Equivalent Hyperboloid (Ellipsoid) and Its Application

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***Abstract*—Analogous to the equivalent paraboloid, the equivalent hyperboloid (ellipsoid) for dual quadratic surface reflector antennas is derived. The condition that the equivalent reflector is center fed is also derived. The numerical example shows that a reflector geometry that satisfies this center fed condition is a good initial antenna geometry to design a shaped dual reflector antenna.**

***Index Terms*—Reflector antennas.**

I. INTRODUCTION

It is the purpose of this paper to present the equivalent hyperboloid (or ellipsoid) for dual quadratic surface reflector antenna system and the geometrical condition for which the equivalent reflector is center fed and also to evaluate the merit of using this new antenna geometry to design shaped reflector antennas. The format of the solution presented in this papaer will be similar to that in the literature for comparison. Of course, the result of this analysis will be identical to that of other publications (such as [1], [2]), when the main reflector becomes a paraboloidal surface. This derivation is based on geometry of the antenna and the paraxial ray optics approximation. The graphic presentation provides insight into the formation of the equivalent surface. The example shows that a reflector geometry that satisfies the center fed condition is a good initial antenna geometry to design a low cross-polarization shaped dual-reflector antenna.

II. EQUIVALENT SURFACE

The antenna geometries under consideration are shown in Figs. 1 and 2. D and O are two foci of the main reflector and O and F are two foci of the subreflector. The reflectors are confocused. The main reflector shown in the figures is a hyperboloid. It can also be an ellipsoid. Four significant parameters in the figures are

$$\rho_m = \frac{(1+e_m)f_m}{1+e_m\cos\theta_m} \tag{1}$$

$$\rho_o = \kappa_m \frac{f_m\left(1+e_m^2+2e_m\cos\theta_m\right)}{(1-e_m)(1+e_m\cos\theta_m)} \tag{2}$$

$$\rho_1 = \frac{(1-e_s)f_s}{1-e_s\cos\theta} \tag{3}$$

and

$$\rho_2 = \kappa_s \frac{f_s\left(1+e_s^2-2e_s\cos\theta\right)}{(1+e_s)(1-e_s\cos\theta)} \tag{4}$$

where the subscripts m and s stand for the main and the subreflectors, respectively, f's are the focal lengths, e's are the eccentricities and $\kappa_{m,s} = 1$ for an ellipsoid, $\kappa_{m,s} = -1$ for a hyperboloid. Reference [3] provides a good reference to the expressions.

Representing the reflectors by lenses, the focusing behavior of the local ray $CABF$ in Fig. 1 is presented in Fig. 3. This lens representation is a powerful tool in the analysis of a reflector system. Because the lenses unfold the ray path, the measure of the ray path length becomes straightforward. In Fig. 3, A is the location of the main reflector, B is the location of the subreflector, and F is the location of the feed. To replace two lenses (at A and B) by a single lens and to maintain the relationship between the input/output ray, the equivalent lens (surface), E as shown, must be located at the intersection of the right circular cone at the feed and the right circular cone of the output surface (at D). In the parlance of optics, this equivalent surface preserves the angular magnification of the original optical system. As a word of caution, this equivalent surface does not preserve the dimension of reflector aperture. As shown in Fig. 3, the aperture of the lens at E is smaller than that of the lens at A. From the trigonometric relation in Fig. 3 which is the case of a hyperboloidal main reflector and an ellipsoidal subreflector, one obtains

$$\frac{R_1}{R_2} = \frac{ED}{AD} = \frac{\rho_o-\rho_m-\rho_1-\rho_2+\rho_{\rm eq}}{\rho_o} \tag{5}$$

$$\frac{R_2}{R_3} = \frac{AO}{OB} = \frac{\rho_m}{\rho_2} \tag{6}$$

and

$$\frac{R_3}{R_1} = \frac{BF}{EF} = \frac{\rho_1}{\rho_{\rm eq}}. \tag{7}$$

Multiplying (5)–(7), $\rho_{\rm eq}$ results

$$\rho_{\rm eq} = \frac{\rho_m\rho_1(\rho_o-\rho_m-\rho_1-\rho_2)}{-\rho_1\rho_m+\rho_o\rho_2}. \tag{8}$$

Similar expressions can be obtained for three other reflector arrangements, i.e., ellipsoid/ellipsoid, ellipsoid/hyperboloid, and hyperboloid/hyperboloid. The results can be summarized as

$$\rho_{\rm eq} = \frac{\rho_m\rho_1(\kappa_m\rho_o+\rho_m+\rho_1+\kappa_s\rho_2)}{\rho_1\rho_m+\kappa_m\rho_o\rho_2}. \tag{9}$$

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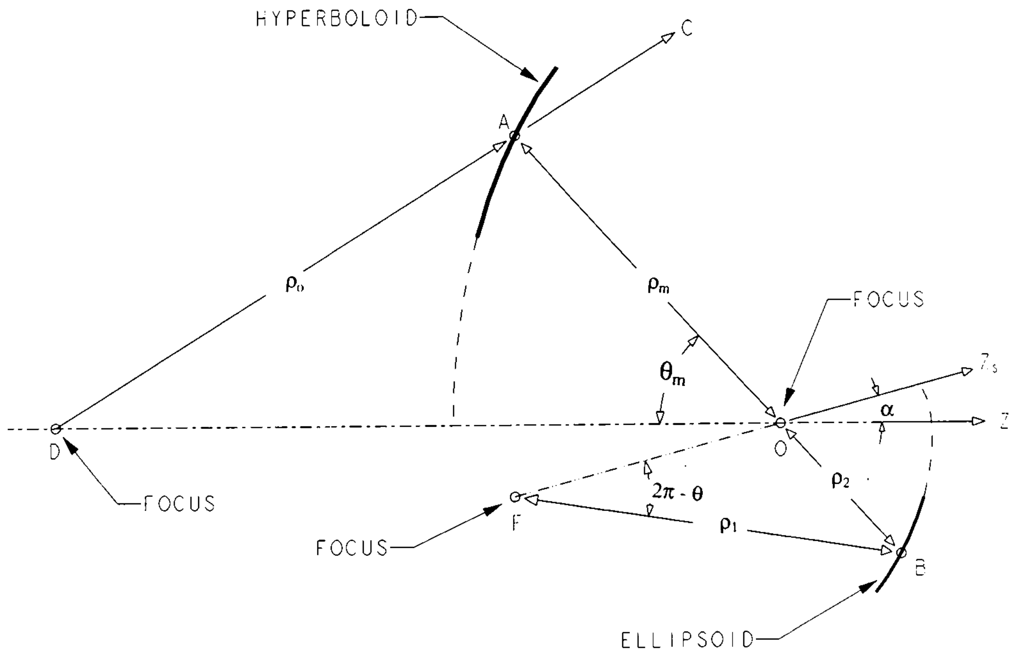

Fig. 1. Offset hyperboloidal/ellipsoildal dual-reflector geometry.

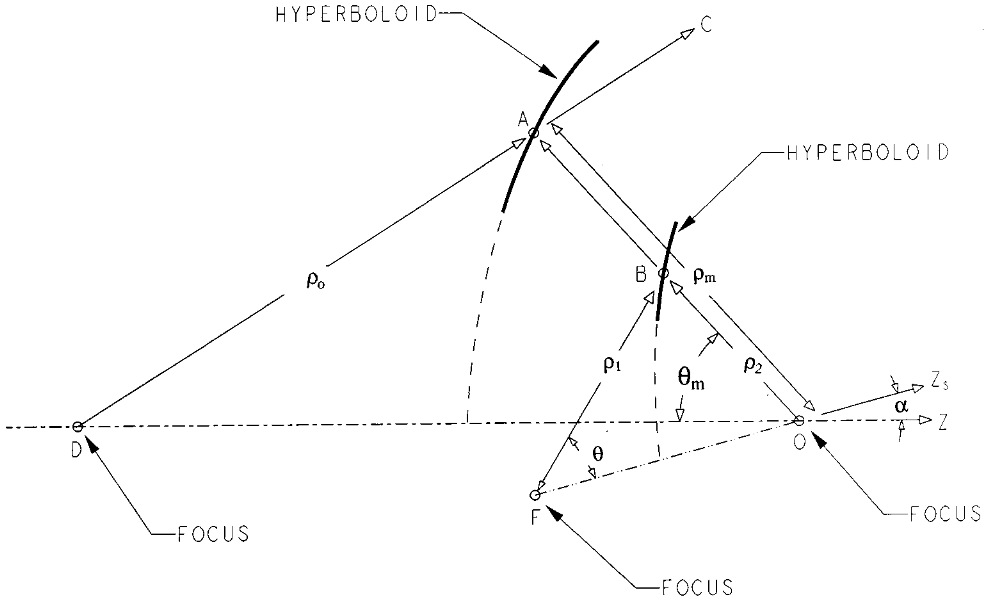

Fig. 2. Offset hyperboloidal/hyperboloidal dual-reflector geometry.

This is the expression of the equivalent surface (lens). When $\rho_o \Rightarrow \infty$, (9) reduces to left-hand side (LHS) of (14) in [2]. Substituting (1)–(4) in to (9) and using the trigonometric relation in Figs. 1 and 2

$$\rho_2 \sin(\alpha + \theta_m) = -\kappa_s \rho_1 \sin(\theta) \tag{10}$$

the equivalent surface results in

$$\rho_{\text{eq}} = \frac{F}{A\cos\theta + B\sin\theta + D} \tag{11}$$

where

$$F = \kappa_s \left(\frac{2f_m}{(1-e_m)} + \frac{2f_s}{(1+e_s)}\right)\left(1-e_m^2\right)\left(1-e_s^2\right),$$
$$A = 2e_m\left(1+e_s^2\right)\cos\alpha - 2e_s\left(1+e_m^2\right)$$
$$B = -2e_m\left(1-e_s^2\right)\sin\alpha$$

and

$$D = \left(1+e_m^2\right)\left(1+e_s^2\right) + \kappa_s\left(1-e_m^2\right)\left(1-e_s^2\right) - 4e_s e_m \cos\alpha.$$

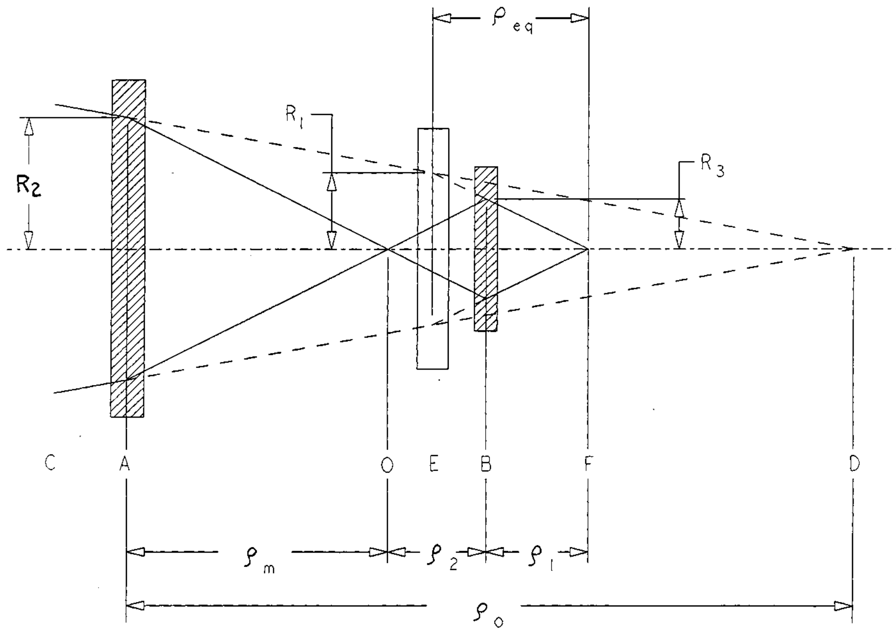

Fig. 3. The lens representation of a hyperboloidal/ellipsoidal dual-reflector antenna.

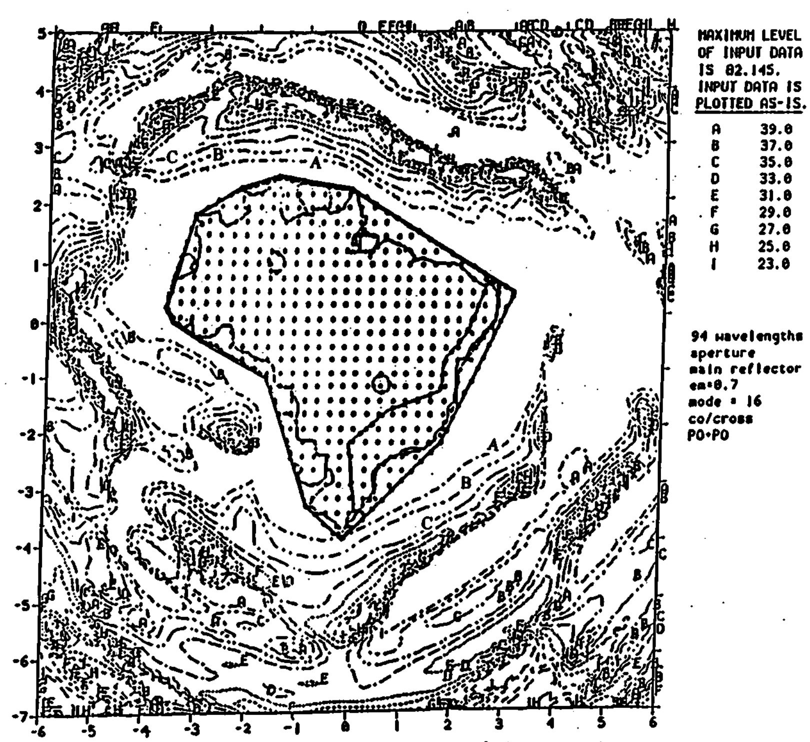

Fig. 4. co/cross-polarization ratio of the Brazil coverage antenna.

Equation (11) can be rewritten as

$$\rho_{\text{eq}} = \frac{F/D}{1 + (\sqrt{A^2 + B^2})/D \cos(\theta - \beta)} \tag{12}$$

where

$$\tan\beta = \frac{B}{A} = \frac{-e_m\left(1 - e_s^2\right)\sin\alpha}{e_m\left(1 + e_s^2\right)\cos\alpha - e_s\left(1 + e_m^2\right)}. \tag{13}$$

When $e_m = 1$, which is the case of a paraboloidal main reflector, (13) reduces to (25) of Rusch's [2]. Equation (12) represents a quadratic surface with

$$\text{eccentricity, } e_{\text{eq}} = |(\sqrt{A^2 + B^2})/D| \tag{14}$$

$$\text{focal length, } f_{\text{eq}} = \frac{F}{\sqrt{A^2 + B^2} + D}. \tag{15}$$

Equation (12) also shows that the axis of the equivalent surface is in the direction of $\theta = \beta$. Thus, the feed axis must be in the direction of β if the equivalent reflector is to be center fed.

III. APPLICATION

The geometry of a shaped dual-reflector antenna customarily evolves from the classical Gregorian (or Cassegranian), which satisfies the Mizugutch condition [1]. The reasons for choosing this initial antenna geometry are partially due to lack of knowledge and partially due to convenience. Many shaped dual-reflector antennas were designed with such a process [4]. The principal polarization contours are, in general, satisfactory. The cross-polarization level, however, worsens as the coverage area becomes larger. Typically, the co/cross-polarization ratio within the coverage area is limited to $\sim$34 dB. Since both a shaped reflector and a hyperboloid (or an ellipsoid) are not collimating devices, it is more logical to use a hyperboloid (or an ellipsoid) as an initial condition instead of a paraboloid. The Fig. 4 is contour of the co/cross polarization ratio of the Brazil coverage shaped reflector antenna, which is designed from the initial condition of an ellipsoidal main reflector which satisfied the center fed condition i.e., (13). Within 99.9% of the coverage area, co/cross-polarization ratio of 39 dB has been achieved. In this analysis, the chief ray of the antenna geometry is pointing toward the center of the coverage and it is also defined as the Z axis of the output coordinate.

The design and computations were performed by the use of TICRA's software POD, a physical optics dual-reflector shaping program. The X polarized Gaussian feed (-15 dB taper at 25 degrees) is assumed in this example.

IV. CONCLUSION AND REMARKS

The condition (13) for which a dual quadratic surface reflector antenna generates low cross polarization is presented. The paper also points out that the equivalent hyperboloid (ellipsoid) does not preserve the dimension of the main reflector, which is the major deviation from the concept of the equivalent paraboloid. Although the example presented in the paper is over simplified by not having a real feed, it clearly show the advantage of the use of the new antenna configurations to design a shaped reflector. The performance advantage obtained, of course, may be reduced as the complexity of the coverage shape increases.

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