Errata

Corrections to "Asymptotic Solutions for the Scattered Fields of Plane Wave by a Cylindrical Obstacle Buried in a Dielectric Half-Space"

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***Abstract*—The purpose of this communication is to point out multiple errors in a previously published work and to present the erroneous parts of the work in an error free form. The typographical errors cause serious problems in the understanding of this interesting and stimulating work. It is also observed that the plots given in the published paper also contain errors. Some accurate plots are given for the same data as given in the previously published work. The plots given in this communication agree very well with the corresponding method of moments results. Efforts are also made to extend previous work by evaluating the far-zone scattered field in the lower dielectric half-space.**

***Index Terms*—Asymptotic diffraction theory, buried object detection, cylindrical scatterers.**

I. Introduction

The problem of scattering of electromagnetic waves from an obstacle buried in dielectric half-space geometry has been undertaken by many authors. The work by Hongo and Hamamura[1] is most interesting and stimulating among all of these. Unfortunately, there are many typographical errors in the published paper. Some of these typographical errors cause serious problems in the understanding of their work. In this communication, their work is presented in an error-free form so that interested readers can easily understand Hongo and Hamamura's important work. Efforts are also made to extend their work to calculate the far-zone scattered field expression in region below the buried obstacle. For convenience, equations and figures in this communication are labeled with same equation and figure numbers as given in work by Hongo and Hamamura and only erroneous parts of the paper are discussed.

It is important to note that, field plots given in that paper[1] cannot be reproduced by either using erroneous far-zone field expression which is given in the published paper or far-zone field expression given in this communication. Therefore, it is concluded that the field plots given in the paper[1] also contain errors. Far-zone field expression is plotted for the same data as given in the paper[1] for the case of a perfectly conducting circular cylinder using the field expression given in this communication.

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[1]K. Hongo and A. Hamamura, *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 1306–1312, Nov. 1986.

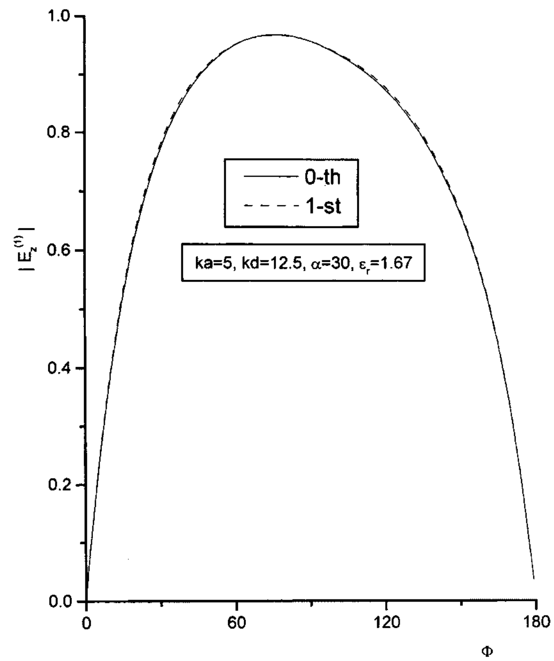

Fig. 3. Comparison between the zeroth and the first-order approximation.

II. Scattering of a Plane Wave by a Strip Embedded in a Dielectric Half-Space

A. Formulation

Referring to the geometry of scattering problem as illustrated in Fig. 1[1], the incident, reflected, and the transmitted waves are expressed as

$$E_z^i = \exp[jk_o(x\cos\alpha + y\sin\alpha)] \tag{2a}$$
$$E_z^r = R_o\exp[jk_o(x\cos\alpha - y\sin\alpha)] \tag{2b}$$
$$E_z^t = T_o\exp[jk(x\cos\beta + y\sin\beta)] \tag{2c}$$

where R_0 and T_0 are the Fresnel reflection and transmission coefficients, respectively.

B. Imposition of the Boundary Condition

The scattered fields generated by a strip in different regions are expressed by a superposition of plane waves spectrum in (4)[1]. The continuity of E_z and H_x at $y = 0$, and the continuity of E_z at $y = -d$ yields

$$f_{\substack{e1\\o1}}(\xi) = f_{\substack{e2\\o2}}(\xi) + f_{\substack{e3\\o3}}(\xi) \tag{6a}$$
$$-\eta_o f_{\substack{e1\\o1}}(\xi) = \eta\left\{f_{\substack{e2\\o2}}(\xi) - f_{\substack{e3\\o3}}(\xi)\right\} \tag{6b}$$
$$f_{\substack{e2\\o2}}(\xi)\exp\left(-\eta\frac{d}{a}\right) + f_{\substack{e3\\o3}}(\xi)\exp\left(\eta\frac{d}{a}\right) = f_{\substack{e4\\o4}}(\xi). \tag{6c}$$

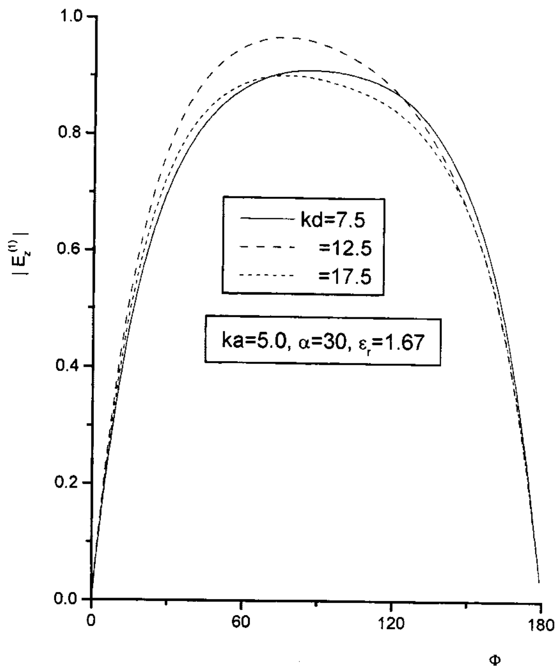

Fig. 5. (a) The effects of angle of incidence on the diffraction pattern when E-polarized plane wave is incident from the free-space.

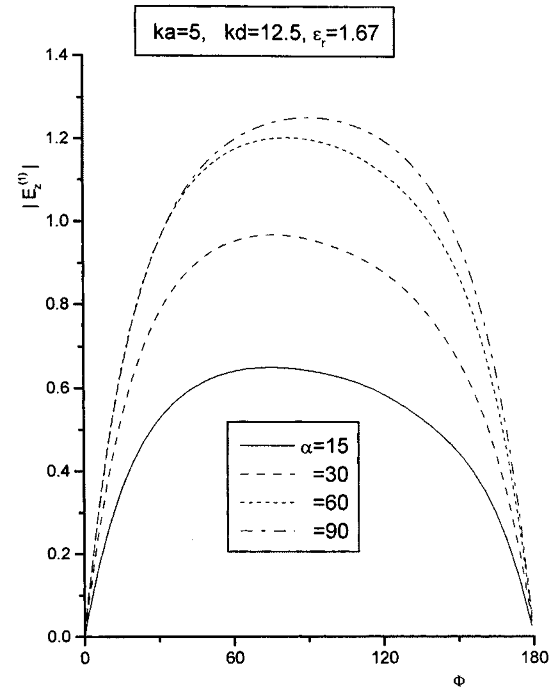

Fig. 5. (b) The effects of separation between the circular cylinder and the boundary of the dielectric half-space on the diffraction pattern when E-polarized plane wave is incident from the free-space.

Similarly, from the continuity of H_x for $|x| \geq a$ and the condition $E_z + E_z^t = 0$ for $|x| \leq a$ in the plane $y = -d$, the following relations are obtained:

$$\sqrt{\frac{\pi x}{2a}} \int_0^\infty \frac{\eta}{a} \left\{ f_{\substack{e2\\o2}}(\xi) \exp\left(-\eta \frac{d}{a}\right) - f_{\substack{e3\\o3}}(\xi) \exp\left(\eta \frac{d}{a}\right) - f_{\substack{e4\\o4}}(\xi) \right\} \times J_{(\mp)1/2}\left(\frac{x\xi}{a}\right) \sqrt{\xi}\, d\xi = 0, \quad \text{for } |x| \geq a \tag{7a}$$

$$\sqrt{\frac{\pi x}{2a}} \int_0^\infty \left\{ f_{e4}(\xi) J_{-1/2}\left(\frac{x\xi}{a}\right) + f_{o4}(\xi) J_{1/2}\left(\frac{x\xi}{a}\right) \right\} \sqrt{\xi}\, d\xi = -T_0 \exp[-jkd \sin\beta + jkx\cos\beta], \quad \text{for } |x| \leq a. \tag{7b}$$

In order to reduce dual integral equations (6) and (7) to a matrix equation, following is the accurate representation of spectrum functions in terms of unknown expansion coefficients:

$$f_{\substack{e2\\o2}}(\xi) \exp\left(-\eta \frac{d}{a}\right) - f_{\substack{e4\\o4}}(\xi) = \exp(-jkd\sin\beta) \sum_{m=0}^{m=\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi). \tag{8}$$

The unknown functions $f_{ei}(\xi)$ and $f_{oi}(\xi)$ in terms of expansion coefficients A_{em} and A_{om} are obtained as

$$f_{\substack{e1\\o1}}(\xi) = \frac{-2\eta}{\eta + \eta_0} \exp\left(-\eta\frac{d}{a} - jkd\sin\beta\right) \times \sum_{m=0}^{\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi)$$

$$f_{\substack{e2\\o2}}(\xi) = -\frac{\eta - \eta_0}{\eta + \eta_0} \exp\left(-\eta\frac{d}{a} - jkd\sin\beta\right) \times \sum_{m=0}^{\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi)$$

$$f_{\substack{e3\\o3}}(\xi) = -\exp\left(-\eta\frac{d}{a} - jkd\sin\beta\right) \times \sum_{m=0}^{\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi)$$

$$f_{\substack{e4\\o4}}(\xi) = -\left[1 + \frac{\eta - \eta_0}{\eta + \eta_0} \exp\left(-2\eta\frac{d}{a}\right)\right] \times \exp(-jkd\sin\beta) \sum_{m=0}^{\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi).$$

It is obvious after observing the unknown functions $f_{\substack{e2\\o2}}(\xi), f_{\substack{e3\\o3}}(\xi)$ and $f_{\substack{e4\\o4}}(\xi)$ that the scattered field expression for the half-space $y < 0$ may be written as a combination of two terms as given in

$$E_z = E_z^c + E_z^{ci} \tag{8a}$$

where

$$E_z^c = -\sqrt{\frac{\pi x}{2a}} \int_0^\infty \exp(-jkd\sin\beta) \times J_{-1/2}\left(\frac{x\xi}{a}\right) \sum_{m=0}^{\infty} \frac{1}{\eta} A_{em} J_{em}(\xi) \times \exp\left\{\pm(y+d)\frac{\eta}{a}\right\} \sqrt{\xi}\, d\xi + \text{CO}$$

$$E_z^{ci} = -\sqrt{\frac{\pi x}{2a}} \int_0^\infty \frac{\eta - \eta_0}{\eta + \eta_0} \exp\left(-\eta\frac{2d}{a} - jkd\sin\beta\right) \times J_{-1/2}\left(\frac{x\xi}{a}\right) \sum_{m=0}^{\infty} \frac{1}{\eta} A_{em} J_{em}(\xi) \times \exp\left\{(y+d)\frac{\eta}{a}\right\} \sqrt{\xi}\, d\xi + \text{CO}.$$

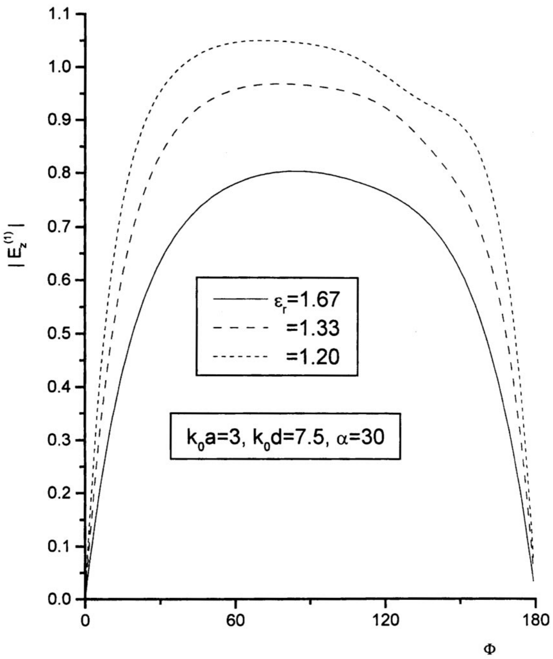

Fig. 5. (c) The effects of dielectric constant of the medium on the diffraction pattern when E-polarized plane wave is incident from the free-space.

First term E_z^c in (8a) corresponds to the field scattered directly by the obstacle in the half-space $y < 0$. Last term E_z^{ci} in the expression (8a) corresponds to that part of the scattered field which after reflection from the dielectric interface $y = 0$ contributes to the total scattered field in the region $y < 0$.

C. Approximate Solution for the Expansion Coefficients

Matrix equation (11) in the paper[1] is written in an error-free form

$$\begin{aligned}[G_e][A_{em}] &= T_o[J_{en}] - [M_e][A_{em}] \\ [G_o][A_{om}] &= jT_o[J_{on}] - [M_o][A_{om}] \end{aligned} \tag{11}$$

where $[A_{em}], [A_{om}], [J_{en}]$ and $[J_{on}]$ are the column matrices with elements $A_{em}, A_{om}, J_{2n}(\kappa \cos\beta)$ and $J_{2n+1}(\kappa\cos\beta)$, respectively. Matrices $[G_e], [G_0], [M_e]$, and $[M_o]$ have dimensions $G(2n, 2m)$, $G(2n+1, 2m+1), M(2n, 2m)$, and $M(2n+1, 2m+1)$, respectively. Using the saddle point method of integration, the asymptotic solution of elements of matrix $[M(m,n)]$ is

$$M(m,n) = C(2kd)P_d(D)Q\left(\frac{\pi}{2}\right) \tag{13a}$$

where

$$Q(\theta) = J_m(\kappa\cos\theta)J_n(\kappa\cos\theta) \times \frac{\kappa\sin\theta - \sqrt{\kappa_0^2 - \kappa^2\cos^2\theta}}{\kappa\sin\theta + \sqrt{\kappa_0^2 - \kappa^2\cos^2\theta}} \tag{13b}$$

$$P_d(D) = \sum_{m=0}^{m=\infty} \frac{(1+4D^2)(9+4D^2)\ldots([2m-1]^2+4D^2)}{(-j8kd)^m m!} \tag{13c}$$

$$C(x) = \sqrt{\frac{\pi}{2x}}\exp\left[-j\left(x+\frac{\pi}{4}\right)\right]. \tag{13d}$$

It is important to note that the matrix $[M(n,m)]$ may be written as

$$[M] = C(2kd)P_d(D)R(\theta)[J_n(\kappa\cos\theta)][J_m(\kappa\cos\theta)]^T.$$

Applying the iterative method to (12) in[1], the zeroth-order approximation solutions $A_{em}^{(0)}$ and $A_{om}^{(0)}$, and the first-order correction $A_{em}^{(1)}$ and $A_{om}^{(1)}$ in which terms proportional to $(kd)^{-1/2}$ are retained are given by

$$\begin{aligned}\left[A^{(o)}_{\substack{em\\om}}\right] &= T_o\delta_{\substack{en\\on}}\left[G_{\substack{e\\o}}\right]^{-1}\left[J_{\substack{en\\on}}\right] \\ \left[A^{(1)}_{\substack{em\\om}}\right] &= -\left[G_{\substack{e\\o}}\right]^{-1}\left[M_{\substack{e\\o}}\right]\left[A^{(0)}_{\substack{em\\om}}\right]\Big|_{\theta=(\pi/2)}.\end{aligned} \tag{15a}$$

Substituting the values of $[M_{\substack{e\\o}}]$ and $[A^{(o)}_{\substack{em\\om}}]$ and then taking the transpose of both sides of above expression yields the following:

$$\begin{aligned}\left[A^{(1)}_{\substack{em\\om}}\right]^T &= -T_o\delta_{\substack{en\\on}}C(2kd)P_d(D)R(\theta)f_{\substack{e\\o}}(\beta,\theta) \\ &\quad\times\left[J_{\substack{en\\on}}(\kappa\cos\theta)\right]^T\left[G_{\substack{e\\o}}\right]^{-1}\Big|_{\theta=(\pi/2)}.\end{aligned} \tag{15b}$$

D. Far-Field Expression for the Scattered Field

A far-field expression for the scattered field in region I can be obtained from (4a) by applying the saddle-point method of integration. Substituting (15) for A_{em} and A_{om} in the far-field expression for the scattered yields

$$\begin{aligned}E_z^{(I)} &= C(k_0\rho)T(\phi)T_0[f(\phi',\beta) \\ &\quad - C(2kd)P_d(D)R(\theta)f(\theta,\phi')f(\beta,\theta)|_{\theta=(\pi/2)}]\end{aligned} \tag{16a}$$

where

$$\begin{aligned}T(\phi) &= \frac{-2\kappa_o\sin\phi}{\kappa_o\sin\phi + \sqrt{\kappa^2-\kappa_o^2\cos^2\phi}} \\ &\quad\times\exp\left\{-j\sqrt{\kappa^2-\kappa_0^2\cos^2\phi}\frac{d}{a} - j\kappa\frac{d}{a}\sin\beta\right\}.\end{aligned} \tag{16b}$$

In the above equations, $f(\phi',\beta)$ represents the plane wave pattern function of the diffracted field by an isolated strip in a homogeneous medium with dielectric constant k. It is important to note that Fig. 2[1] deals with the physical interpretation of zeroth-order approximate solution and first-order approximate solution for the scattered field.

Field scattered by the buried cylindrical obstacle in a region $y < 0$ is given by (8a) of this communication. Using the steepest decent method of integration and substituting the approximate results of expansion coefficients, the dominant contribution to the far-zone by the buried obstacle in region $y < -d$ is obtained as

$$\begin{aligned}E_z^c &= C(k\rho)T_0\exp(-jkd\sin\beta)\exp(-jkd\sin\phi)[f(\phi,\beta) \\ &\quad - C(2kd)P_d(D)R(\theta)f(\theta,\phi)f(\beta,\theta)|_{\theta=(\pi/2)}] \\ E_z^{ci} &= \exp(-jkd\sin\beta)C(k\rho)T_0R(-\phi)\exp(jkd\sin\phi)[f(-\phi,\beta) \\ &\quad - C(2kd)P_d(D)R(\theta)f(\theta,-\phi)f(\beta,\theta)|_{\theta=(\pi/2)}].\end{aligned} \tag{17}$$

III. Numerical Results and Discussion

The pattern function of a perfectly conducting cylinder of radius a, when it is excited by a plane wave at certain incident angle θ_1, is given as

$$f(\theta_2,\theta_1) = \frac{-2j}{\pi}\sum_{n=-\infty}^{n=\infty}\frac{J_n(ka)}{H_n^{(2)}(ka)}\exp\{jn(\theta_2-\theta_1+\pi)\}.$$

The circular cylinder is placed in a homogeneous medium having propagation constant k.

In Fig. 3, comparison between the zeroth order approximation and the first order approximation (both with same definitions as given in the paper[1]) is made for the case of a perfectly conducting circular cylinder. It is seen from this comparison that contribution to scattered field of term with order $(kd)^{-3/2}$ is negligibly small. So the contribution of this term may be neglected to calculate the far-zone scattered field.

Some accurate scattered field patterns for the case of a perfectly conducting circular cylinder are given using the same data as given in the paper[1]. It may be noted that the results obtained in this communication are in good agreement with the results obtained by Naqvi [1] using method of moments.

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[1] Q. A. Naqvi, "Scattering of Electromagnetic Waves from a Buried Cylinder," Ph.D. Thesis, Quaid-i-Azam University, Islamabad, Pakistan, 1997.