(Rayleigh fading).} \qquad (19b)$$

Equation (19a) corresponds to an equation derived by the authors in [5, eq. (11)] [where $\sigma^2 \to s^2/2$] and (19b) can be seen in [7].

III. Spatial Correlation Characteristics for Narrow-Band Signal

The spatial correlation characteristics for a narrow-band signal can be calculated by (18) and (19) for propagation environments expressed by the angular profile (average power profile) of incoming waves. We will now find the correlation characteristics in the following three cases.

A. Case Where Scattered Waves Arrive with a Uniform Angular Distribution—Case 1

Here, the case will be considered where a large number of scattered waves arrive with a uniform angular distribution in addition to the direct wave as shown in Fig. 3. The angular profile of the scattered waves Ω_s in (16) is given by

$$\Omega_s(\theta) = \frac{s^2}{2\pi} \qquad (20)$$

hence,

$$\rho_{a,x}(\Delta x;\, f_c) = J_0(k\Delta x)\exp(-jk\Delta x \cos\theta_b). \qquad (21)$$

Here,

J_0; 0th order first-kind Bessel function

k wave number defined by $k = 2\pi f_c/c$.

The characteristics are given by the following equation:

$$\rho_{\delta P}(\Delta x) = \frac{2\left\{J_0(k\Delta x)\cos(k\Delta x\cos\theta_b)\right\} + s^2\left\{J_0(k\Delta x)\right\}^2}{2+s^2} \quad \text{(Nakagami–Rice fading)} \qquad (22)$$

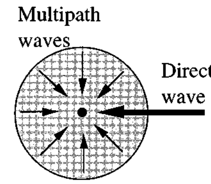

Fig. 3. Angular profile of incident waves whose multipath-wave component is distributed omnidirectionally (case 1).

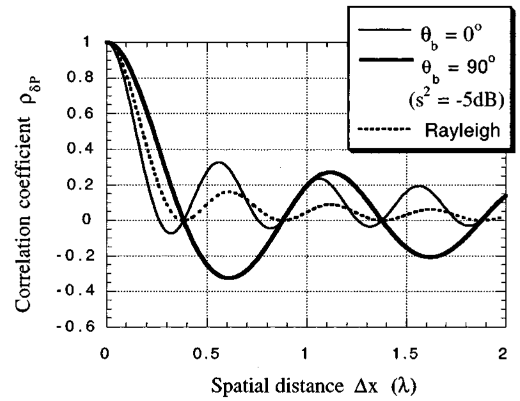

Fig. 4. Spatial correlation characteristics of Nakagami–Rice fading where multipath waves are distributed omnidirectionally (case 1).

$$= \left\{J_0(k\Delta x)\right\}^2 \qquad \left(1 \ll s^2\text{: Rayligh fading}\right) \qquad (23a)$$

$$= J_0(k\Delta x)\cos(k\Delta x\cos\theta_b) \qquad \left(1 \gg s^2\right). \qquad (23b)$$

Fig. 4 shows the case where, in a Nakagami–Rice fading environment of $s^2 = -5$ dB, the baseline angle θ_b, taking the incident direction of the direct wave as a reference, is 0° or 90°. The case of Rayleigh fading (23a) is also shown for comparison. Defining the distance for which the correlation coefficient becomes 0.5 as a correlation distance and the area enclosed by the correlation distance as the correlation area, the correlation area in the case of Rayleigh fading is a circle with a radius of 0.18 λ (λ: wavelength of carrier frequency). In Nakagami–Rice fading, the correlation area seems an ellipse having its short axis in the direction $\theta_b = 0°$ and its long axis in the direction $\theta_b = 90°$. Fig. 5 shows the correlation distance as a function of the parameter s^2 for $\theta_b = 0°$ and 90°. Based on the definition of (6), we are looking here at the correlation of power fluctuations and at $s^2 < -20$ dB there is no further dependence on s^2. However, in the region where the reflected wave component is sufficiently small as in this case, there is not much decrease of the signal intensity and fading is not a problem so it is not meaningful to discuss the area where the reflected wave power is so small.

B. Case Where Scattered Waves Have a Normal Distribution and Direct Wave Is Located at Their Center—Case 2

This is the case $\theta_0 = 0°$ in the coordinate system shown in Fig. 6. A condition is that the angular spread (standard deviation σ_θ) of the reflected waves is sufficiently small compared to 2π. This corresponds to a typical radio propagation environment for

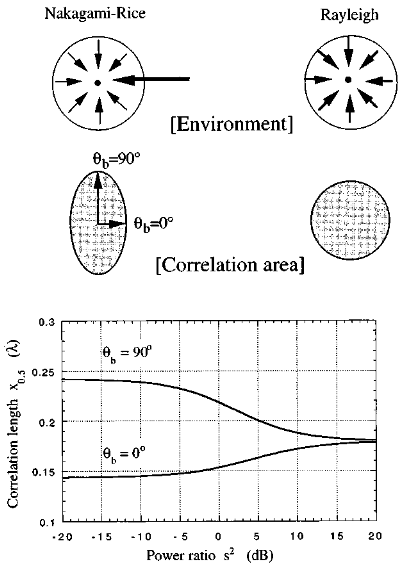

Fig. 5. Correlation length as a function of multipath power ratio (s^2: multipath power/direct wave power) (case 1).

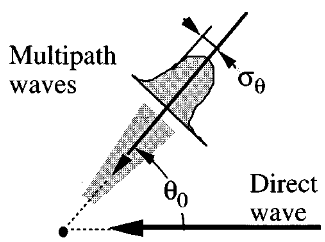

Fig. 6. Angular profile of incident waves whose multipath-wave component has a Gaussian profile (cases 2 and 3).

base-station reception in LOS communications. An angular profile for multipath waves excluding the direct wave component as shown in Fig. 6 is

$$\Omega_s(\theta) = \frac{s^2}{\sqrt{2\pi}\sigma_\theta} \exp\left\{-\frac{(\theta-\theta_0)^2}{2\sigma_\theta}\right\}. \tag{24}$$

As $\theta_0 = 0°$, the correlation characteristics are given by the following equation:

$$\rho_{\delta P} = \frac{2\exp\left\{-\frac{(\sigma_\theta k \Delta x \sin\theta_b)^2}{2}\right\} + s^2 \exp\left\{-(\sigma_0 k\Delta x \sin\theta_b)^2\right\}}{2+s^2} \quad \text{(Nakagami–Rice fading)}. \tag{25}$$

For Rayleigh fading (namely, $s^2 \gg 1$), the equation can be simplified to

$$\rho_{\delta P} = \exp\left\{-(\sigma_\theta k\Delta x \sin\theta_b)^2\right\} \quad \text{(Rayleigh fading)}. \tag{26}$$

The formula can be seen in [3].

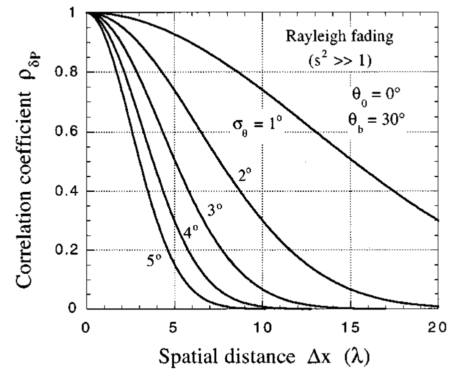

Fig. 7. Spatial correlation statistics for Rayleigh fading whose multipath-wave component has a Gaussian profile.

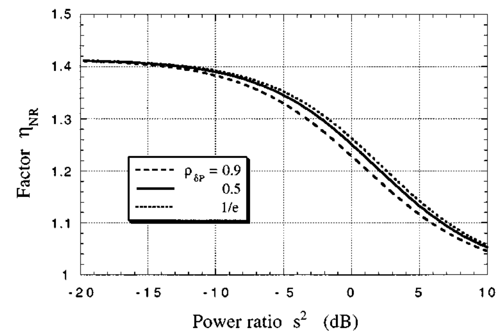

Fig. 8. Conversion factor η_{NR} for obtaining correlation coefficient of Nakagami–Rice fading from that of Rayleigh fading in "case 2."

Writing the Δx, which gives $\rho_{\delta P}$ as $x_{\rho,\,\mathrm{Rayleigh}}$

$$x_{\rho,\,\mathrm{Rayleigh}}(\rho_{\delta P},\,\theta_b,\,\sigma_\theta) = \frac{\sqrt{-\ln\rho_{\delta P}}}{\sigma_\theta k \sin\theta_b}. \tag{27}$$

Similarly, for a Nakagami–Rice fading environment (25), writing the Δx, which gives $\rho_{\delta P}$ as $x_{\rho\mathrm{NR}}$, the relation with $x_{\rho,\,\mathrm{Rayleigh}}$ is obtained from the following equation through a simple calculation:

$$x_{\rho\mathrm{NR}}\left(\rho_{\delta P},\,\theta_b,\,\sigma_\theta,\,s^2\right) = \eta_{\mathrm{NR}} x_{\rho,\,\mathrm{Rayleigh}} \tag{28}$$

$$\eta_{\mathrm{NR}}(\rho_{\delta P}, s^2) = \sqrt{\frac{2\ln\left(\frac{-1+\sqrt{1+(2+s^2)s^2\rho_{\delta P}}}{s^2}\right)}{\ln\rho_{\delta P}}}. \tag{29}$$

$x_{\rho,\,\mathrm{Rayleigh}}$ itself is a function of many parameters, but it is interesting to note that the factor $\eta_{\mathrm{NR}} \equiv x_{\rho,\,\mathrm{NR}}/x_{\rho,\,\mathrm{Rayleigh}}$, which converts the calculation results of Rayleigh fading to Nakagami–Rice fading, does not depend on θ_b or σ_θ.

Fig. 7 shows an example calculation of Rayleigh fading correlation characteristics ($\theta_b = 30°$; $\theta_0 = 0°$). Fig. 8 shows the coefficient η_{NR} in (29) relative to threshold values 0.9, 0.5, $1/e$ of the correlation coefficient. From the figure, it is seen that the correlation distance in a Nakagami–Rice fading environment depends on the parameter s^2 and is longer by up to 40% (i.e., 140% in total) compared to Rayleigh fading.

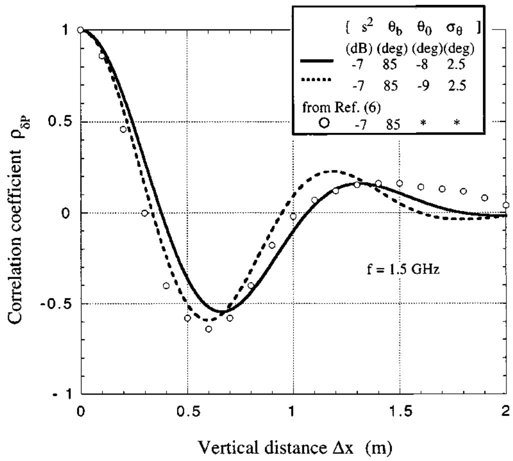

Fig. 9. Spatial correlation characteristics with application to multipath fading due to sea reflection in maritime satellite communications at an elevation angle of 5° (case 3).

C. Case Where Scattered Waves Have a Normal Distribution But Direct Wave Arrival Direction Does Not Coincide with the Center of the Normal Distribution (Case 3)

This is the case $\theta_0 \neq 0$ in Fig. 6 and is typical of sea-reflection fading in maritime mobile-satellite communications or aeronautical mobile-satellite communications [10]. In this case

$$\rho_{a,x}(\Delta x;\,\theta_b,\,\sigma_\theta) = \exp\left[j2k\Delta x \sin\left(\frac{\theta_0}{2}\right)\sin\left(\theta_b - \frac{\theta_0}{2}\right) - \frac{\{k\sigma_\theta \Delta x \sin(\theta_0 - \theta_b)\}^2}{2}\right]. \quad (30)$$

The spatial correlation characteristics are obtained by substituting this expression into (19a).

Fig. 9 shows a result of calculation for space diversity assessment in maritime mobile-satellite communication systems where sea-reflection fading becomes serious at low elevation angles. The parameter values are set assuming the condition for an elevation angle of 5°, namely, $s^2 = -7$ dB, $\theta_b = 85°$, $\sigma_0 = 2.5°$, $\theta_0 = 8°$ (solid line), and $\theta_0 = -9°$ (broken line). The reason why it does not give $\theta_0 = -10°$ corresponding to the specular reflection is that it takes account of the analytical result [10] that in the case of incoherent scattering, the center of the arrival direction of scattered waves is shifted slightly toward the horizon. The results of this figure agree closely with those of a more rigorous theoretical analysis [6]. The slight discrepancy may be due to the fact that the arrival angular profile obtained from the theoretical analysis is somewhat different from the normal distribution assumed in the model.

IV. Bandwidth Dependence

In the case of wide-band signals, the basic equations are (13) instead of (19a) and (14) instead of (18), which were used for narrow-band signals. From (14) and (17), in the case of wide-band signals, information regarding the delay profile is also required. For the delay profile of the scattered wave part, an exponential function generally used for mobile communications is employed. This is given by the following

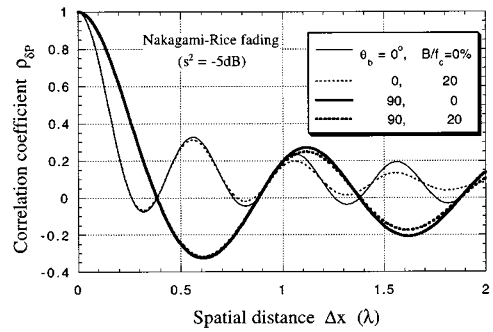

Fig. 10. Spatial correlation characteristics for wide-band signal (case 1).

equation where $\sigma_{\tau,R}$ is the delay spread of multipath waves (excluding the direct wave)

$$p_s(\tau) = \frac{s^2}{\sigma_{\tau,R}} \exp\left(-\frac{\tau}{\sigma_{\tau,R}}\right) u(\tau) \quad (31)$$

where u is the step function giving 1 for $u > 0$ and 0 for $u < 0$.

When scattered waves arrive with a uniform angular distribution from all directions, the case discussed in Section III-A (Case 1), we examine two situations: 1) where the relative bandwidth is sufficiently narrow and 2) where it is comparatively wide ($B/f_c = 0.2$; namely, 20%). Fig. 10 shows calculation results; the solid lines represent narrowband (three curves in Fig. 4) and the broken lines represent wide-band (20%). When $s^2 = -5$ dB, the results depend on the orientation of the baseline (θ_b), the two cases $\theta_b = 0°$ and $90°$ being shown here similarly to Fig. 4. By comparing the solid lines and broken lines in the figure, it is seen that within this order of bandwidth, there are almost no wide-band dependent effects in the spatial correlation characteristics. This can also be predicted to some extent from the simulation results of Nakabayashi *et al.* [11]. In the wide-band case, delay spread $\sigma_{\tau,R}$ of $5/B$ was assumed, but the dependence of the spatial correlation on the delay spread is negligibly small for the exponential delay profile given in (31). The spatial correlation characteristics do depend on the frequency f_{RF}, but the difference is small within a relative bandwidth of about 20% and the calculated amounts are average values for the in-band frequency range.

Since the frequency bandwidth dependence of spatial correlation characteristics is small, the narrow-band model is sufficient for calculating spatial correlation lengths (distance differences with a correlation coefficient of 0.5) or correlation areas.

V. Conclusions

We derived theoretical formulas to calculate spatial correlation characteristics, focusing on a Nakagami–Rice fading environment which is a widely applicable propagation environment that includes Rayleigh fading. We also calculated spatial correlation characteristics for a typical fading environment appearing in LOS mobile radio systems. From this analysis, we clarified the differences of correlation characteristics for Nakagami–Rice fading and Rayleigh fading and the differences of spatial correlation characteristics between wide-band signals and narrow-band signals. The following conclusions are obtained by summarizing the results.

1) In a Nakagami–Rice fading environment, where there is a stationary wave (= direct wave) and scattered waves arriving with a uniform angular distribution from all directions, the correlation area (= area for which the correlation coefficient is 0.5 or higher for all baseline directions) shows an elliptical area having its short axis in the direction of the direct wave and its long axis in a direction perpendicular to the direction of the direct wave.
2) When there is a stationary wave and scattered waves having an angular profile of a normal distribution whose center direction is the same as that of the stationary wave, the correlation length is always larger than that of a Rayleigh fading environment having the same angular profile and has a maximum of the order of 140%.
3) Spatial correlation characteristics do not largely depend on frequency bandwidth up to approximately 20% of the carrier frequency. A narrow-band model is sufficient for computing the spatial correlation characteristics within this range.

Analysis of the spatial correlation characteristics for Nakagami–Rice fading environments presented in this paper will doubtless provide basic information in considering fading countermeasures such as space diversity and adaptive array antennas in LOS radio communications systems such as indoor wireless LAN's, mobile-satellite communications systems and mobile access systems operating at higher frequency bands.

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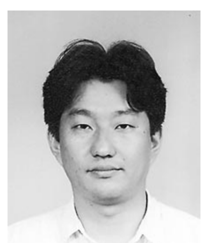

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