Letters

Efficient and Accurate Computation of an Annular Slot on a Dielectric Half-Space

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***Abstract*—An efficient and accurate method of analysis for an annular slot radiating on a dielectric half-space is presented. The result involves neither singularity nor any numerical integration and, therefore, is computationally very efficient and easy to implement numerically. The method is very rigorous and general and should find applications in other related configurations.**

***Index Terms*—Apertures, half-space, slot antennas.**

I. Introduction

Slot antennas have been studied and used extensively because of their inherent merits of low profile, lightweight, ease of fabrication, and compatibility with monolithic-microwave integrated-circuit (MMIC) integration. Recently, attention has been paid to the study of a slot on a dielectric half-space [1], [2]. The problem is of practical interest since the half-space can be used to model an electrically large dielectric lens, which efficiently couples incident energy to an active device placed at the antenna feed point. An annular slot antenna placed on a dielectric half-space was studied by Tong and Blundell [2]. This configuration can be easily modified to give a magic slot radiator, which can be used in the design of a quasi-optical balanced mixer. Another advantage of the configuration is that circularly polarized fields can be easily generated by adding a quadrature source displaced 90° in space, or simply by shorting one point [3].

In [2], the Hankel transform was used to solve the problem. The method was rigorous and elegant, but assuming a lossless dielectric half-space, singular integrals were found which caused inconvenience in numerical implementation. In addition, the upper limits of the integrals were of infinite extent, making the implementation even more inconvenient. Using the well-known spatial Green's function of the form e^{-jkR}/R the second problem can be avoided, but the first problem still remains. A simple solution is to use the concept of equivalent radius [2], [3] and utilize the well-established result for a circular loop antenna [4]. This method, however, is not general, but limited to a narrow slot only. In this letter, a simple result that requires no numerical integration is presented. It starts with the mode-matching method [5] in spherical coordinates. The formulation is very rigorous and general, though only the narrow slot case is considered here for simplicity. The result is simply a regular series without any singularity, therefore, the numerical implementation is very easy and straightforward. The result is compared with previously calculated and measured results [2] and excellent agreement are obtained.

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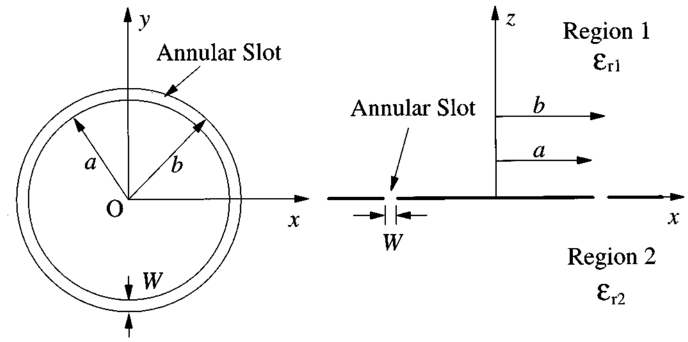

Fig. 1. Geometry of the configuration. (a) Top view. (b) Side view.

II. Theory

Fig. 1 shows the geometry of the configuration, where the annular slot has an inner radius a, outer radius b, and width $W = b - a$, and is excited at $x = 0$. The wavenumbers of regions 1 and 2 are denoted by k_1 and k_2, respectively. To begin with, the equivalence principle is used, which enables each region to be considered separately. Image theory is then invoked so that the equivalent magnetic current in the slot can be solved in free-space conditions. The mode-matching method [5] is used to obtain the Green's functions for H_ϕ in the two regions. The Green's functions are common to both regions, except that in each region the Green's function has its own constitutive parameters. Employing the moment method, we expand the magnetic current as $M(\phi) = \sum_{q=1}^{N} V_q \cos(q-1)\phi$, where V_q are expansion coefficients to be determined. Because of the orthogonal property of the basis functions $\cos(q-1)\phi$ the summation over m (azimuthal variation) vanishes and only the summation over n (order of Bessel and Hankel functions) remains. Moreover, the orthogonal property causes the admittance matrix elements Y_{pq} to vanish for $p \neq q$, resulting in a diagonal matrix. Consequently, the expansion coefficients V_q are simply equal to $1/Y_{qq}$, where

$$\begin{aligned} Y_{qq} = {} & \frac{\pi \Delta_{q-1}^2}{W^2} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \cdot \frac{(n-q+1)!}{(n+q-1)!} \\ & \cdot \left\{ \frac{1}{\Delta_{q-1}} \left[P_n^q(0)\right]^2 \left[\frac{\alpha_1^{\rm TM}(n)}{\eta_1} + \frac{\alpha_2^{\rm TM}(n)}{\eta_2} \right] \right. \\ & \left. + (q-1)^2 \left[P_n^{q-1}(0)\right]^2 \left[\frac{\alpha_1^{\rm TE}(n)}{\eta_1} + \frac{\alpha_2^{\rm TE}(n)}{\eta_2} \right] \right\} \end{aligned} \tag{1}$$

with $\Delta_m = 2$ for $m = 0$ and 1 for $m \neq 0$ and $P_n^m(x)$ being the associated Legendre function of the first kind with order m and degree n. At $x = 0, P_n^m(0)$ is given by

$$P_n^m(0) = \begin{cases} (-1)^{(n+m)/2} \cdot \frac{1 \cdot 3 \cdot 5 \cdots (n+m-1)}{2 \cdot 4 \cdot 6 \cdots (n-m)} & n+m \text{ even} \\ 0, & n+m \text{ odd} \end{cases} \tag{2}$$

where the denominator is set to one when $n = m$. In (1), for $j = 1, 2$ we get (3), shown at the top of the next page

$$\alpha_j^{\rm TE}(n) = \frac{-jW}{k_j} + \frac{1}{k_j^2} \left[\hat{J}_n(k_j a) \hat{H}_n^{(2)}(k_j a) \right.$$

$$\alpha_j^{\mathrm{TM}}(n) = \frac{1}{k_j^2}\sum_{i=1}^{N_1}\left[\begin{array}{l} I_n^{(1)}\left(a, a+\frac{2i-1}{2N_1}W\right) I_n^{(2)}\left(a+\frac{i-1}{N_1}W, a+\frac{i}{N_1}W\right) \\ +I_n^{(2)}\left(a+\frac{2i-1}{2N_1}W, a+W\right) I_n^{(1)}\left(a+\frac{i-1}{N_1}W, a+\frac{i}{N_1}W\right)\end{array}\right] \tag{3}$$

$$+ \hat{J}_n(k_j b)\hat{H}_n^{(2)}(k_j b) - 2\hat{J}_n(k_j a)\hat{H}_n^{(2)}(k_j b)\Big] \tag{4}$$

where for $l = 1, 2$

$$I_n^{(l)}(\alpha, \beta) = \int_{k_j\alpha}^{k_j\beta} \hat{f}_n^{(l)}(x)\,dx \tag{5}$$

with $\hat{f}_n^{(1)}(x) = \hat{J}_n(x)$ and $\hat{f}_n^{(2)}(x) = \hat{H}_n^{(2)}(x)$ being the Schelkunoff-type [5] spherical Bessel function of the first kind and Hankel function of the second kind, respectively. α_j^{TE} and α_j^{TM} $(j = 1, 2)$ are associated with the TE and TM modes of the fields, respectively, and η_j are dielectric wave impedances. Denote $\hat{f}_n^{(3)}(x) = \hat{Y}_n(x)$ as the Schelkunoff-type spherical Bessel function of the second kind, we have $I_n^{(2)} = I_n^{(1)} - jI_n^{(3)}$. The integrals $I_n^{(l)}$ $(j = 1, 3)$ are evaluated by virtue of the following recurrence formula:

$$I_n^{(l)}(\alpha, \beta) = -\frac{2n-1}{n-1}\left[\hat{f}_{n-1}^{(l)}(k_j\beta) - \hat{f}_{n-1}^{(l)}(k_j\alpha)\right] + \frac{n}{n-1}I_{n-2}^{(l)}(\alpha, \beta) \tag{6}$$

It was found that $I_n^{(3)}$ could be calculated in a straightforward manner, with initial values $I_0^{(3)}(x_1, x_2) = -(\sin x_2 - \sin x_1)$ and $I_1^{(3)}(x_1, x_2) = -[\mathrm{Ci}(x_2) - \mathrm{Ci}(x_1)] + (\cos x_2 - \cos x_1)$, where $\mathrm{Ci}(x)$ is the cosine integral for which a simple formula is readily available [6]. For $I_n^{(1)}$, however, backward recurrence should be used to obtain stable results. This is not surprising, as it is well known that backward recurrence is also required for $\hat{J}_n(x)$, the integrand of $I_n^{(1)}$. It should be mentioned that the summation in (3) converges very rapidly for a narrow slot $(\sqrt{(\varepsilon_{r1} + \varepsilon_{r2})/2}k_0 W \ll 1)$, and only a few terms are sufficient for accurate results. Note that Y_{qq} can be calculated without the need for any numerical integration. Furthermore, it is a regular series that can be calculated easily and straightforwardly. After Y_{qq} are found, the input impedance can be easily obtained from $Z_{\mathrm{in}} = \sum_{q=1}^{N} 1/Y_{qq}$.

III. Results

To validate the theory, the result is compared to that in [2]. Since six azimuthal modes were used for the magnetic current in [2], $N = 6$ was used for the moment method in this letter. It was found that Y_{qq} converged very well in 70 modal terms for the parameters of [2]. Furthermore, excellent convergence resulted by using $N_1 = 4$ in (3). The program was written in Fortran 77 and run on a SunSPARC 20 Model 612 workstation. The average computational time for each frequency point was extremely short, at only 6.9 ms. A comparison of the present and previous calculation is shown in Fig. 2, along with the previous measurement. It is interesting to note that the present result, in general, has a better agreement with the measurement, especially for $k_0 a > 1.3$. Part of the reason is that the singularity of the integrand has been handled analytically in the present theory. Moreover, as the present theory

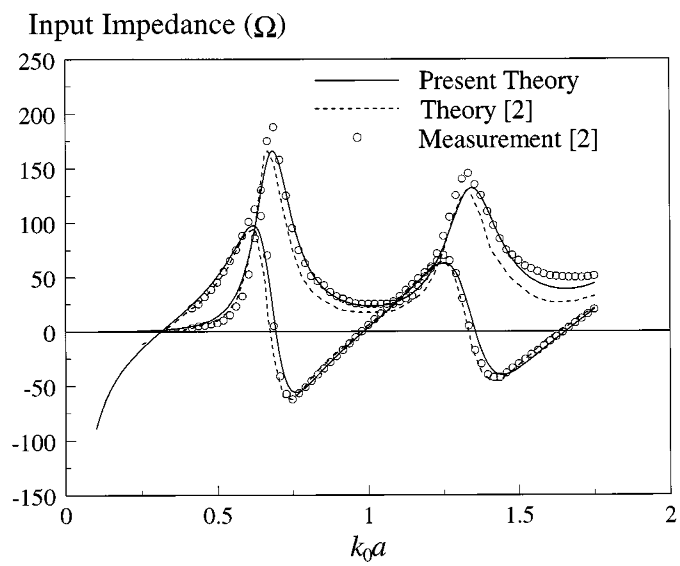

Fig. 2. Comparison of the present and previous results: $a = 19.25$ mm, $b = 10.75$ mm, $W = 1.5$ mm, $\varepsilon_{r1} = 4$, and $\varepsilon_{r2} = 1$.

does not involve any numerical integration, errors arising from possible convergence problems were avoided. Other results for $\varepsilon_{r1} = 2.1, 4.0,$ and 12.0 with $\varepsilon_{r2} = 1$ were generated, which, again, agreed very well with those of [2]. The results, however, are omitted here for brevity. The approach can be extended to study a wide annular slot. In this case, the radial current has to be included in the formulation and a total of four Green's functions $G_{M_\beta}^{H\alpha}$ $(\alpha, \beta = r, \phi)$ are required, where $G_{M_\beta}^{H\alpha}$ denotes the Green's function of H_α due to a magnetic point current M_β.

Finally, it should be mentioned that the result is useful for other configurations as well, such as for the cavity-backed annular slot antenna [3].

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