Radiation Patterns of Sources Placed Near the Truncation of a Semi-Infinite Dielectric Structure: The Demonstration Case of a Magnetic Line Current

Vladimir Volski and Guy A. E. Vandenbosch

***Abstract*—Numerical methods are presented to estimate the radiation pattern of a magnetic line current placed near the truncation of a dielectric structure. The problem is solved using the integral equation technique, expansion wave concept, and physical optics approximation. The comparison between different methods permits to understand better the possibilities of each of the methods.**

***Index Terms*—Dielectric slabs, electromagnetic diffraction, half-plane.**

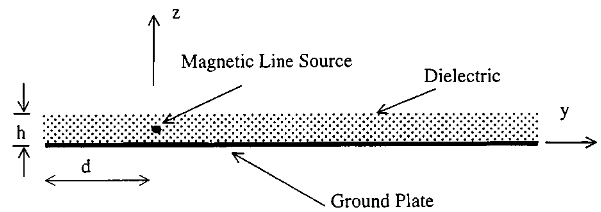

Fig. 1. Geometry of truncated structure.

I. Introduction

THE fast development of modern high-frequency structures demands more and more accurate descriptions of these objects. The truncation of a dielectric structure is one of the examples that is physically present in many antennas, but usually it is ignored in software models. However, the truncation can affect considerably the computed parameters, especially the radiation pattern. The direct solving approach involves the volume polarization current technique [1], which requires huge computer resources. Its application is rather difficult in many cases. An alternative approach is based on the postulates of the geometrical theory of diffraction (GTD). The diffraction coefficient can be estimated by involving the method of physical optics (PO) [2]–[4] or its calculation is based on the general impedance boundary condition techniques [5]. The third group of methods uses the integral equation solving procedure [6]–[9] to define the diffraction coefficients. All methods to solve this problem have their own advantages and disadvantages but their main lack is that the accuracy depends on the substrate parameters due to the assumptions made. It is not easy to estimate the real accuracy because a highly accurate solution is not available yet. The comparison of different techniques applied to the same problem is very interesting. The radiation pattern of a line magnetic source placed near the truncation of a dielectric structure is very suitable for this purpose. It is a two-dimensional problem that can be solved in a rather easy way by involving numerical methods. The fields in the infinite structure are known and different approximations can be made by using the PO approximation for equivalent volume and/or surface polarization currents to take the truncation into consideration. In [6], we have used an integral equation technique to find diffraction coefficients for the surface or space wave. As it was shown, this technique yields accurate results. A similar procedure can be applied to find the diffraction pattern of a magnetic line current placed near the truncation. In this paper, we compare the different possible solution techniques.

Manuscript received November 25, 1998; revised April 11, 1999.
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Publisher Item Identifier S 0018-926X(00)01651-3.

II. Expansion Wave Solving Procedure

In order to make the analysis of a truncated dielectric structure more understandable, we will first discuss some properties of the infinite structure. Consider an infinite dielectric structure excited by an x-directed magnetic line source placed at ($z = z_0$, $y = 0$) (Fig. 1). For reasons of simplicity, we restrict the parameters of the dielectric in such a way that no more then one surface wave can propagate and we assume that the substrate has no losses. These restrictions are not fundamental. A field component in the dielectric can be expressed in terms of Fourier integrals

$$F_t(y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f_t(\eta, z, z_0) e^{-j\eta y} \, d\eta \tag{1}$$

where $t = x, y, z$, $F = E, H$, and $f = e, h$ is a spectral field component which can be obtained following well-known procedures [10], [11]. The field distribution along the x axis is assumed to be constant. In [12], it was proposed to describe the field along the dielectric by using the expansion wave concept (EWC). The main idea of this concept is that the Green's functions in the spatial domain can be approximated as a sum of functions, which are determined by the spectral singularities (pole and branch point). By using this concept, the z components of the electric field can be rewritten

$$E_z(y, z) \approx \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(R^P e_P(z) \frac{1}{\eta^2 - P^2} + R^B e_B(z) \frac{1}{\gamma + \gamma_P} \right) e^{-j\eta y} \, d\eta \tag{2}$$

where

P	pole position;
γ	$= \sqrt{\eta^2 - k_0^2}$;
γ_P	$= \sqrt{P^2 - k_0^2}$;
k_0	$= 2\pi/\lambda$;
$R^{P,B}$	amplitudes of surface or space expansion waves which are determined numerically;
$e_{P,B}(z)$	wave distribution along the z axis.

This distribution has been proven to be independent of the source position z_0 (this follows from the source-free transmission line equation in the layered medium). More details about this can be found in [12]. The first term includes the surface wave contribution and the second term is necessary to describe the space wave contribution. Equation (2) has such a form that R^B is smooth when the layered structure is changed from homogeneous to nonhomogeneous. Approximation (2) is only valid along the air-dielectric interface and in the dielectric. The first term for $y > 0$ is

$$E_z^{\text{SUW}}(y, z) = \frac{1}{2\pi} R^P e_p(z) \int_{-\infty}^{+\infty} \frac{1}{\eta^2 - P^2} e^{-j\eta y}\, d\eta = -jR^P \frac{e_p(z)}{2P} e^{-jPy}. \tag{3}$$

To our knowledge, the second term in (2) cannot be evaluated analytically, but it is possible to find a good approximation for large and intermediate distances (see Appendix)

$$\begin{aligned} E_z^{\text{SPW}}(y, z) &= \frac{1}{2\pi} R^B e_B(z) \int_{-\infty}^{+\infty} \frac{1}{\gamma + \gamma_P} e^{-j\eta y}\, d\eta \\ &\approx R^B e_B(z) \frac{e^{-jk_o y - j\pi/4}}{\sqrt{2\pi k_0 y}} \\ &\quad \cdot \left[1 + \alpha \sqrt{\frac{k_0 y}{2}} e^{-j(\alpha^2 k_0 y/2)} \right. \\ &\quad \left. \cdot \left[jC\left(\frac{\alpha^2 k_0 y}{2}\right) - S\left(\frac{\alpha^2 k_0 y}{2}\right) \right] \right] \end{aligned} \tag{4}$$

where

$$C(x) = \int_x^{+\infty} \frac{\cos t}{\sqrt{t}}\, dt;\ S(x) = \int_x^{+\infty} \frac{\sin t}{\sqrt{t}}\, dt;\ \alpha = \frac{\gamma_P}{k_0}.$$

All other field components can be derived from (2) by using Maxwell's equations. Now let us consider a truncated dielectric structure. The magnetic line current is placed at the distance d from the truncation edge (Fig. 1). It excites surface and space waves, which propagate along the air-dielectric interface up to the truncation. Following the Keller geometric theory of diffraction, a truncation excites waves that propagate in all directions. The amplitudes of these waves are determined by the incident field at the truncation point. The distribution of the incident field at the truncation plane can be easily computed by using (3) and (4). In [6], we describe a method that enables us to get a diffraction coefficient from the known incident field components at the truncation plane. By using this technique and GTD, the radiation pattern can be computed as

$$H_x(\theta) = \left(H_x^{WT}(\theta) + D^{\text{SPW}}(\theta) + D^{\text{SUW}}(\theta)\right) \frac{e^{-jk_0 r}}{\sqrt{r}} \tag{5}$$

where H_x^{WT} is the radiation pattern of the line current without truncation and $D^{\text{SPW,SUW}}$ is a diffraction pattern for the incident space/surface wave that can be computed by the method proposed in [6]. In the EWC the distribution of the space wave in z-direction in free-space is assumed to be constant which evidently is not true. So a lack of this concept when used to model truncations is that the distribution of the space wave ($e_B(z)$) in the z-direction is not describing the real field distribution at the truncation plane ($y = -d$) because it does not include the real position of the source. As a consequence, the total diffraction pattern is not described properly in all directions. It has a discontinuity at $\theta = 180°$ (Fig. 3). The distribution which is used in the EWC corresponds more or less to a source position at infinity ($y = +\infty$), which means that the cylindrical wave becomes a plane wave. The error is expected to be of the same order as the differences between uniform and nonuniform coefficients of the GTD [13].

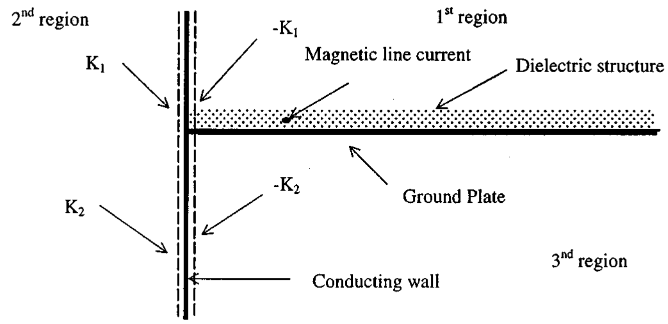

Fig. 2. Equivalent current model.

III. Integral Equation Solving Procedure (IE)

The real source position can be included by modifying the procedure described in [6]. By using an equivalence principle, it is possible to introduce an unknown surface magnetic current backed by a conducting wall in the plane perpendicular to the dielectric structure Fig. 2. Now the space is split into three regular regions where there are no truncations. The condition of continuity of tangential magnetic field components yields an integral equation for the magnetic current

$$\begin{aligned} H_x^2(K_1) + H_x^2(K_2) &= H_x^1(-K_1) + H_x^1(K_0), \qquad z > 0 \\ H_x^2(K_1) + H_x^2(K_2) &= H_x^3(-K_2), \qquad z < 0 \end{aligned} \tag{6}$$

where $H(K)$ means the field created by magnetic current K the superscript refers to the region number. This equation is solved by the Galerkin method. The set of basis functions is discussed in [6]. Around the truncation the magnetic current can be described by rooftop functions and away from the truncation its behavior is determined by cylindrical waves which are known except for the relevant amplitudes. The use of two types of basis

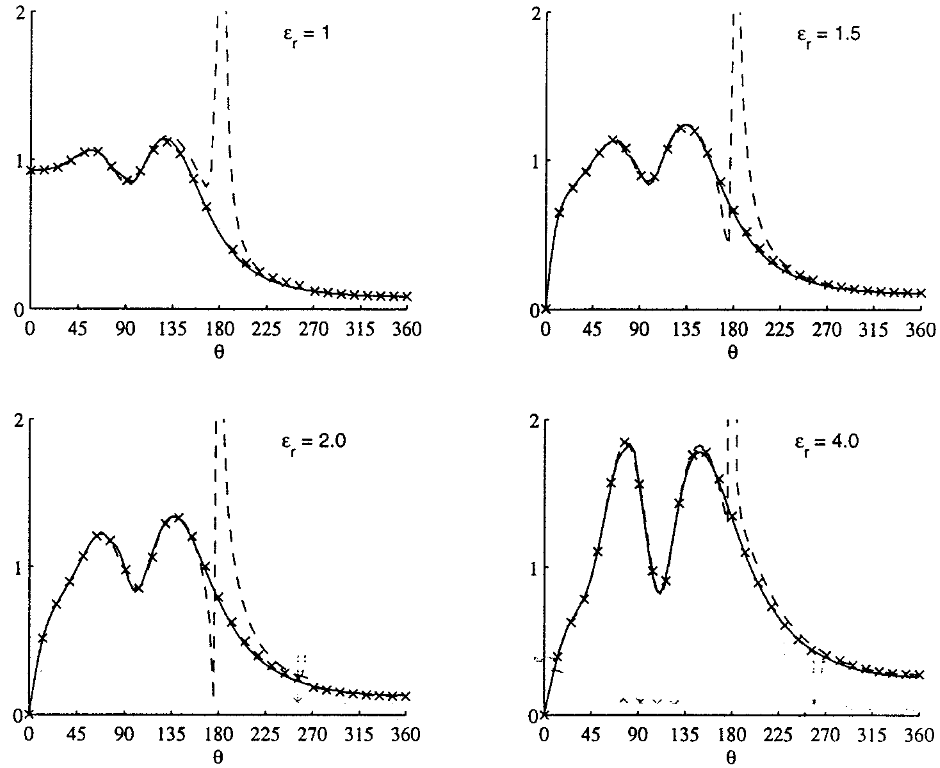

Fig. 3. The radiation pattern of a magnetic line source placed at the distance 0.9λ from the truncation of a semi-infinite dielectric structure (permittivity ε_r; thickness $h = 0.1\lambda$) computed by different methods (—: integral equation solving procedure, - - - : expansion wave solving procedure, and x x x: modified expansion wave solving procedure).

functions permits to describe the unknown current that flows up to infinity by a finite number of basis functions. The solution yields a radiation pattern. It is worth to mention that this method can be used directly to calculate the field excited by the line current. This method is the most accurate one but it requires more computer resources and the necessity to solve (6) for each of the magnetic line current positions.

IV. Modified Expansion Wave Solving Procedure (MEWC)

Now we will try to overcome some problems of (5) by modifying the procedure to calculate the diffraction coefficient for the space wave. Its calculation is based on an equation similar to (6)

$$\begin{aligned} H_x^2(K_1) + H_x^2(K_2) &= H_x^1(-K_1) + 2H_x^{\text{inc}}, \qquad z > 0 \\ H_x^2(K_1) + H_x^2(K_2) &= H_x^3(-K_2), \qquad z < 0 \end{aligned} \tag{7}$$

where $H_{x\infty}^{\text{inc}}$ is the incident wave. The index ∞ means that it concerns an expansion space wave component (its distribution along z does not depend on the source position). The unknown current is written in the form

$$K_1 = K_\infty^{\text{PO}} + \Delta K_\infty \tag{8}$$

where $K_\infty^{PO} = -n \times E_\infty^{\text{inc}}$ is a PO contribution which is known. For the space wave diffraction pattern there is a discontinuity in the direction $\theta = 180^\circ$ caused only by the PO contribution. A rather evident way to avoid it is to put in (8) the real electric field distribution excited by line current without the surface wave component

$$K_1 = K_d^{\text{PO\_}SPW} + \Delta K_\infty \tag{9}$$

where ΔK_∞ is known from the solution of the integral equations in the original EW procedure of (8), $K_d^{\text{PO\_SPW}} = K^{\text{PO}} - K_d^{\text{PO\_SUW}}$ is a PO component and SPW/SUW means space/surface wave component. The known PO components are

$$\begin{aligned} K^{\text{PO}} &= -\, n \times E^{WT}(K_0) \\ K^{\text{PO\_SUW}} &= -\, n \times E_\infty^{\text{SUW}} \end{aligned} \tag{10}$$

where $E^{WT}(K_0)$ refers to the electric field excited by a line current in the structure without truncation and E_∞^{SUW} is the surface wave contribution to the electric field, which is calculated by using (3). We have to remember that the surface wave contribution is taken already into consideration in (5) and so it must be removed in (9) because here we are only concerned with the space wave contribution. The approximation (9) is based on the assumption that the radiation current ΔK is determined by the field around the truncation point ($y = -d, z \sim 0$), which is a well-known fact. The next step requires the calculation of the field excited by the magnetic current in (9). Normally, this field cannot be calculated analytically because it is necessary to calculate Fourier integrals, but this can be done numerically by using standard integrating routines. The current presentation (9) changes also the expression (5) for the radiation pattern. After

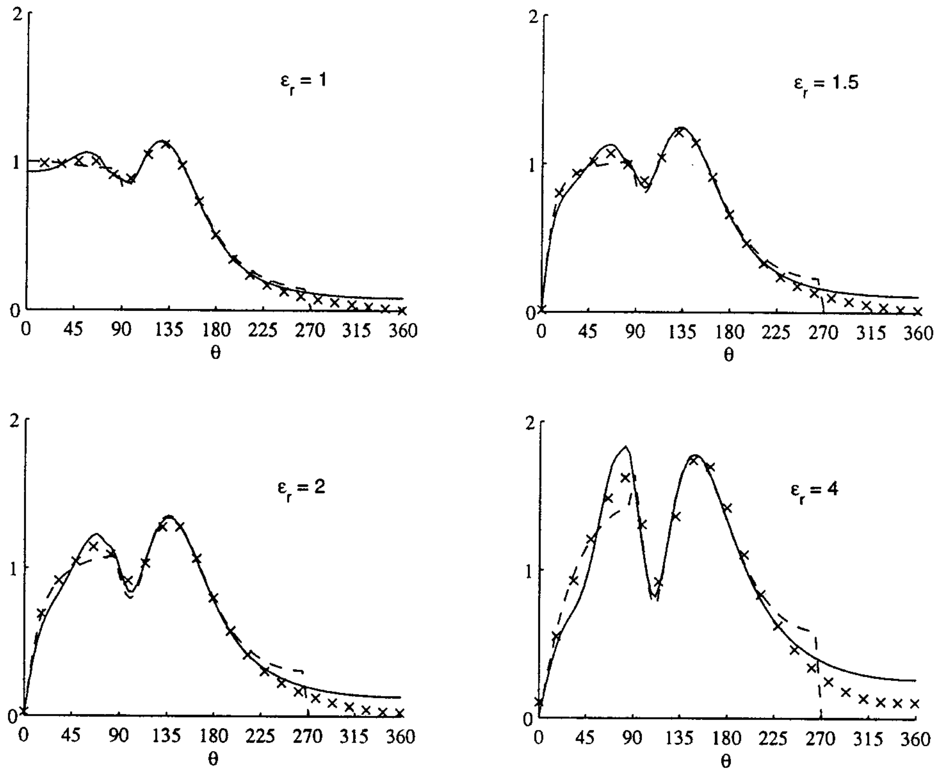

Fig. 4. The radiation pattern of a magnetic line source placed at the distance 0.9λ from the truncation of a semi-infinite dielectric structure (permittivity ε_r; thickness $h = 0.1\lambda$) computed by different methods (—: integral equation solving procedure, - - - : surface physical optics solving procedure, and x x x: volume physical optics solving procedure).

splitting the space into three regions there is no direct contribution anymore of the primary source to the radiation pattern in regions 2 and 3, so the radiation pattern in region 2 has to be calculated by using the following expression:

$$H_x(\theta) = \left(D^{\mathrm{SPW\_mod}}(\theta) + D^{\mathrm{SUW}}(\theta)\right) \frac{e^{-jk_0 r}}{\sqrt{r}} \qquad (11)$$

and in regions 1 and 3 we can use (5).

V. Numerical Results

The radiation pattern of the magnetic line source placed near the truncation of a dielectric structure can be calculated by three methods: integral equation method, expansion wave method, and modified expansion wave method. The comparison is presented in Fig. 3 for the line current at $d = 0.9\lambda$ from the edge, four values of the relative permittivity ($\varepsilon_r = 1, 1.5, 2, 4$), and the thickness of the dielectric 0.1λ. The line current is placed in the middle of the layer 0.05λ above the ground plate. The agreement between all methods is good in the first and third region ($\theta < 140^\circ$, $\theta > 220^\circ$). The discontinuity in the direction $\theta = 180^\circ$ is generally determined by the space wave contribution and it always exists in the case of the unmodified expansion wave concept. This discontinuity is successfully overcome by using the modified expansion wave concept. The contribution of the space wave is quite important and it cannot be ignored. The traditional approach (taking only the surface wave contribution into account) yields quite different results. Even in cases where the contribution of the surface wave becomes more considerable (when the thickness or the permittivity of the dielectric is increased), the space wave has to be taken into account. Only when the line current is placed far from the truncation edge is it possible to ignore the space wave contribution. The minimal distance depends on the substrate parameters.

VI. Surface Physical Optics Solution Procedure (SPO)

Another approach to solve this problem is simpler and does not require the solution of any integral equation. The equivalent PO currents are assumed to be the same as in the infinite dielectric structure. The currents in Fig. 2 are assumed to be

$$K_1 = K^{\mathrm{PO}};\ K_2 = 0. \qquad (12)$$

This approximation is in fact closer to a different type of ground plate as in our original structure truncation (the wedge angle of the truncation is 90° instead of 0°), but the difference is expected to be rather small for angles less than 180°. It is much easier to calculate the radiation field in this case. The radiation pattern computed by this method is plotted in Fig. 4 by dashed line. The geometry of the truncation is the same as it was in Fig. 3. The result obtained by IE is also plotted on this figure in solid line. This PO approximation yields a reasonable result for the observation angles $90^\circ < \theta < 270^\circ$.

VII. Volume Physical Optics Solution Procedure (VPO)

The last approximation involves the equivalent volume and surface electric currents. The dielectric can be replaced by an electric volume polarization current

$$J_d^{\mathrm{PO}} = j\omega\varepsilon_0(\varepsilon_r - 1)E^{WT}(K_0) \tag{13}$$

and the ground plane is replaced by a surface electric current

$$J_g^{PO} = -z \times H^{WT}(K_0) \tag{14}$$

where $E^{WT}(K_0)$, $H^{WT}(K_0)$ denotes the electric/magnetic field due to the line current in the structure without truncation. The total field includes contributions from all three components: line current, volume polarization current, and surface electric current

$$H_x(\theta) = \left(H_x(K_0, \theta) + H_x(J_g^{\mathrm{PO}}, \theta) + H_x(J_d^{\mathrm{PO}}, \theta)\right) \frac{e^{-jk_0 r}}{\sqrt{r}}. \tag{15}$$

In this expression, $H(X)$ means the far field excited by a current X in free-space.

We mention that this PO method (15) does not require the necessity to calculate the real PO current distribution (13), (14). It is possible to calculate directly the radiation pattern. The radiation field is expressed in terms of Fourier integral, which is evaluated numerically. The comparison between the IE technique and this PO approximation is presented in Fig. 4 by x-marks. The agreement is quite good for the angles $90° < \theta < 270°$. However the differences are increasing with the permittivity of the substrate and it is not recommended to use this method in this case.

VIII. Conclusion

The radiation pattern of a magnetic line source placed near the truncation of a dielectric structure has been calculated by using different methods. The application of PO solutions seems to be rather effective but it does not yield accurate solutions for all observation angles and its accuracy is decreasing with the increasing of the substrate thickness/permittivity. The last conclusion is explained by the fact that a PO method does not take into consideration any reflection of surface waves from the truncation of the dielectric structure and the boundary condition for the field at the truncation is not satisfied. So methods that are based on the solution of the integral equations (IE, EWC, MEWC) are more capable to describe the effects caused by the truncation for all substrate parameters and all observation angles. The comparison between different methods permits to understand better the advantages and the restrictions of each method. This study is useful for better modeling and understanding the effects caused by truncations.

Appendix

In this Appendix, we present the approximations that we have made during the derivation of (4). Let us consider instead of (4) the following integral:

$$I(\alpha, y, \Delta) = \int_{-\infty}^{+\infty} \frac{1}{\gamma + \alpha} e^{-j\eta y - \gamma\Delta}\, d\eta \tag{A.1}$$

where $\gamma = \sqrt{\eta^2 - 1}$. This integral becomes identical with the (4) type integral when $\Delta = 0$. We define a new function U as

$$U(\alpha, y, \Delta) = I_\Delta^{-1} = \int_{-\infty}^{+\infty} -\frac{1}{\gamma(\gamma+\alpha)} e^{-j\eta y - \gamma\Delta}\, d\eta. \tag{A.2}$$

By using

$$\int_{-\infty}^{+\infty} \frac{e^{-j\eta y - \gamma\Delta}}{\gamma}\, d\eta = -j\pi H_0^{(2)}\left(\sqrt{y^2 + \Delta^2}\right)$$

[14] and $I = U'_\Delta$ we obtain a differential equation for the new function U

$$U'_\Delta - \alpha U = -j\pi H_0^{(2)}\left(\sqrt{y^2 + \Delta^2}\right). \tag{A.3}$$

The equation (A.3) is solved in the ordinary way. The function U is presented as

$$U(\Delta) = v(\Delta)u(\Delta) \tag{A.4}$$

where $v(\Delta) = e^{\alpha\Delta}$ is a solution of (A.3) with the right part equal to zero. After the substitution of (A.4) into (A.3) one obtains

$$u(\Delta) = \int \left(-j\pi H_0^{(2)}\left(\sqrt{y^2+\Delta^2}\right) e^{-\alpha\Delta}\right) d\Delta + C. \tag{A.5}$$

The constant C is equal to zero because it follows from (A.1), (A.3), and (A.4) that

$$\lim_{y\to\infty} u(\Delta) = 0.$$

The position of the truncation point is not to close to the source. This means that the value of y in (A.5) is not small for the case in which we are interested in so we can use an approximation for the Hankel function

$$H_0^{(2)}(\sqrt{y^2+\Delta^2}) \approx \sqrt{\frac{2j}{\pi y}} e^{-j(y + (1/2)(\Delta^2/y))}. \tag{A.6}$$

After the substitution of (A.6) into (A.5), we derive that

$$u(\Delta) \approx -j\sqrt{\frac{2\pi j}{y}} e^{-jy} e^{-j(\alpha^2 y/2)} \int_{+\infty}^{\Delta - j\alpha y} \exp\left(-j\frac{1}{2y}\xi^2\right) d\xi. \tag{A.7}$$

The analytical approximation of the integral (A.1) is obtained by using (A.4) and (A.7) as

$$\begin{aligned} I(\Delta = 0) &= \lim_{\Delta\to 0} U_\Delta \\ &= -j\sqrt{\frac{2\pi j}{y}} e^{-jy} \left(1 + \alpha e^{-j(\alpha^2 y/2)} \int_{+\infty}^{-j\alpha y} \cdot \exp\left(-j\frac{1}{2y}\xi^2\right) d\xi\right). \end{aligned} \tag{A.8}$$

Equation (4) follows from (A.8).

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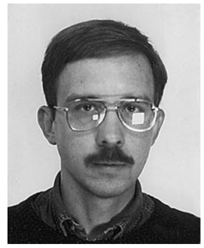

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