Two-Step Inverse Scattering Method for One-Dimensional Permittivity Profiles

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***Abstract*—A numerical method to invert the dielectric permittivity profile from the Riccati equation using the Newton–Kantorovich iterative scheme is described. Instead of handling the equations in terms of usual geometrical depth, we determine the profile as a function of the electromagnetic path length since the convergence and the stability of the solution are found to be significantly better in this case. The initial profile used as a starting point for the inversion is obtained by another method employing successive reconstruction of dielectric interfaces and homogeneous layers in a step-like form. This method, though not always accurate, is fast and well suited for the approximate reconstruction of the profile, thus creating ideal starting conditions for the previous approach. As a result, the computation time is considerably reduced without using any *a priori* information. The approach is applicable to both continuous and discontinuous profiles of high contrast and exhibits a good stability of the solution with respect to noisy input data. A lossy medium profile can also be inverted provided the overall thickness of the inhomogeneous slab and the background permittivity are known.**

***Index Terms*—Inverse scattering, nonhomogeneous media.**

I. Introduction

VARIOUS methods have been used to reconstruct one-dimensional permittivity profiles from electromagnetic reflection coefficient data. The first-order Born and Rytov approximations assuming that the medium acts as a small perturbation on the incident wave [1]–[3], can be applied to accurate imaging of the quasi-homogeneous objects and produce qualitative images in other cases. Nevertheless, the methods of this kind are mostly used in practice because of their simplicity and stability.

A large number of investigations have been carried out using the Gel'fand–Levitan–Marchenko theory [4]–[7]. Unfortunately, this in principle exact approach is actually very difficult to implement due to considerable mathematical complexity. This leads sometimes to failures especially when faced with the discontinuous profiles of high contrast [6]. A nonlinear approximation of the Riccati equation allowing its solution in closed form with subsequent inversion yields in some cases very accurate reconstructions [8].

Iterative numerical methods based on the exact equations are also widely used in microwave imaging [9]–[12]. They do not principally have contrast limitations. Unfortunately, because of ill-posedness of the inverse problem, the convergence, and the stability of solution essentially depend on the actual contrast values, deteriorating for highly contrasted discontinuous profiles. To improve the reliability of the solution and the convergence rate, as much as possible of *a priori* knowledge of the object under test should be included in the inversion procedure. However, this is not always convenient in practice.

The purpose of this paper is to improve the convergence and the stability of the iterative optimization scheme for complicated highly contrasted dielectric profiles. No *a priori* information of the reconstructed profile is used in a lossless case, whereas minimal additional information is needed when conductive losses are taken into account. Besides, angular dependency of the reflected signal is excluded from consideration. Although multiangle measurements [9], [13], [14] yield additional input data allowing to get more reliable reconstruction and to retrieve, e.g., material dispersion, their implementation results in large and expensive antenna arrangements. Furthermore, the measurement routine becomes time consuming. This is not always appropriate for many practical applications such as ground penetrating radar, nondestructive testing in civil engineering, etc. Consequently, the consideration is restricted here to the case of normal incidence only.

The reconstruction of dielectric half-space is performed using a new two-step approach. The Newton–Kantorovich iterative method applied to the Riccati equation is used as the basic reconstruction algorithm. It is shown that a linear integral equation to obtain the next iterate to the profile is more accurate in case if the derivation is accomplished in terms of electromagnetic path length rather than in usual spatial coordinate. This allows improving the convergence and the stability of the solution. The initial profile needed for the iterative procedure is obtained by another method employing discrete reconstruction. At this step, the permittivities and the thicknesses of the layers can be determined one after another by minimizing the maximum of the reflection coefficient in the frequency band of operation. The method yields the exact reconstruction for simple one- and two-layered profiles, if the reflection data are given in a wide enough frequency band. Otherwise, the inversion is approximate. Nevertheless, the difference between the reconstructed profile and the exact one is quite small. Hence, good starting conditions are created for the previous approach. Only a few iterations are needed now to complete the reconstruction, saving the computation time.

The numerical simulations demonstrate good accuracy, fast convergence, and robustness of the algorithm for complicated continuous and discontinuous profiles of high contrast. For a lossy medium, simultaneous determination of the permittivity

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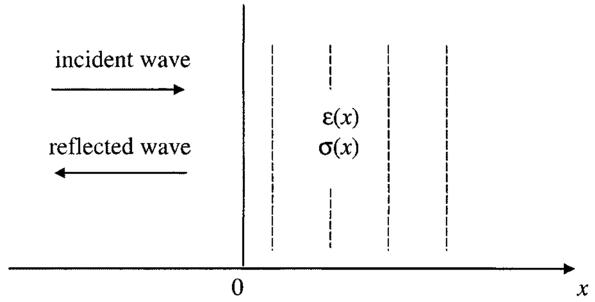

Fig. 1. Geometry of the problem.

and the conductivity profiles is possible, when the overall thickness of the inhomogeneous slab and the background permittivity are known.

II. Inverse Scattering Theory

A. Newton–Kantorovich Method

The geometry of the problem is shown in Fig. 1. Consider the reflection of a normally incident time-harmonic electromagnetic wave from an inhomogeneous layered medium of unknown complex permittivity $q(k, x) = \varepsilon(x) - j\eta_0\sigma(x)/k$, where k is the free-space wavenumber, η_0 is the characteristic impedance of free-space, $\varepsilon(x)$ and $\sigma(x)$ denote medium permittivity and conductivity, respectively. The problem of interest is to find the complex permittivity from knowledge of the reflection coefficient given at a number of frequencies. The complex reflection coefficient $r(k, x)$ satisfies the Riccati nonlinear differential equation [3]

$$\frac{dr(k, x)}{dx} = 2jk\sqrt{q(k, x)}r(k, x) + \frac{1 - r^2(k, x)}{4q(k, x)}\frac{dq(k, x)}{dx} \tag{1}$$

which can be integrated from infinity to $x = 0$ to yield the observed reflection coefficient. Alternatively, starting from the given reflection data $R(k)$ at $x = 0$, integration of (1) from $x = 0$ to infinity with the boundary condition

$$r(k, 0) = \frac{R(k) - r_{01}}{1 - R(k) \cdot r_{01}} \tag{2}$$

must yield zero reflection coefficient at some large depth, where the permittivity does not vary any more. The formula (2) describes transformation of the reflection coefficient at the possible discontinuity at $x = 0$, with $r_{01}=(1 - \sqrt{q(k, 0)})/(1 + \sqrt{q(k, 0)})$ being the Fresnel reflection coefficient at the interface.

Introducing a new variable, electromagnetic path length (more strictly, it coincides with the electromagnetic path length in a lossless case)

$$z = \int_0^x \sqrt{\varepsilon(x')}\, dx' \tag{3}$$

(1) can be rewritten in the form

$$\frac{dr(k, z)}{dz} = 2jk\sqrt{\frac{q(k, z)}{\varepsilon(z)}}r(k, z) + \frac{1 - r^2(k, z)}{4q(k, z)}\frac{dq(k, z)}{dz}. \tag{4}$$

Since the solution to this nonlinear equation in closed form with subsequent inversion is impossible, some optimization technique is to be applied. In this work, an iterative Newton–Kantorovich procedure [15] is used. According to it, a forward problem is first solved for some initial profile $q^*(k, x) = \varepsilon^*(x) - j\eta_0\sigma^*(x)/k$. The integration of (4) for this profile with the boundary condition (2) yields the reflection coefficient $r^*(k, z)$. It is, in general, nonzero at some depth z_0, which must be larger than the total electromagnetic path length in the inhomogeneous part of half-space. To eliminate this discrepancy, a small variation $\Delta q(k, z) = \Delta\varepsilon(z) - j\eta_0\Delta\sigma(z)/k$ is added to the initial profile function causing a corresponding change of the reflection coefficient $\Delta r(k, z)$.

Substituting these variations into (4), we get the following equation for $\Delta r(k, z)$

$$\frac{d\Delta r}{dz} = \left(2jk\sqrt{\frac{q^*}{\varepsilon^*}} - \frac{r^*}{2q^*}\frac{dq^*}{dz}\right)\Delta r + \frac{jkr^*}{\sqrt{\varepsilon^* q^*}}\left(\Delta q - \frac{q^*}{\varepsilon^*}\Delta\varepsilon\right) + \frac{1 - {r^*}^2}{4}\frac{d}{dz}\left(\frac{\Delta q}{q^*}\right) \tag{5}$$

where higher order terms are neglected and the arguments of functions are omitted. The solution to this linear differential equation can be written in closed form, yielding for $z = z_0$

$$\Delta r(k, z_0)=\Delta r(k, 0)+\int_0^{z_0} e^{f(k, z)-f(k, z_0)} \cdot \left[\frac{jkr^*}{\sqrt{\varepsilon^* q^*}}\left(\Delta q-\frac{q^*}{\varepsilon^*}\Delta\varepsilon\right)+\frac{1 - {r^*}^2}{4}\frac{d}{dz}\left(\frac{\Delta q}{q^*}\right)\right]dz \tag{6}$$

where

$$\Delta r(k, 0) = \frac{\dfrac{\Delta q(k, 0)}{\sqrt{q^*(k, 0)}}\dfrac{1 - R(k)}{1 + R(k)}}{\left[\sqrt{q^*(k, 0)} + \dfrac{1 - R(k)}{1 + R(k)}\right]^2} \tag{7}$$

is derived from (2), and

$$f(k, z) = -\int_0^z \left[2jk\sqrt{\frac{q^*}{\varepsilon^*}} - \frac{r^*}{2q^*}\frac{dq^*}{dz'}\right] dz'. \tag{8}$$

Integrating the last term in (6) by parts and using (2), we obtain the following:

$$\Delta r(k, z_0)=\frac{1-{r^*}^2(k, z_0)}{4}\frac{\Delta q(k, z_0)}{q^*(k, z_0)}+\int_0^{z_0} \frac{jke^{f(k, z)-f(k, z_0)}}{2\sqrt{\varepsilon^* q^*}} \left[(1 + r^*)^2\Delta q - 2r^*\frac{q^*}{\varepsilon^*}\Delta\varepsilon\right] dz. \tag{9}$$

Finally, employing the condition $\Delta r(k, z_0) = -r^*(k, z_0)$, the desired integral equation to calculate the variation of the permittivity and the conductivity profiles can be written in the form

$$\begin{aligned}\int_0^{z_0} & \frac{e^{f(k,z)-f(k,z_0)}}{2\sqrt{\varepsilon^* q*}} \\ & \cdot \left[jk\left(1 + r^{*^2} + 2jr^*\frac{\eta_0\sigma^*}{k\varepsilon^*}\right)\Delta\varepsilon(z)\right. \\ & \left. +\eta_0(1+r^*)^2\Delta\sigma(z)\right] dz \\ & + \frac{1-r^{*^2}(k, z_0)}{4}\frac{\Delta\varepsilon(z_0) - j\eta_0\Delta\sigma(z_0)}{q^*(k, z_0)} \\ = & -r^*(k, z_0). \end{aligned} \tag{10}$$

Note that the term taking into account a possible existence of the discontinuity at $x = 0$ though present in (6) disappears in the final equation (10).

The linear integral equation (10) is solved by a commonly used technique, including expansion of the unknown $\Delta\varepsilon(z)$ and $\Delta\sigma(z)$ by some basis functions (Fourier basis is used in this work) and transformation of (10) into a matrix equation to find the expansion coefficients

$$\boldsymbol{A}\cdot\boldsymbol{P} = \boldsymbol{Q} \tag{11}$$

where $\boldsymbol{P}$ is a column of the expansion coefficients and $\boldsymbol{Q}$ is a column of reflection coefficients in the right-hand side of (10). The least square solution to (11) with a standard Tikhonov regularization [15] is given by

$$\boldsymbol{P} = [\boldsymbol{A}^*\boldsymbol{A} + g\boldsymbol{I}]^{-1}\boldsymbol{A}^*\boldsymbol{Q} \tag{12}$$

where g is a regularization parameter. Once $\Delta\varepsilon(z)$ and $\Delta\sigma(z)$ are found, the next iterate to the profile is obtained as follows:

$$q^{(1)}(k, z) = q^*(k, z) + \Delta\varepsilon(z) - j\eta_0\Delta\sigma(z)/k \tag{13}$$

and the whole procedure is repeated until the convergence criterion is satisfied. Then, a coordinate transformation (3) is applied to obtain the final profile as a function of the physical distance.

The inversion is performed here in terms of the electromagnetic path length rather than in the real spatial coordinate because of a better accuracy of the resultant equations. Indeed, while taking variation of (4) to obtain (5), the second-order term $2jk\Delta r\cdot\Delta(\sqrt{q/\varepsilon})$ among others is neglected. This term is small in case of low losses and vanishes totally for a lossless profile. On the other hand, if the derivation is done in terms of a real distance, a much larger term $2jk\Delta r \cdot \Delta(\sqrt{q})$, as seen from (1), is to be neglected. A comparison of the reconstructions performed using the described approach and its counterpart derived using the spatial coordinate showed apparent advantage of the presented formulation with respect to the convergence and the stability of solution. Moreover, the effect of ill-posedness of the problem is reduced so that the regularization term in (12) in most cases can be omitted without a remarkable deterioration of reconstruction. This is entirely impossible if the problem is considered with the use of spatial coordinate.

The reconstruction depth z_0 can be chosen in principle arbitrarily. Naturally, it must exceed the electromagnetic path length of the inhomogeneous slab to be reconstructed. Besides, the reconstruction depth must not be too large to avoid significant spatial compressing of inhomogeneities resulting in loss of resolution.

Despite of the good convergence of the solution in the proposed approach, quite a lot of computation time can be saved by a careful selection of an initial guess used as a starting point for the iterative technique. In this paper, the initial profile is obtained using another approach based on a successive reconstruction of dielectric interfaces and homogeneous layers.

B. Discrete Reconstruction Method

The reflection of electromagnetic wave from an inhomogeneous half-space can be also treated using a concept employing 2×2 scattering matrices [16]. Accordingly, the medium in the region $x > 0$ is represented as a stack of homogeneous layers. The scattering matrix of the stratified half-space is given by

$$\boldsymbol{S} = \boldsymbol{I}_{01}\boldsymbol{L}_1\boldsymbol{I}_{12}\boldsymbol{L}_2\boldsymbol{I}_{23}\cdots \tag{14}$$

where

$$\boldsymbol{I}_{i(i+1)} = \frac{1}{1+r_{i(i+1)}}\begin{pmatrix} 1 & r_{i(i+1)} \\ r_{i(i+1)} & 1\end{pmatrix} \tag{15}$$

is the matrix of interface between two adjacent layers numbered i and $i+1$

$$\boldsymbol{L}_i = \begin{pmatrix} e^{j\beta_i} & 0 \\ 0 & e^{-j\beta_i}\end{pmatrix} \tag{16}$$

is the matrix of homogeneous layer with the number i and

$$\begin{aligned} r_{i(i+1)} = \frac{\sqrt{q_i} - \sqrt{q_{i+1}}}{\sqrt{q_i} + \sqrt{q_{i+1}}}; \qquad \beta_i = kd_i\sqrt{q_i}; \\ q_i = \varepsilon_i - j\eta_0\sigma_i/k \end{aligned} \tag{17}$$

where

- ε_i denote the permittivity of layer with the number i;
- σ_i denote the conductivity of layer with the number i;
- d_i denote the thickness of layer with the number i.

The medium in the region $x < 0$ is assumed to be a free-space with $q_0 = 1$. The reflection coefficient is calculated from (14) as

$$R = S(2, 1)/S(1, 1). \tag{18}$$

This formulation of the problem is entirely equivalent to that described in preceding section, because any continuous profile can be represented with the desired accuracy by a large enough number of homogeneous layers.

The inversion principle valid for the lossless step-like profiles is based on the behavior of the maximum of modulus of the reflection coefficient. In case the profile contains only one interface [i.e., only $\boldsymbol{I}_{01}$ term is left in (14)], the modulus of the reflection coefficient is $|r_{01}|$ and does not depend on frequency. If another interface is added to the profile, the matrix (14) becomes $\boldsymbol{S} = \boldsymbol{I}_{01}\boldsymbol{L}_1\boldsymbol{I}_{12}$ and the maximum of modulus of the reflection coefficient can be easily derived from (14)–(18) as

$$\max_2 = \frac{|r_{01}| + |r_{12}|}{1 + |r_{01}| \cdot |r_{12}|} \geq |r_{01}| \tag{19}$$

the equality being reached when $|r_{12}| = 0$ or when the second interface is absent. For a three-step profile (i.e., the profile containing two layers on substrate) the expression (19) can be extended to

$$\max_3 = \frac{\dfrac{|r_{01}| + |r_{12}|}{1 + |r_{01}| \cdot |r_{12}|} + |r_{23}|}{1 + \dfrac{|r_{01}| + |r_{12}|}{1 + |r_{01}| \cdot |r_{12}|} \cdot |r_{23}|} \geq \frac{|r_{01}| + |r_{12}|}{1 + |r_{01}| \cdot |r_{12}|} \tag{20}$$

and the equality is reached again if $|r_{23}| = 0$. In the general case, an addition of one more step to the step-like profile results in increasing the maximum of modulus of the reflection coefficient and the increase is the larger, the higher is the step. Hence, the principle of reconstruction can be formulated by inverting this statement: the parameters of the layers are to be chosen successively so that to minimize the maximum of modulus of the reflection coefficient for the remaining region in the frequency band of operation. Starting from the matrix $\boldsymbol{I}_{01}$, we obtain from (14), (15) the expression to find the permittivity ε_1

$$\max |R_1| = \max \left| \frac{R - r_{01}}{1 - R \cdot r_{01}} \right| \to \min \tag{21}$$

where R_1 is the reflection coefficient for the profile with the first interface being excluded. At the next step, matrices $\boldsymbol{L}_1$ and $\boldsymbol{I}_{12}$ are reconstructed simultaneously using the minimax criterion

$$\max |R_2| = \max \left| \frac{R_1 \exp(2jkd_1\sqrt{\varepsilon_1}) - r_{12}}{1 - R_1 \exp(2jkd_1\sqrt{\varepsilon_1}) \cdot r_{12}} \right| \to \min \tag{22}$$

where R_2 is the reflection coefficient for the profile without the first layer and the second interface. This is an optimization problem for two variables d_1 and ε_2. If the cost function (22) has several minima, the one corresponding to the smallest thickness d_1 is to be chosen. Consequently, the unknown d_2 and ε_3 are determined at the next step of reconstruction, etc. The procedure is stopped when the reflection coefficient becomes zero at all frequencies or when its maximum does not go down any more.

The method yields exact reconstruction for lossless layered profiles, if the reflection data are given in a wide enough frequency band. In other cases, the reconstruction is approximate because the frequency of maximal reflection can be outside the given frequency band. However, a discrepancy between the given profile and the reconstructed one is usually small. Besides, this approach is very fast. Though it requires a set of optimization problems to be solved, the cost function (22) is simple. Thus, the whole reconstruction is performed in a time considerably shorter than needed for one iteration of the Newton–Kantorovich method. Therefore, this approach if does not cope with the reconstruction by itself, creates ideal starting conditions for the previous method. Thus, the methods are complementary and allow to save a lot of computation time when used one after another.

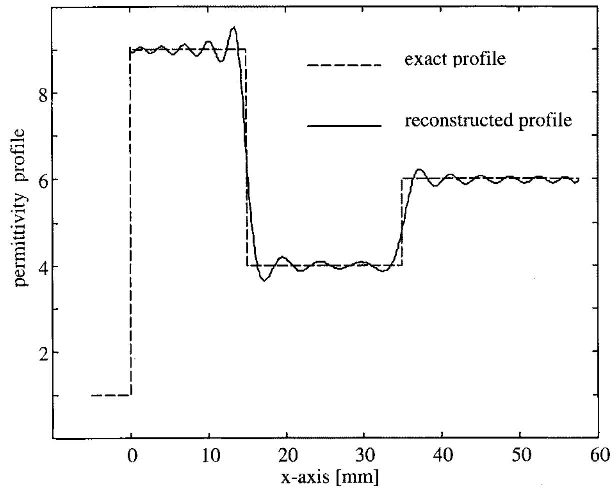

Fig. 2. Profile reconstruction using the Newton–Kantorovich method.

III. NUMERICAL EXAMPLES

This section is devoted to some numerical results, obtained by means of the described reconstruction algorithms. Only highly contrasted profiles are considered. The reflection coefficient data are simulated by numerical integration of the Riccati equation (1) for a continuous profile and by the matrix calculations using formulas (14)–(18) for discontinuous profiles. All the synthetic data are calculated here for the frequencies in the range of 0.5–15 GHz in steps of 0.5 GHz. As usual in the frequency domain techniques, the signal bandwidth determines the spatial resolution, whereas the largest possible depth of reconstruction depends on the frequency step [17]. Therefore, necessary corrections can be done easily if needed. The Newton–Kantorovich iterative reconstruction algorithm is stopped when the mean value of the modulus of reflection coefficient at the reconstruction depth over all frequencies is less than 0.01.

Fig. 2 represents a two-layered lossless profile on a substrate. The Newton–Kantorovich procedure converges to the final reconstructed profile shown in Fig. 2 in five iterations when started from a constant initial guess with $\varepsilon = 9$. Another choice of the constant initial guess results in the increase of the required number of iterations. By using the discrete reconstruction method, the profile is inverted exactly in a few seconds of microcomputer time.

A more complicated highly contrasted four-layered profile on a substrate is shown in Fig. 3. An approximate reconstruction is obtained in this case by the discrete reconstruction method. Nevertheless, all the four layers are reconstructed quite correctly. Using this profile as the starting point, the Newton- Kantorovich method completes the reconstruction in six iterations. It is worth noting that the third layer (air gap) is reconstructed quite well

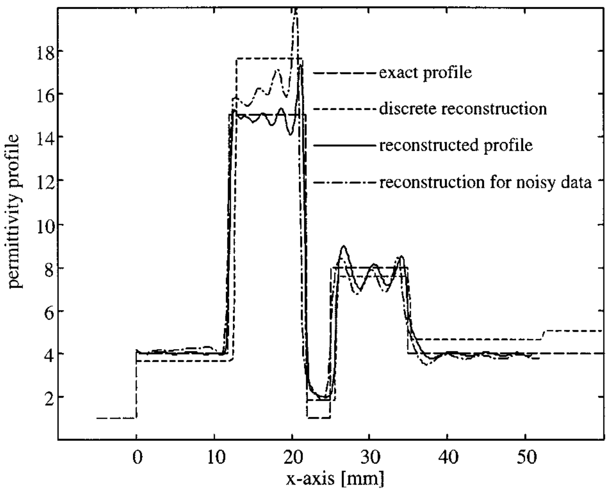

Fig. 3. Reconstruction of a discontinuous profile using the two-step approach for exact and noisy simulated data.

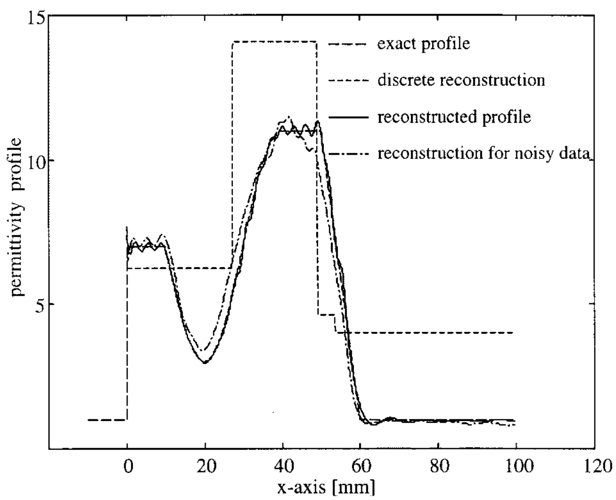

Fig. 4. Reconstruction of a continuous profile using the two-step approach for exact and noisy simulated data.

though its thickness is about seven times less than the shortest wavelength in the input data. The effect of measurement errors is estimated by adding to the real and imaginary parts of the reflection coefficients a random signal distributed uniformly over the interval $[-0.02 \quad +0.02]$. The solution retains good stability as seen from Fig. 3.

Although the discrete reconstruction method is developed for the step-like profiles, it can be applied to the continuous ones as shown in Fig. 4. It can be seen that the quality of the reconstruction is not good. However, starting from this profile, only ten iterations of the continuous method are needed, still saving a lot of computation time compared to an unsuccessful choice of initial guess. The robustness of the approach is also demonstrated here with the magnitude of the random signal being increased to ± 0.05.

Lossless profiles have been considered to this point. Finally, let's consider a two-layered profile on a substrate, while the second layer is lossy as shown in Fig. 5. First, a lossless profile is attempted to be reconstructed using the input data without any knowledge of the thickness of the layers. Indeed, the lossless

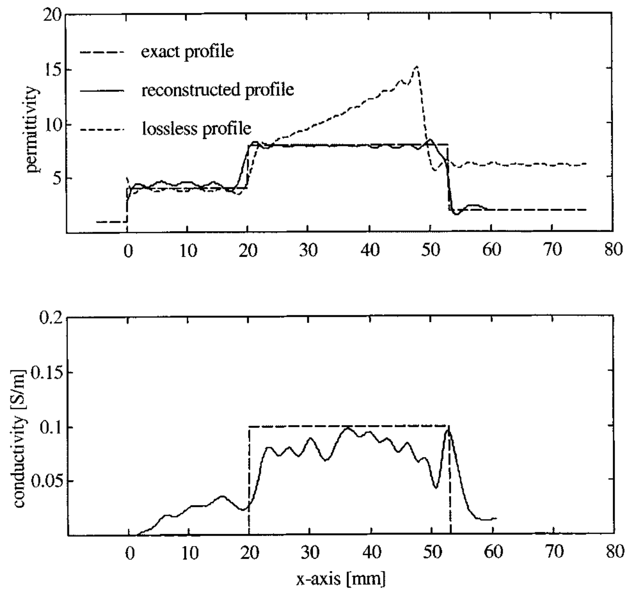

Fig. 5. Reconstruction of the lossy medium profile using the Newton–Kantorovich method, when the slab thickness and the background permittivity are known.

profile shown in Fig. 5 has almost precisely the same reflection coefficient at all given frequencies as the real lossy profile (moduli of discrepancies are less than 0.005). Hence, at least if a reasonable measurement error is assumed, the reconstruction of both the permittivity and the conductivity using the reflection data for a normally incident wave is impossible because of lack of information. That is why the scanning of the angle of incidence together with the frequency sweep are used when the lossy profiles are to be reconstructed [9], [13], [14]. Unfortunately, the measurement apparatus when an angle of incidence is varied is quite complicated. However, the reconstruction of lossy profiles is possible in case of normal incidence, if a total thickness of the inhomogeneous layer and the background permittivity are known. Under these additional constraints imposed on the solution of the integral equation (10), the proposed method yields an accurate reconstruction of the permittivity profile and a qualitative reconstruction of the conductivity profile as shown in Fig. 5.

IV. Conclusions

A new two-step approach to the one-dimensional inverse scattering is proposed. At the first step, the unknown profile is reconstructed in a step-like form using minimax criterion for the modulus of reflection coefficient in the frequency band of operation. The method is valid for accurate reconstruction of simple stratified profiles. In other cases, it provides good starting conditions for the Newton–Kantorovich approach applied to the numerical inversion of the Riccati equation. Thus, a lot of computation time is saved. Besides, *a priori* information is not needed when lossless profiles are reconstructed.

The convergence and the stability of the solution are improved considerably by handling the Riccati equation in terms of the electromagnetic path length instead of the usual spatial coordinate. This is explained by a better accuracy of the linear integral equation used for the calculation of the next iterate to the profile. The approach is well suited for the reconstruction of highly contrasted and discontinuous profiles and exhibits a good stability with respect to the noisy input data.

The reflection coefficient data obtained for the case of a normal incidence only are shown to be insufficient to reconstruct both the permittivity and the conductivity profiles. When the thickness of the inhomogeneous layer and the background permittivity are assumed to be known, the permittivity profile is reconstructed accurately whereas for the conductivity profile the method yields qualitative reconstruction.

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