Correspondence

Comments on "Statistics of the Scattering Cross Section of a Small Number of Random Scatterers"

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***Index Terms*—Complex radar targets, electromagnetic propagation in random media, random scatters, scattering cross section.**

In [1], Daba and Bell have used a random vectors model to investigate the statistical behavior of the scattering cross section, when the number of scatterers is small. Specifically, they have considered the sum of n random vectors with lengths A_i's and angles Φ_i's. Using a phasor notation, the ith vector can be represented by $A_i \exp(j\Phi_i)$, where $j = \sqrt{-1}$. Thus, the abovementioned vector sum can be written as $\sum_{i=1}^{n} A_i \exp(j\Phi_i)$. In [1], n is a deterministic variable, A_i's are positive deterministic variables and Φ_i's are independent random variables, distributed uniformly on $[0, 2\pi)$. Based on the above assumptions, Daba and Bell have derived closed-form expressions for the probability density function (PDF) of $S_n = |\sum_{i=1}^{n} A_i \exp(j\Phi_i)|^2$, defined in [1, eq. (2)], using two different approaches. In [1, sec. III-A] and for different $A_i s$, a recursive method is developed; while in [1, sec. III-B] and for equal A_i's, an orthonormal Laguerre polynomial representation is used. However, the following issues concerning these two methods should be mentioned.

1) It seems that the recursive method described in [1] has been used previously in [2], for solving exactly the same mathematical problem, i.e. the PDF of S_n.
2) Laguerre series expansion for the PDF of S_n presented in [1] holds not only for positive identical constants A_i's, but also is applicable to the general case in which A_i's are dependent positive random variables [3], [4].
3) In [1], the coefficients of the Laguerre series expansion for the PDF of S_n are determined via Monte-Carlo simulations, assuming positive identical constants A_i's. However, it can be shown that under the more general condition of dependent positive random A_i's, these coefficients can be computed exactly, via closed-form formulas [3], [4].
4) In [1], by the Kolmogorov–Smirnov nonparametric test, it is only confirmed that a Laguerre series expansion, with a finite number of terms, can closely approximate the PDF of S_n. However, to manipulate a series with infinite number of terms efficiently, it is important to have a quantitative relationship between the truncation error and the number of terms used. This important issue in not addressed in [1]. For a similar random vector problem, this subject is discussed completely in [3] and necessary relationships are derived between the truncation error and the number of terms used. These results are partly reported in [5] and [6].

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Authors' Reply

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***Index Terms*—Random scattering, scattering cross section.**

I. Introduction

Abdi and Nader-Esfahani raise four concerns with respect to our paper [1].

Abdi and Nader-Esfahani point out that we did not reference Simon [2] for the part of the paper presenting the recursive method for finding the density function of the scattering cross section. However, we did cite two earlier references that make use of this technique [3], [4]—the first of the same mathematical problem (random walks in the plane) and the second specifically related to the problem of radar scattering. We agree that [2] would have been an excellent additional reference on a related problem in communications.

They further point out that our Laguarre series expansion holds not only for positive identical scattering amplitudes A_i, but also for the case where the A_j are dependent random variables. A careful reading of our paper will show that we did not assume the A_j were identical. We only assumed they took on fixed (nonrandom) values $A_1, \cdots, A_k$. We only used identical values in the examples presented for ease of illustration. In [5], the results are shown to hold for A_j that are independent identically distributed (i.i.d.) random variables. We did not show that it held for dependent sequences of random variables, but because the Gram–Charlier technique hinges on the fact that the real and imaginary parts of $\sum_{j=1}^{k} A_j e^{i\phi_j}$ are asymptotically Gaussian, we would expect the result to hold for any random sequence $\{A_j\}$ satisfying the α-*mixing* property or the conditions of the Lindeberg–Feller form of the central limit theorem [6, sec. 27]. Still, we thank Abdi and Nader-Es-

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fahani for pointing out this generalization as dependent sequences did not arise in the problem we were modeling, so we did not pursue this direction.

Abdi and Nader-Esfahani report that since the publication of our paper, they have derived closed-form solutions for the Laguerre expansion coefficients. Having not seen the work, we cannot comment on it, but this is a significant accomplishment and we look forward to its publication [7].

Abdi and Nader-Esfahani point out that we determined the closeness of fit using the Kolmogorov–Smirnov distance. This is correct, as it is the most appropriate measure of goodness of fit of the observed data to the statistical distributions of the scatterers. However, if the goal is to manipulate infinite series (e.g., differentiate them), we agree that expressions for the truncation error as a function of the number of terms may be valuable. Abdi and Nader-Esfahani have apparently derived expressions for the truncation error. Having not seen these expressions, we cannot comment on them. We would like to point out, however, that in using these distributions for statistical inference, Kolmogorov–Smirnov distances can be used to compute the probability that the observed data did not come from the distribution in question. This cannot be done with expressions for truncation error.

It appears that Abdi and Nader-Esfahani have significantly extended our work. We look forward to seeing its publication [7].

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