A Truncated Floquet Wave Diffraction Method for the Full Wave Analysis of Large Phased Arrays—Part I: Basic Principles and 2-D Cases

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***Abstract*—This two-part sequence deals with the formulation of an efficient method for the full wave analysis of large phased-array antennas. This is based on the method of moments (MoM) solution of a fringe integral equation (IE) in which the unknown function is the difference between the exact solution of the finite array and that of the associated infinite array. The unknown currents can be interpreted as produced by the field diffracted at the array edge, which is excited by the Floquet waves (FW's) pertinent to the infinite configuration. Following this physical interpretation, the unknown in the IE is efficiently represented by a very small number of basis functions with domain on the entire array aperture. In order to illustrate the basic concepts, the first part of this sequence deals with the two-dimensional example of a linearly phased slit array. It is shown that the dominant phenomenon for describing the current perturbation with respect to the infinite array is accurately represented in most cases by only three diffracted-ray-shaped unknown functions. This also permits a simple interpretation of the element-by-element current oscillation, which was recently described by other authors. The second part of this paper deals with the appropriate generalization of this method to three-dimensional (3-D) arrays.**

***Index Terms*—Electromagnetic diffraction, Floquet expansions, phased-array antennas.**

I. Introduction

THE electromagnetic modeling of large finite arrays as well as the scattering by finite periodic structures is an important topic for a large variety of engineering applications and has been the object of many recent investigations [1]–[10]. A rigorous analysis based on an element-by-element method of moments (MoM) becomes computationally difficult when the size of the array increases. If the structure is taken as infinite, the numerical effort is reduced to that of a single periodic cell solution [1]. This approximation leads to reasonable results in predicting the input impedance of elements far out from the edges. However, for near-edge elements it is significantly inaccurate. Furthermore, for wide-beam angle scanning the effects of truncation can be relevant also for elements very far from the edges.

The present method is focused on the prediction of the current distributions on the array radiating elements (including the ones close to the edge of the array) retaining an extremely small number of unknowns. This method is based on the MoM solution of an appropriate "fringe" integral equation (IE) in which the unknown function is the difference between the exact current distribution on the truncated array and the current distribution pertinent to the infinite array. This unknown function can be interpreted as due to the edge diffracted field excited by the Floquet waves (FW's) relevant to the infinite, periodic continuation of the actual array. Following this physical interpretation, the unknown of the IE is efficiently represented by a few entire domain basis functions, which are shaped as FW diffracted rays.

In order to simplify the description of the overall method, the paper is split in two parts. In this paper, Part I, the basic concepts are illustrated with reference to a specific two-dimensional (2-D) example of a linearly phased array of slits in an infinite metallic plane. The physical insight gained by the investigation of this prototype problem is useful to highlight the basic mechanisms that dominate the edge perturbation, thus allowing a neat selection criterion for the number of unknowns in the MoM scheme. This will be fundamental for the successive three-dimensional (3-D) generalization carried out in Part II. For the present 2-D case, only three basis functions in the unknown description of the fringe IE are sufficient for providing the same accuracy as that obtained from an element by element full wave analysis.

The example presented here allows for a clear interpretation of the global element by element current oscillation produced by edge perturbations recently described by Hansen and Gammon [9] by means of a model based on the Gibbs phenomenon. Although interesting, this latter model does not seem applicable in general cases since it does not predict the change of the oscillation period close to one edge occurring for scanning array with more than a half-wavelength period. By using the present approach, this effect will be shown to be associated with the interference between evanescent FW's and their relevant diffracted fields; a simple formula is provided to extend that given in [9] to arbitrary spacings.

A summary of the methods presented in literature for large arrays in comparison with the present approach is in order. The technique proposed in [2] and improved in [3], accounts for the edge effects by a windowing method. This is based on constructing the active Green's function to be used in the MoM procedure by the radiation of an array of elementary dipoles whose amplitude and phase are dictated exclusively by the excitation. This approach is convenient since it requires the same number of unknowns as those for solving the infinite array. However, it sometimes leads to incongruence in predicting the effects of

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truncation, especially when studying aperture arrays on ground planes [see, Section IV].

In [4], an interesting hybrid method incorporating an FW expansion in a MoM scheme is proposed for a 2-D array of strips. In this approach, the distribution of the radiating currents in the central portion of the array is supposed to be the same as that in the absence of truncations. In the next step, these currents are used for exciting the electric field on the elements close to the edge. Although very good results can be obtained for near broadside scan, this method fails when the effects of the truncation do not rapidly vanish away from the edges, as occurs in wide-beam scanning.

In [6], an enlightening and elegant method for prediction of scattering from a 2-D finite periodic strip grating is analyzed; the numerical solution for the current on the strips is obtained with an approximation that is equivalent to the windowing approach, i.e., assuming that the current on each strip is independent of the strip location within the array. This position-independent current is taken to be that on the central element of a similar finite, but smaller grating; this latter is analyzed with an element-by-element MoM with a reduced computational cost. This approach is a clever way of exploiting a very efficient version of the element by element 2-D MoM devised by the same authors [10] and is an alternative implementation of the basic windowing approach, which differs from [2] and [3] for the choice of the position-independent current, which is obtained from the infinite periodic problem in [2] or via active Green's function in [3]. However, the use of (indeed pioneering) asymptotic constructs in [6] might lead to equivocate techniques and objectives, presented in [6] and in the present work. In [6], the asymptotic analysis is focused on the scattering problem by a strip grating both in frequency and, most remarkably, in time with a consequent relaxation in the accuracy required in the determination of the currents on the array elements. Consequently, the asymptotic expressions were not used there to construct these currents, but only to observe the far field. The present work is instead primarily concerned with the efficient, though accurate, evaluation of the currents on the array elements, not only for the accurate assessment of the truncation effects on the radiation pattern, but especially for the determination of the antenna input parameters (impedance and/or scattering parameters). The important issue of the accurate and fast evaluation of the field radiated by the array currents in both near and far zone is outside the scopes of this paper; instead, it is dealt with in detail in some companion works [8], [11], [12]. The use of uniform asymptotic techniques here is employed for finding efficient array-global basis functions for the successive rigorous MoM solution. The method is then substantially different from those approaches that use the windowing of the active Green's function concept [2], [3], [6].

This Part I is organized as follows. In Section II, the IE's are formulated for the slit-array problem. The truncated FW description presented in Section III prepares the successive introduction of the diffracted ray basis functions defined in Section IV, that are used for the expansion of the unknowns in the MoM solution of the IE. In Section V the validation is carried out meanwhile gaining further insight on large scanning effects.

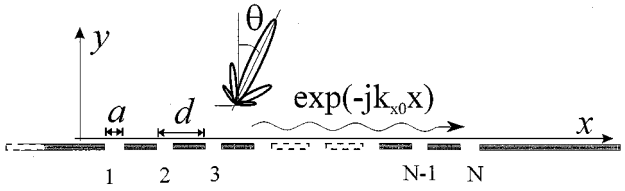

Fig. 1. Geometry of an array of slits on an infinite ground plane. The slits are fed by magnetic aperture fields $H_z^{\rm inc}(x) = H_z^{\rm inc} \exp(-jkx \sin\theta)$ where x is restricted to the slit regions. The main beam of the array is oriented in direction θ with respect to the normal.

II. Integral Equations for a Slit Array

The geometry of the problem consists of an array of N thin slits (denoted by $n = 1, 2, \cdots, N$) etched on an infinite ground plane (Fig. 1). The size of the slits and the interelement period are denoted by a and d, respectively; a reference system is introduced, with its x, y, and z axes oriented along the array, along the normal to the interface, and along the slits, respectively. The first slot is placed at $x = d$. We assume a simple impressed magnetic field of the kind

$$H^f(x) = e^{-jk_{xs}x} \tag{1}$$

that gives rise to beam radiation in direction $\theta = \sin^{-1}(k_{xs}/k)$ from broadside, k being the free-space wavenumber. In order to formulate the problem as an IE, let us denote by A the portion of the x axis occupied by the slits, and define the characteristic function χ_A of the array, which is unity on A and zero elsewhere

$$\chi_A(x) = \sum_{n=1}^{N} \mathrm{rect}(x - nd);\ \mathrm{rect}(x) = \eta(x) - \eta(x-a) \tag{2}$$

in which $\eta(x)$ is the Heaviside unit step function. By applying the equivalence principle, each slit is replaced by a perfectly conducting surface with two unknown magnetic current distributions $\pm M(x')\chi_A(x')$ on its opposite faces that are of equal amplitude and opposite sign to ensure the continuity of the tangential component of the electric field at the interface. The usual IE is now considered, which expresses the enforcement of the appropriate continuity through the apertures of the total (impressed plus radiated) magnetic field

$$2\chi_A(x) \int_A G_M^H(x;\ x') M(x')\, dx' = \chi_A(x) H^f(x) \tag{3}$$

where G_M^H is the grounded half-space Green's function of the tangential magnetic field at x for a magnetic source placed at x'. The factor two on the left-hand side (LHS) of (3) arises from the sum of the two equal field contributions from the upper and lower side of the ground plane. Next, it is convenient to introduce an infinite array, that coincides with the actual (finite) array over its extent and realizes its regular infinite periodic continuation. In this infinite array we denote by A^* the regions occupied by the slots outside the actual finite array, i.e., for $x < 0$ and

$x > Nd$. Enforcing the IE for the infinite array problem whose unknown magnetic current is denoted by $M_\infty(x')$ yields

$$2\chi_\infty(x)\int_{-\infty}^{\infty} G_M^H(x;\,x')M_\infty(x')\,dx' = \chi_\infty(x)H^f(x) \quad (4)$$

where $\chi_\infty = \chi_{A+A^*} = \chi_A + \chi_{A^*}$ is the characteristic function of the infinite slot array. It is useful to introduce the difference M_d between the magnetic current M on the finite array and the solution M_∞ to the infinite array problem

$$M_d = (M - M_\infty)\chi_A. \quad (5)$$

Note that M_d extends only over the actual, finite array. The problem can be reformulated in terms of this difference M_d by inserting (5) into the original (3) and explicitly enforcing (4) that defines M_∞. In doing this it is useful to observe that in all problems the total field continuity condition must hold on each slot so that (4) is unchanged by multiplication by χ_A, thus obtaining

$$\chi_A(x)\int_A G_M^H(x;\,x')M_d(x')\,dx' = \chi_A(x)H_{\rm ext}(x) \quad (6)$$

where

$$H_{\rm ext}(x) = \int_{A^*} G_M^H(x;\,x')M_{\rm ext}(x')\,dx' \quad (7)$$

is the field radiated by the sources

$$M_{\rm ext} = M_\infty\chi_{A^*} \quad (8)$$

which are the array sources external to A.

The first step of our procedure is the solution of the infinite array problem for M_∞; next, we find the unknown $M_d(x')$ via the MoM applied to (6), employing a very convenient scheme; while this scheme will have a rigorous formulation and the appropriate generalization to 3-D problems in Section II, the key to the solution strategy will emerge from the physical interpretation of the problem; to this end it is important to understand the meaning of (6).

It is apparent that (6) represents the same boundary conditions as the original IE (3), but with a different forcing term produced by the external sources $M_{\rm ext}$; we stress that $M_{\rm ext}$, after solving in a traditional way the IE (4) for the infinite array is a known term. Therefore, M_d corresponds to the electric field E_{yd} found in the slits of the real finite array in the presence of the forcing term $H_{\rm ext}$. On the other hand, the forcing term $H_{\rm ext}$ of (6) is the field radiated by that part of the infinite array that has been suppressed to obtain the actual problem from the infinite one. This expresses the fact that the deformation of the finite-array solution M_d with respect to the infinite-array solution M_∞ is such as to compensate for the absence of the radiation contribution from the suppressed part $M_{\rm ext}$ of the infinite array.

As it will be described later on, $H_{\rm ext}$ can be represented as the radiation on the region $\sum \equiv x \in (0,\,(N+1)d)$ of FW's distributed on the complementary aperture $\sum^* \equiv x \in (\infty,\,0)\bigcup((N+1)d,\,\infty)$. This field can be asymptotically interpreted and represented in terms of diffracted rays originated at the endpoint of $\sum^*$. As discussed next, the same representation will be applied to M_d to obtain a suitable set of basis functions to be used in a MoM scheme. Due to this interpretation and using a terminology which is typical of the physical theory of diffraction [13], (6) will be denoted as a *fringe* IE.

The first step in the outlined strategy requires the understanding of the features of the field radiated by truncated FW distributions.

III. Truncated FW's

Let us consider first the infinite array of magnetic sources $M_\infty\chi_A$. Owing to the periodicity, its magnetic field $H_\infty(x)$ can be represented by superposition of FW's

$$H_\infty(x) = \sum_{-\infty}^{\infty} H_{m\infty}\,e^{-jk_m x} \quad (9)$$

propagating along x with wavenumbers

$$k_{xm} = k_{xs} + \frac{2\pi m}{d}, \qquad m = 0,\,\pm1,\,\pm2,\,\cdots. \quad (10)$$

The FW's with phase velocity along x greater $(k_{xm} < k)$ or less $(k_{xm} > k)$ than the speed of light, are denoted by homogeneous FW's (HFW's) or evanescent FW's (EFW's), respectively. The latter are exponentially attenuated in the direction normal to the array.

The field $H_{\rm ext}(x)$ radiated by the external sources $M_\infty\chi_{A^*}$ of the array [i.e., the forcing term of (6)] can be obtained by the radiation integral of the FW equivalent current distributions over the two semi-infinite apertures $x < 0$ and $x > (N+1)d$. This may be rigorously and simply formalized by using the Poisson summation formula as it is widely discussed in [5] and [7]. When the radiation integral of each semi-infinite FW aperture is evaluated asymptotically, it yields a field representation in terms of its stationary phase point, which recovers the FW itself restricted to a certain region of space, plus its relevant contributions at the endpoint of the integration domain. These latter may be interpreted as a diffracted field arising from the edge of the aperture. When observing at a point lying on the array for $x \in (d,\,Nd)$ the stationary phase point never occurs in the external integration domain so that the field is represented only in terms of the diffracted ray fields. Fig. 2(a) and (b) represent the diffracted rays for the dominant HFW ($m = 0$) and for the first EFW ($m = -1$), respectively. This latter is assumed to be propagating backward with respect to the HFW. The rays depicted by longer arrows denote larger amplitudes of the diffracted field, and the dark regions denote the transition regions of the diffracted rays. As discussed in [8], these latter have a parabolic or elliptic contours for HFW or EFW, respectively. The two radiating apertures are associated with the suppressed part of the array and, therefore, their endpoints, where the diffracted rays start, are displaced externally one period from the first and the last source of the actual array. A convenient asymptotic representation of the external forcing field for $x \in (0,\,L)$ is given in terms of the diffracted contribution $h_{\rm ext}^+(x)$ and $h_{\rm ext}^-(x^*)$ associated with the left $(x = 0)$ and right $(x = (N+1)d)$ endpoints, respectively

$$H^{\rm ext}(x) \sim h_{\rm ext}^+(x) - h_{\rm ext}^-(x*) \quad (11)$$

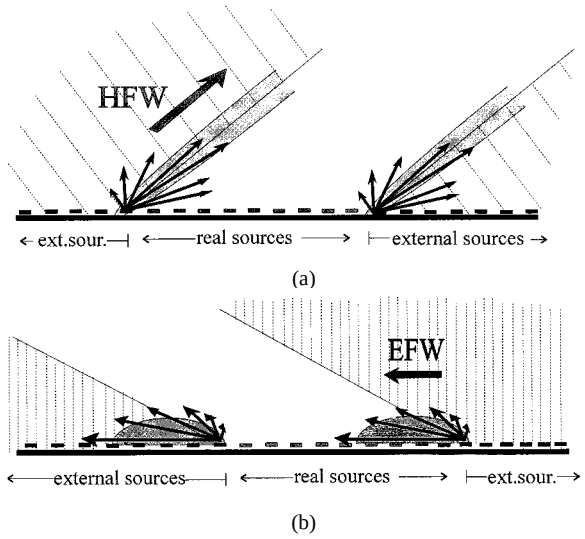

Fig. 2. Truncated FW's picture for a finite array of magnetic currents on the infinite ground plane. (a) and (b) correspond to the total field for the dominant HFW ($m = 0$) and for the first EFW ($m = -1$), respectively. Diffracted rays arising from the truncation compensate for the FW discontinuity occurring at the shadow boundaries (SB's). Longer arrows denote larger amplitudes of the diffracted rays; the dark zones denote the transition regions of the same diffracted rays. When the observation point is on the array plane, the stronger diffracted ray is that relevant to HFW and to EFW for the edge at left and right, respectively. This allows the interpretation of the result in Fig. 4.

where $x^* = (N+1)d - x$ and

$$h^{\pm}_{\text{ext}}(x) = I^{\pm}v(x) + I^{\pm}_0 u_0(x) + I^{\pm}_1 u_1(x) + I^{\pm}_{-1}u_{-1}(x). \quad (12)$$

In (12)

$$I^{\pm} = \frac{I\,\text{sinc}\left(\frac{1}{2}ka\right)}{1 - e^{-jd(k\mp k_{x0})}} \quad (13)$$

$$I^{\pm}_m = \frac{I\,\text{sinc}\left(\frac{1}{2}k_{xm}a\right)k\sqrt{2k}}{2d\sqrt{k \pm k_m}(k \mp k_m)^2} \qquad (m = 0,\ 1,\ -1) \quad (14)$$

where $I = 2jkaM_{0\infty}/(\zeta\sqrt{2\pi j})$

$$v(x) = e^{-jkx\sqrt{kx}} \quad (15)$$

and

$$u_m(x) = \frac{e^{-jkx}}{kx\sqrt{kx}}\mathcal{F}_s(x(k - k_{xm})). \quad (16)$$

The function

$$\mathcal{F}_s(y) = 2jy(1 - \mathcal{F}(y)) \quad (17)$$

where

$$\mathcal{F}(y) = 2j\sqrt{y}\,e^{jy}\int_{\sqrt{y}}^{\infty} e^{-jt^2}\,dt;\ -\frac{3\pi}{2} < \arg(y) \le \frac{\pi}{2} \quad (18)$$

is the slope transition function of the uniform theory of diffraction (UTD) [14]. The above expressions may be derived from [8, eq. (17)] by means of straightforward algebraic manipulations. Three u_m-terms have been used in (12); this corresponds to extracting three poles close to the saddle point in the Van der Waerden asymptotic evaluation of the relevant spectral diffraction integral defined in [8, eq. (10)].

The slope transition function in (17) tends to unity for large arguments so that for $k \neq k_m u_m(x)$ possesses an asymptotic behavior like $(kx)^{-3/2}$. Therefore, the diffracted field from one endpoint contains an asymptotically dominant term $v(x)$ of order $(kx)^{-1/2}$ plus three terms $u_m(x)(m = -1, 1, 0)$ of higher order $((kx)^{-3/2})$ associated with the FW's with propagation constants k_{xm}. These latter contributions become asymptotically dominant [of order $(kx)^0$] when k_{xm} approaches k; that is, when the pertinent FW approaches its cutoff from the homogeneous to the evanescent regime. The inclusion of a mathematical description of this transition, which is provided here by the UTD transition function, is important to preserve the solution when approaching scan-blindness angles.

We stress again the fact that the caustic point of the diffracted rays is displaced of one period from the first and the last source. Consequently, since the observation point is always inside $\sum$, the value of x in (11) is always large enough to justify the second-order asymptotic approximation in (12), which is sufficiently accurate for $x > 0.3\lambda$ as demonstrated in [8].

IV. Global Basis Functions and MoM Solution of the Fringe Integral Equation

For the sake of simplicity, the slit width is considered to be small in terms of a wavelength so that the magnetic current distribution on each slit can be described with only one basis function $c(x)$, which is chosen to be uniform, i.e., $c(x) = rect(x)$. The magnetic current of the infinite array has then the form

$$M_{\infty}(x) = \sum_{n=-\infty}^{\infty} M_{0\infty}c(x - nd)\,e^{-jk_{xs}nd} \quad (19)$$

where $M_{0\infty}$ is a constant which is found by applying a MoM procedure to (4). We consider now the unknown $M_d(x)$ of the fringe IE that physically represents the x-component of the electric field. By assuming this field as associated to a diffracted wave propagating toward x, the amplitude of this x-component must necessarily decay as $(kx)^{-3/2}$. For this reason, and taking inspiration from the representation (14) we expand $M_d(x)$ as

$$M_d(x) = \chi_A(x)\sum_{m=-1}^{1}[M^{+}_m u_m(x) + M^{-}_m u_m(x^*)] \quad (20)$$

where $x^* = (N+1)d - x$, i.e., in terms of the global basis functions $u_m(x)$ in (16) shaped as the second order of $h^{\pm}_{\text{ext}}(x)$. The six unknowns $M^{\pm}_m(m = -1,\ 0,\ 1)$, are found by applying a MoM-Galerkin method to the fringe IE (6). Note that the number of unknowns does not depend on the number of elements of the array. Furthermore, it will be seen from numerical examples that in most cases only three unknowns in place of six are sufficient to accurately solve the problem.

Before proceeding further, consider the familiar windowing method (WM) when applied as in [2] to the example discussed here. Since the present Green's function imposes a vanishing tangential (x-directed) electric field all over the plane of the slits, no *magnetic current* correction is predicted by WM with

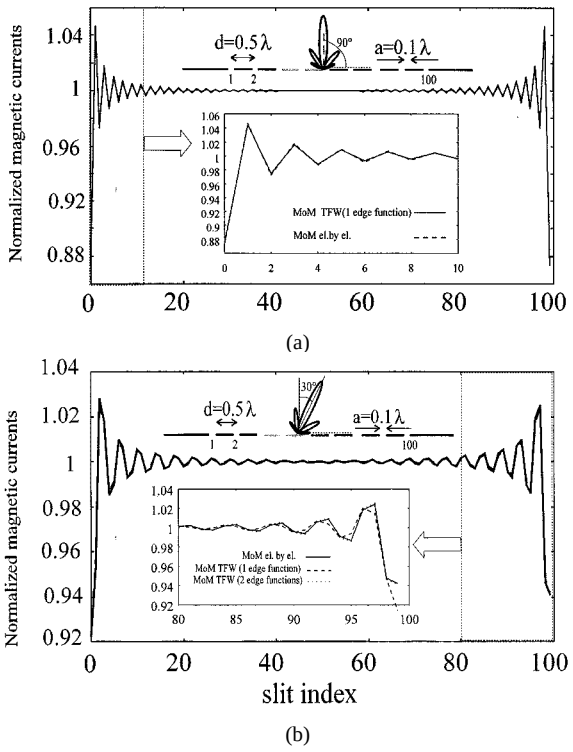

Fig. 3. Normalized amplitude of the magnetic currents versus the slit index for an array of 100 slits; $d = 0.5\lambda$, $a = 0.1\lambda$. (a) $\theta' = 90°$. (b) $\theta' = 30°$. Solid line—element-by-element solution; dashed line—TFW-MoM solution.

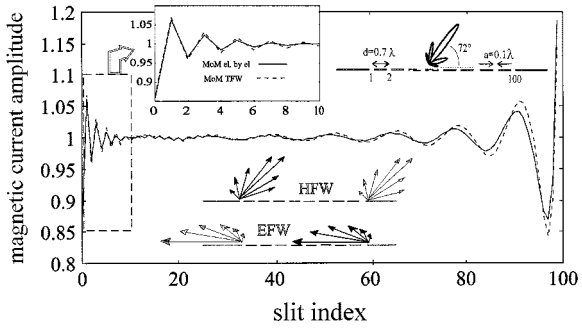

Fig. 4. Normalized amplitude of the magnetic current distribution for $d = 0.7\lambda$ and $a = 0.1\lambda$ against the slit index; the main beam is at $\theta = 18°$. This amplitude oscillates around the solution of the infinite array according to (9) and (10) for the left and right side, respectively. The most important diffraction contribution is that from the HFW for the end-point left and from the first EFW for the end-point right.

respect to the case of an infinite array. Consequently, the input impedance of each aperture calculated by WM differs from that for the infinite array only because of the perturbation of the *magnetic fields* on the slits without accounting for the perturbation of the *magnetic current*. This yields significant inaccuracy not only for this specific problem, but for all the arrays formed by apertures on ground plane. The problem is, however, less critical for the case of layered structures [3], where the pertinent Green's function enables WM to predict a first-order correction with respect to the infinite distribution for both the electric currents and the electric fields.

V. Illustrative Examples

The procedure described above (denoted hereinafter as truncated FW MoM or TFW-MoM) is compared with an ordinary formulation of the MoM which uses element by element unknowns. The various curves presented in the following show the amplitude of the normalized magnetic currents $M/M_{0\infty} = (M_d + M_{0\infty})/M_{0\infty}$ as a function of the slit index. The slits are small in terms of a wavelength $(a = 0.1\lambda)$, so that the global distribution is significantly described by only one sample per slit. A curve composed by straight segments is plotted to give continuity to the discrete values, dashed, and continuous line denoting our method and the ordinary MoM method. These curves oscillate slit by slit around the solution for the infinite array (which is unity due to the normalization) well depicting the effect of the interference between the FW fields and the relevant diffracted rays.

A. Selection of the Unknowns

An array composed of 100 slits periodically spaced $(d = 0.5\lambda)$ is considered in Fig. 3. We will denote by 1 and 2 the edges on the left and right sides, respectively. Fig. 3(a) and (b) are associated with beam angles $\theta = 0°$ and $\theta = 30°$, respectively. For the case in Fig. 3(a), the TFW solution has been obtained by using only one diffracted wave per edge in the expansion of M_d, i.e., in (21) M_1^+ and M_1^- are forced to zero and only M_0^+ and M_0^- are retained as unknowns. Thus, a 2×2 linear system has been solved to obtain the TFW solution, versus a 100×100 linear system pertinent to the element-by-element MoM approach. Despite this, excellent agreement has been found.

For the case of Fig. 3(b) $(\theta = 30°)$ a third curve is presented (dotted line), that corresponds to the TFW solution in which one more basis function is used for the edge at right, i.e., M_d is represented by three unknowns: M_0^+, M_0^-, M_{-1}^- in (20). This means we have selected one diffracted ray for the left edge (due to the HFW with $m = 0$) and two diffracted rays for the right edge [one due to HFW $(m = 0)$ and one due to first EFW $(m = -1)$]. This provides an excellent improvement of the TFW solution with respect to the already acceptable result obtained from including only one diffracted ray per edge (dashed line); indeed increasing the beam-scanning, the EFW gradually approaches its cutoff condition, thus rendering increasingly important the inclusion of its diffracted ray in the M_d expansion.

B. Change of Periodicity in the Current Oscillation

In Fig. 4, we consider an array of 100 elements with $d = 0.7\lambda$, $a = 0.1\lambda$, and $\theta = 18°$. For the present case, only one HFW occurs, and its radiation integral produces the main beam in direction θ. To obtain an accurate prediction of the TFW-MoM solution again three unknowns [M_0^+, M_0^-, M_{-1}^- in (20)] have been used. The curves oscillate slit by slit with a different period close to the different endpoints. The diffracted rays associated with the HFW and the dominant EFW pertinent to this geometry are those illustrated in Fig. 2 (reported in the inset for convenience). Following the TFW representation one can interpret the oscillations of the magnetic current amplitude as established by the interference between each FW aperture field and its corresponding diffracted ray, that propagates with the speed of light

along the same direction. In particular, the magnetic currents toward the left edge are dictated by the interference between the dominant HFW and its diffracted field arising from the endpoint left; the period of this oscillation is then

$$p_1 = \frac{2\pi}{|k - k_{xs}|} = \frac{\lambda}{(1 - \sin\theta)}. \tag{21}$$

This is the same expression provided by Hansen and Gammon in the last equation of [9] on the basis of the Gibbsian model. Note that this latter is based on a *spectral truncation* description of the oscillation while the present method invokes a *spatial truncation* and relevant diffraction. The precise relationship between the two methods is at present under investigation.

At the right side the oscillation is dictated by the interference between the EFW ($m = -1$) and its diffracted ray from the right-side endpoint, where the latter is stronger (see the inset of Fig. 4); the period of this oscillation is then

$$p_2 = \frac{2\pi}{|k - k_{x(-1)}|} = \frac{\lambda}{\left(-1 - \sin\theta + \dfrac{\lambda}{d}\right)}. \tag{22}$$

For the present case, where $d = 0.7\lambda$ and $\theta = 18°$, the $m = -1$ EFW is close to its cutoff, where it turns from evanescent to homogeneous, so that its phase velocity approaches the speed of light, i.e., the same as that of its diffracted ray. For this reason, the period of the oscillations at the right side is quite large. Note that for $d = 0.5\lambda$ (which corresponds to the cases of Fig. 3) one obtains $p_1 = p_2$, i.e., the same oscillations on both sides *for all values of* θ, which correspond to the case presented by Hansen and Gammon in [9]. Equation (22) is then the appropriate generalization to general spacing of the last equation in [9].

C. Special Cases

Fig. 5 shows results for a beam angle $\theta = 24°$, which is close to the scan-blindness angle $\theta = 26°$, which corresponds to the cut-off angle of the FW with $m = -1$. TFW-MoM solution maintains a good accuracy with three unknowns. As expected, for this case a relevant overshoot of the magnetic currents close to the right edge is found, which also produces an evident pattern distortion (see the inset of the same figure) with respect to the case of simple windowing of uniform amplitude currents , as expected.

The example shown in Fig. 6 demonstrates the good accuracy of the TFW-MoM method even for arrays of moderate sizes (10 elements, $d = 0.7\lambda$). Two different beam angles have been selected ($\theta = 10°$, and $\theta = 18°$) and two diffraction terms per edge have been used in the TFW solution. Nonnegligible distortion of the radiation pattern has been found also for the first side lobe (see the inset) with respect to the solution obtained with an abrupt windowing of the infinite array. The radiation pattern calculated from MoM element-by-element and from TFW-MoM superimpose, so that only one curve is reported (solid line).

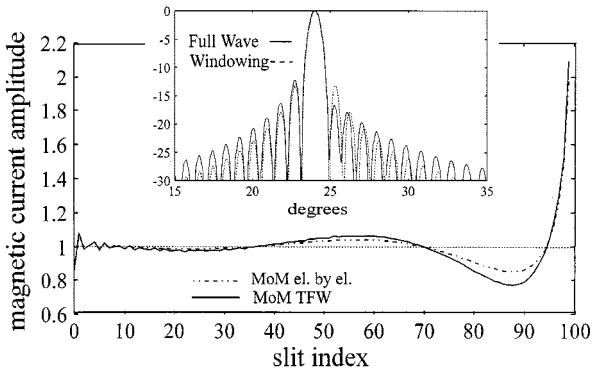

Fig. 5. Normalized amplitude of the magnetic current versus the slit index for an array of 100 slits close to scan-blindness condition ($d = 0.7\lambda$, $a = 0.1\lambda$, $\theta = 24°$). In the inset: radiation patterns.

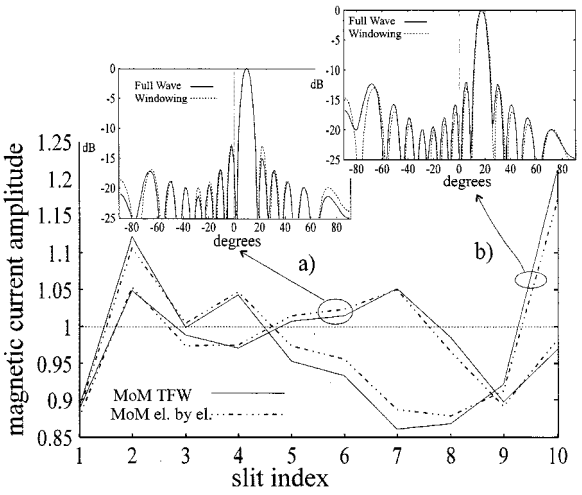

Fig. 6. Normalized amplitude of the magnetic current versus the slit index for an array of 10 slits ($d = 0.7\lambda$, $a = 0.1\lambda$); two diffracted-ray functions have been used for each edge. (a) $\theta = 10°$. (b) $\theta = 18°$. In the inset: radiation patterns.

VI. Concluding Remarks

A formulation has been proposed for the full wave solution of large phased arrays, which provides a drastic reduction of unknowns with respect to the ordinary MoM element-by-element approach. There are two key points of the method: the definition of a convenient fringe IE and the efficient representation of the unknown current in its kernel. The fringe IE is constructed as the difference between the IE pertinent to the actual finite array and to its infinite periodic continuation. This IE expresses the same field continuity as the original one, but applied to a field that is the difference between the currents of the finite and of the infinite array. The reason for dealing with such a quantity is that this field has an intrinsic diffractive nature, thus allowing a simple and efficient representation in terms of diffracted rays associated with truncated FW's. For a slit array on an infinite ground plane, the application of this representation leads to a very small number of terms—no more than three—independently of the array size. Indeed, the phenomenon is substantially described by the diffracted fields relevant to the propagating FW and to the evanescent FW closest to cutoff. The wave interference between the FW and its diffracted field provides an element-by-element current oscillation whose period can be predicted by a simple and intuitive expression also when the phenomenon is dominated at one edge by the EFW diffraction.

Despite the high-frequency nature of the field representation, the method has been demonstrated to be very accurate for edge elements and for moderate array sizes. The inclusion of the UTD transition function in the basis function expression allow for description of the currents also for scan-blindness angle. The guidelines and the physical insight gained in this 2-D analysis will be used for the generalization to actual 3-D-array problems; this generalization is carried out in Part II of this paper.

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