Higher Order Impedance Boundary Conditions for Sparse Wire Grids

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***Abstract*—Higher order impedance boundary conditions designed for modeling wire grids of thin conducting wires are established. The derivation is based on the exact analytical summation of the individual wire fields. This allows to write approximate boundary condition on the grid surface, which connects the averaged electric field and the averaged current (or the electric field and the averaged magnetic fields on the two sides of the grid surface). The condition depends on the tangential derivatives of the averaged current (up to the sixth order). This approach provides an extension of the averaged boundary conditions method (well established for dense grids) to sparse grids. Numerical examples demonstrate very good accuracy of the solutions for the field reflected from grids with the wire separation as large as half of the wavelength.**

***Index Terms*—Impedance boundary conditions, wire grids.**

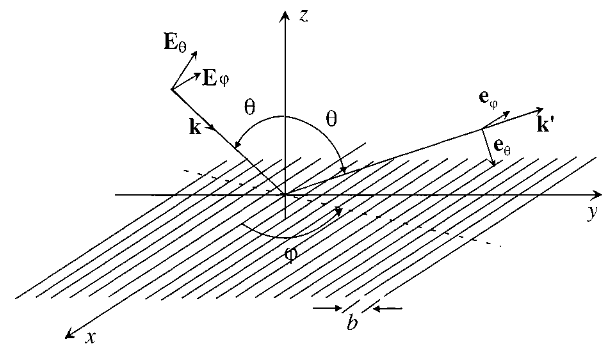

Fig. 1. The geometry of the problem.

I. INTRODUCTION

IT is well known that *approximate boundary conditions* is a very useful tool that can greatly facilitate solutions of various electromagnetic problems. Of course, certain simplifying assumptions about the system are made. This can be, for instance, the assumption of a high contrast of the electromagnetic properties of two media with a common interface. Under this assumption, the Leontovich boundary conditions for a smooth interface between two media can be established [1]. Similarly, electrically thin slabs of various media or layers of high-contrast materials can be modeled by various sheet or impedance conditions [2]–[5]. The other common assumption is that the structure is periodical with the period much smaller than the wavelength. In this case, various *homogenization* approaches are possible, leading to the concept of the surface impedance which connects the averaged fields [6]–[12]. Such models are known for grids of wires [6]–[9] or strips [10]–[12]. In many cases the solution (approximate or even exact) can be easily found for plane wave excitation; that is, in the Fourier domain, e.g., [12]. The main problem is to find the boundary conditions in the physical space so that more complicated excitation or nonplanar structures can be modeled. In this paper, we consider the classical problem of the array of parallel conducting wires. We establish approximate impedance boundary conditions for sparse grids in the physical domain and show that the homogenization is possible until there are no grating lobes. However, to model sparse grids higher order impedance boundary conditions are needed.

Electromagnetic properties of wire grids have been studied for many decades because of their many important technical applications. For grids of parallel thin wires excited by plane waves the exact solution for the wire currents can be easily written in terms of a series of the Hankel functions. This series converges rather slowly, but the convergence can be improved using the Poisson summation rule and some other means, as shown in [13]–[15]. However, the result is still expressed in form of series of nonrational functions of the wave vector and other parameters and can be directly applied only to infinite grids excited by plane waves. For more practical problems, effective boundary conditions are needed which would model the grid properties in the physical domain, not only for Fourier-transformed fields and currents.

Such boundary conditions, so called *averaged boundary conditions*, are known for dense grids [8], [9] (separation between wires is much smaller than the wavelength). These conditions are second-order impedance boundary conditions which can be used under two assumptions: 1) the grid period is much smaller than the wavelength and 2) the wire radius is much smaller than the grid period. In the theory of the averaged boundary conditions the local field exciting a reference wire is approximately calculated using the Euler–MacLaurin summation formula. In order to improve the accuracy of this summation for grids with somewhat larger distances between wires, an improved Euler–MacLaurin formula was suggested in [16] (but no improved boundary condition was given). In the present approach our goal is to find effective boundary conditions that can be used also for sparse grids when the distance between wires becomes comparable with the wavelength. In this study, we start from the exact solution of the corresponding diffraction problem in the Fourier-transformed domain (for plane waves) and then find approximate impedance boundary conditions making the

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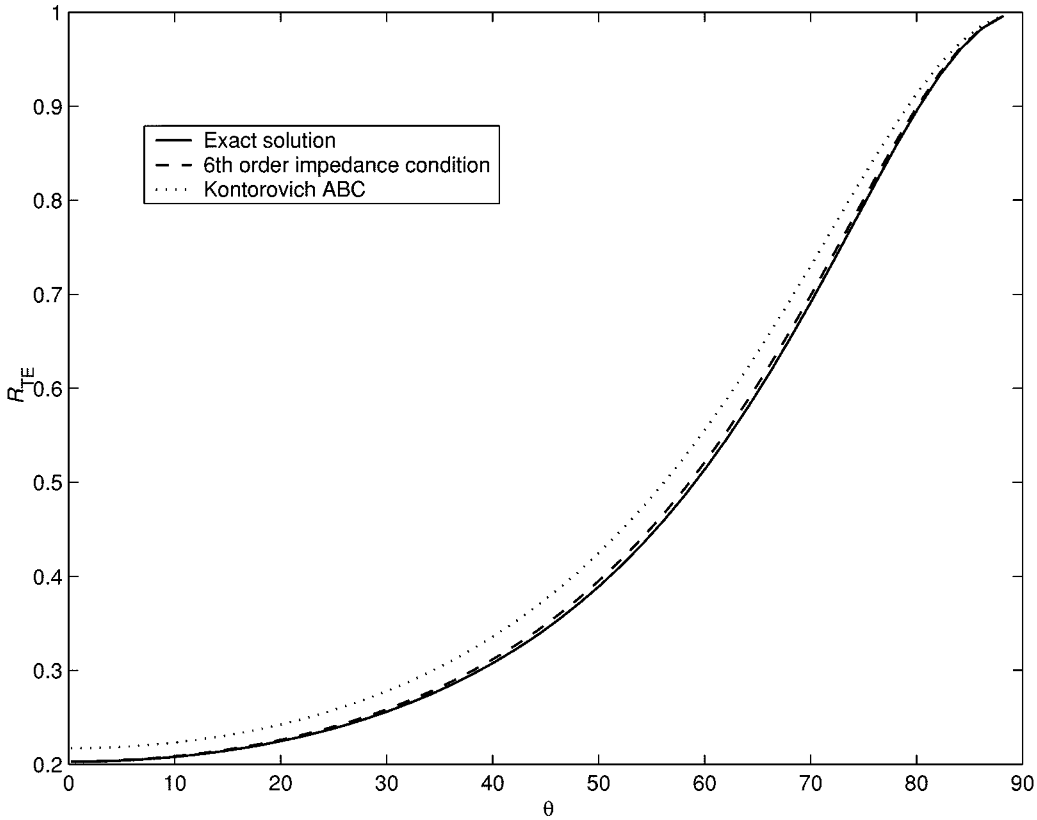

Fig. 2. Absolute value of the copolarized reflection coefficient for TE-polarized incident wave.

appropriate Taylor expansion of the exact result before transforming the fields to the physical space. This way we establish a method that allows to derive higher order boundary conditions of any order. We have explicitly found such conditions with the second-, fourth-, and sixth-order tangential derivatives of the fields. In the second order, our method gives, naturally, the same result as the known approach by Kontorovich [8], [9].

II. Theory

Consider a periodic grid of parallel conducting wires (Fig. 1). The wires can be ideally conducting or lossy wires whose impedance per unit length is Z. Wire radius is r_0, the separation between the wires is b. The grid is excited by a plane wave

$$\mathbf{E}^{\text{ext}} = \mathbf{E}e^{-j\mathbf{k}\cdot\mathbf{r}} \tag{1}$$

so that the currents I_n in the wires depend on the coordinates as

$$I_n = Ie^{-jk_x x}e^{-jk_y y_n} \tag{2}$$

where $y_n = bn$ is the y coordinate of the nth wire. The incidence and reflection angles are equal to θ (the normal incidence corresponds to $\theta = 0$), the azimuthal angle is φ. The components of the wave vector $\mathbf{k}$ of the incident field can be written as

$$\begin{aligned} &k_x = k\sin\theta\cos\varphi, \quad k_y = k\sin\theta\sin\varphi \\ &k_z = -k\cos\theta. \end{aligned} \tag{3}$$

The amplitude of the incident electric field $\mathbf{E}$ can be represented by its components $\mathbf{E}_\theta$ and $\mathbf{E}_\varphi$ connected with the Cartesian components as following (see Fig. 1):

$$\begin{aligned} &\mathbf{E}_\theta = E_x\cos\theta\cos\varphi\mathbf{x}_0 + E_y\cos\theta\sin\varphi\mathbf{y}_0 + E_z\sin\theta\mathbf{z}_0, \\ &\mathbf{E}_\varphi = -E_x\sin\varphi\mathbf{x}_0 + E_y\cos\varphi\mathbf{y}_0. \end{aligned} \tag{4}$$

The suppressed time dependence is of the form $e^{j\omega t}$.

A. Summation of the Wire Fields

Each wire (number n) creates electric field whose component in the direction along the wires (x-direction) is (see, e.g., [17, p. 492])

$$E_{xn}^w = -\frac{\eta}{4k}\left(k^2 - k_x^2\right)I_n H_0^{(2)}\left(\sqrt{k^2 - k_x^2}r_n\right). \tag{5}$$

Here $r_n = \sqrt{y_n^2 + z^2}$, $k = \omega\sqrt{\epsilon_0\mu_0}$ is the wave number, $\eta = \sqrt{\mu_0/\epsilon_0}$ is the wave impedance, and $H_0^{(2)}$ is the Hankel function. As is usually done, we have replaced the wire field by the field of an infinitely thin current line along the wire axis. In this theory, we assume that the wire radius is small compared to the wavelength $(kr_0 \ll 1)$ and it is much smaller than the distance between the wires $(r_0 \ll b)$. To solve for the current in wires we need to know the local field, which acts on a wire surface. This field can be expressed as

$$\begin{aligned} E_x^{\text{loc}} = E_x e^{-jk_x x} - \frac{\eta}{2k}\left(k^2 - k_x^2\right)Ie^{-jk_x x} \\ \times \sum_{n=1}^{\infty}\cos(k_y bn)H_0^{(2)}\left(\sqrt{k^2 - k_x^2}bn\right). \end{aligned} \tag{6}$$

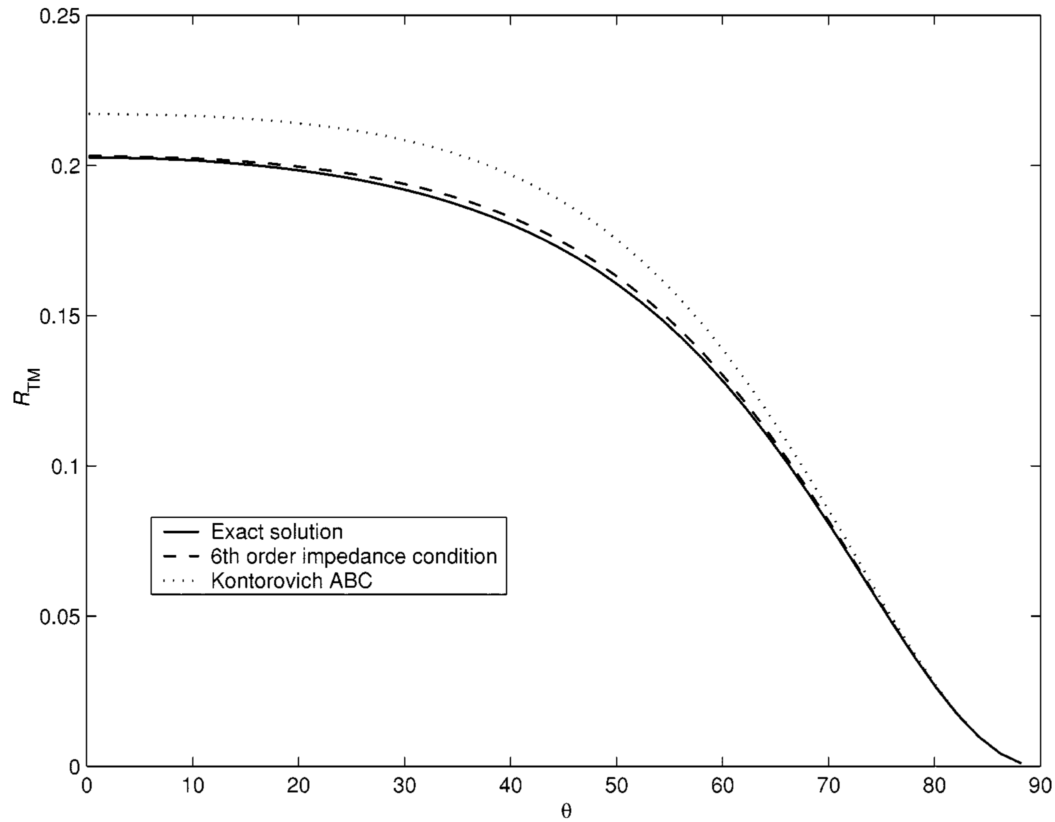

Fig. 3. Absolute value of the copolarized reflection coefficient for TM-polarized incident wave.

The sum of the Hankel functions can be calculated using ([18, 8.521.1, 8.522.3]):

$$\sum_{n=1}^{\infty}\cos(k_y bn)H_0^{(2)}\left(\sqrt{k^2-k_x^2}bn\right) = \frac{1}{|k_z|b}-\frac{1}{2}+\frac{j}{\pi}\left[\log\frac{\sqrt{k^2-k_x^2}b}{4\pi}+C+\frac{1}{2}\sum_{n=-\infty}^{\infty}{}' \times\left(\frac{2\pi}{\sqrt{(2\pi n+k_y b)^2-(k^2-k_x^2)b^2}}-\frac{1}{|n|}\right)\right]. \quad (7)$$

Here $C \approx 0.5772$ is the Euler constant ([18, 9.73]). The prime in the sum denotes that the summation is made over all n except $n = 0$. This is the known result that leads to the exact (valid for thin wires) solution of the plane wave diffraction problem (see, e.g., [13]–[15]).

Now we can write the boundary condition on the surface of one of the wires (wire $n = 0$)

$$E_x^{\mathrm{loc}} + E_{x0}^{w} = ZIe^{-jk_x x}. \quad (8)$$

Here E_{x0}^{w} is the field created by the reference wire current at that wire surface

$$E_{x0}^{w} = -\frac{\eta}{4k}\left(k^2-k_x^2\right)Ie^{-jk_x x}H_0^{(2)}\left(\sqrt{k^2-k_x^2}r_0\right). \quad (9)$$

Under the assumption that $kr_0 \ll 1$, we replace the Hankel function by its asymptotic expression for small arguments

$$H_0^{(2)}\left(\sqrt{k^2-k_x^2}\,r_0\right) \approx 1-j\frac{2}{\pi}\left(\log\frac{\sqrt{k^2-k_x^2}r_0}{2}+C\right) \quad (10)$$

which leads to

$$E_x - \frac{\eta}{2k}(k^2-k_x^2)I\left\{\frac{1}{|k_z|b}+\frac{j}{\pi}\left[\log\frac{b}{2\pi r_0}+\frac{1}{2}\sum_{n=-\infty}^{\infty}{}' \times\left(\frac{2\pi}{\sqrt{(2\pi n+k_y b)^2-(k^2-k_x^2)b^2}}-\frac{1}{|n|}\right)\right]\right\} = ZI. \quad (11)$$

From the last relation, we determine the wire current (it is convenient to introduce surface current density J connected to the current I as $J = I/b$)

$$J = \frac{2}{\eta}\frac{\cos\theta E_x}{(1-\sin^2\theta\cos^2\varphi)(1+j\alpha\cos\theta)+(2/\eta)Zb\cos\theta} \quad (12)$$

where α is called *grid parameter*

$$\alpha = \frac{kb}{\pi}\left[\log\frac{b}{2\pi r_0}+\frac{1}{2}\sum_{n=-\infty}^{\infty}{}' \times\left(\frac{2\pi}{\sqrt{(2\pi n+k_y b)^2-(k^2-k_x^2)b^2}}-\frac{1}{|n|}\right)\right]. \quad (13)$$

Note that the sum in (13) converges very quickly. For dense grids such that $k_y b \ll 2\pi$ and $\sqrt{k^2-k_x^2}b \ll 2\pi$ it gives a very

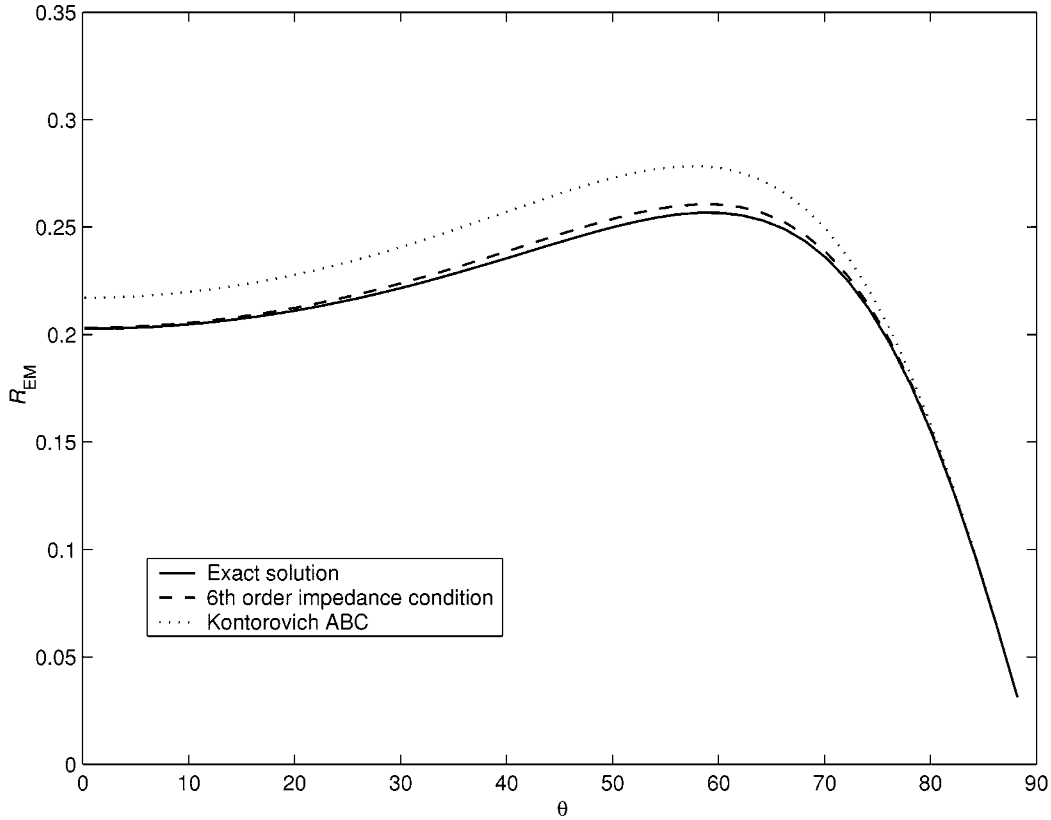

Fig. 4. Absolute value of the cross-polarized reflection coefficient.

small correction and can be neglected. In that case, the averaged current density is given by (12) with α replaced by $\alpha_{\rm ABC}$

$$\alpha_{\rm ABC} = \frac{kb}{\pi}\left(\log\frac{b}{2\pi r_0}\right). \qquad (14)$$

This is a well-known result found by Kontorovich [8], [9] with a different approach (Kontorovich averaged boundary condition approach). As we see, the result of the exact summation differs from the approximate Kontorovich boundary condition by an additional member in the expression for the grid parameter.

B. Boundary Condition

Relation (12) connects the averaged induced current with the incident plane wave field. To establish a relation between the induced current and the incident field for arbitrary incident fields we should transform (12) to the physical space. For arbitrary fields in the physical domain $k_x^2 \rightarrow -\partial^2/\partial x^2$ and $k_y^2 \rightarrow -\partial^2/\partial y^2$. Obviously, to arrive to effective boundary conditions we should find a rational approximation for the grid parameter (13). This can be done using the Taylor expansion of the series in (13). To make the expansion, let us introduce function $f(s,t)$ as the following:

$$f(s,t) = \sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{(n+s)^2-t}} - \frac{1}{n}\right) \qquad (15)$$

where $t = (1-(k_x^2/k^2))(b/\lambda)^2$ and $s = (k_y/k)(b/\lambda)$. Making the Taylor expansion of the sum in (13) we obtain

$$\begin{aligned}
&\frac{1}{2}\sum_{n=-\infty}^{\infty}{}'\left(\frac{2\pi}{\sqrt{(2\pi n + k_y b)^2 - (k^2-k_x^2)b^2}} - \frac{1}{|n|}\right)\\
&\quad = \frac{1}{2}\sum_{n=-\infty}^{\infty}{}'\left(\frac{1}{\sqrt{(n+s)^2-t}} - \frac{1}{|n|}\right)\\
&\quad = \frac{1}{2}[f(s,t) + f(-s,t)]\\
&\quad = f'_t(0,0)t + \frac{1}{2}f''_{ss}(0,0)s^2 + \frac{1}{2}f''_{tt}(0,0)t^2\\
&\qquad + \frac{1}{3}f'''_{sst}(0,0)s^2t + \frac{1}{24}f^{\rm IV}_{ssss}(0,0)s^4 + \cdots \qquad (16)
\end{aligned}$$

$$\begin{aligned}
f'_t(0,0) &= \frac{1}{2}\zeta(3), \quad f''_{ss}(0,0) = 2\zeta(3)\\
f''_{tt}(0,0) &= \frac{3}{4}\zeta(5), \quad f'''_{sst}(0,0) = 6\zeta(5),\\
f^{\rm IV}_{ssss}(0,0) &= 24\,\zeta(5) \qquad (17)
\end{aligned}$$

where $\zeta(x)$ is the Riemann zeta function ([18, 9.522]). With the accuracy to terms of the order $(b/\lambda)^5$ we have for α

$$\begin{aligned}
\alpha = \frac{2b}{\lambda}\Bigg[&\log\frac{b}{2\pi r_0} + \frac{\zeta(3)}{2}\left(1-\frac{k_x^2}{k^2}\right)\left(\frac{b}{\lambda}\right)^2\\
&+ \zeta(3)\frac{k_y^2}{k^2}\left(\frac{b}{\lambda}\right)^2 + \frac{3\zeta(5)}{8}\left(1-\frac{k_x^2}{k^2}\right)^2\left(\frac{b}{\lambda}\right)^4\\
&+ 2\zeta(5)\frac{k_y^2}{k^2}\left(1-\frac{k_x^2}{k^2}\right)\left(\frac{b}{\lambda}\right)^4 + \zeta(5)\frac{k_y^4}{k^4}\left(\frac{b}{\lambda}\right)^4\Bigg]. \qquad (18)
\end{aligned}$$

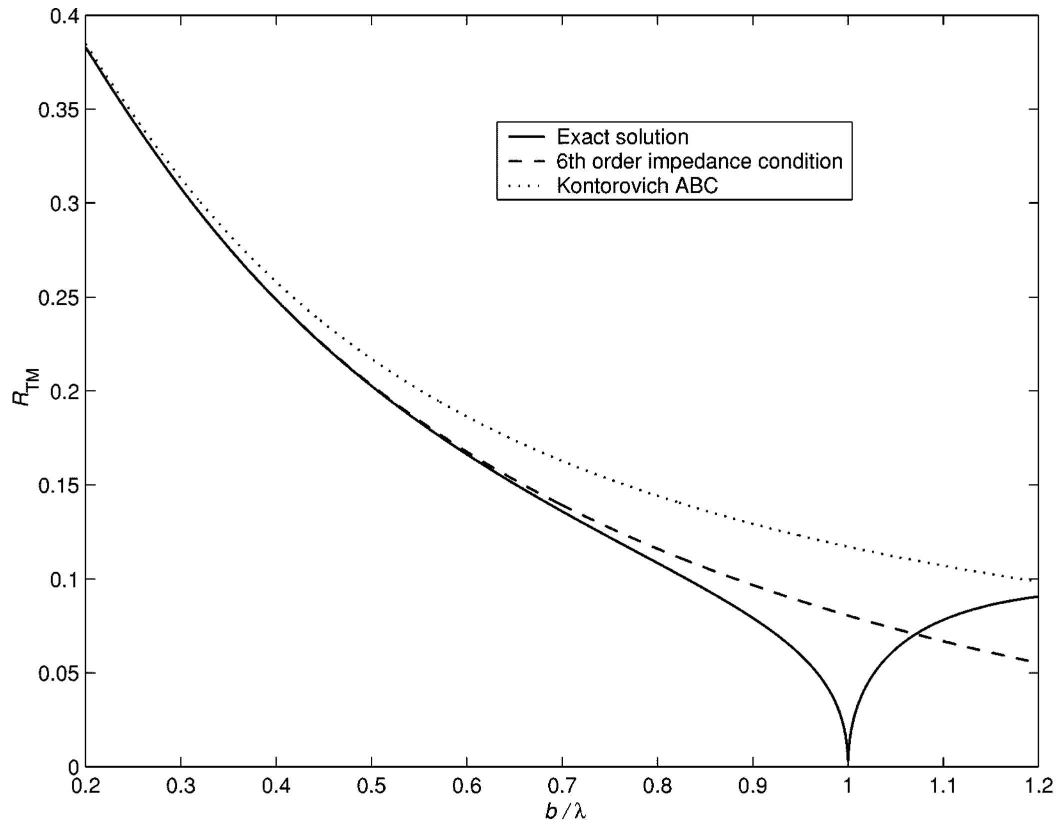

Fig. 5. Absolute value of the copolarized reflection coefficient as a function of the normalized distance between wires for TM-polarized incident wave.

To find the boundary condition which connects the averaged fields, there is no need to perform averaging. It is enough to note that the averaged field equals to the field created by the averaged current $\bar{J}$. Thus, the x component of the total (Fourier-domain) averaged electric field $\bar{E}_x^{\rm tot}$ in the grid plane is the sum of the incident field $E_x^{\rm ext} = E_x e^{-jk_x x} e^{-jk_y y}$ and the plane wave field (its x-component) created by the current sheet $\bar{J} = J e^{-jk_x x} e^{-jk_y y}$

$$\bar{E}_x^{\rm tot} = E_x^{\rm ext} - \frac{\eta}{2} \frac{1 - \sin^2\theta \cos^2\varphi}{\cos\theta} \bar{J}. \tag{19}$$

The line over a symbol denotes the averaged over a grid period quantities. Substitution of the current density from (12) gives for the averaged total field

$$\begin{aligned}\bar{E}_x^{\rm tot} &= \frac{\left[Zb + j\frac{\eta}{2}\alpha(1 - \sin^2\theta\cos^2\varphi)\right] \frac{2}{\eta}\cos\theta E_x^{\rm ext}}{(1 - \sin^2\theta\cos^2\varphi)(1 + j\alpha\cos\theta) + (2/\eta) Zb\cos\theta} \\ &= \left[Zb + j\frac{\eta}{2}\alpha(1 - \sin^2\theta\cos^2\varphi)\right]\bar{J}. \end{aligned}\tag{20}$$

From (3), we recognize that $1 - \sin^2\theta\cos^2\varphi = 1 - k_x^2/k^2$.

Finally, we can make the Fourier transform to the physical space replacing $k_x^2 \to -\partial^2/\partial x^2$ and $k_y^2 \to -\partial^2/\partial y^2$. The result is the boundary condition which connects at the grid plane the x-component of the total field $E_x^{\rm tot}$ with the surface current density $\bar{J}$. It reads

$$\begin{aligned}\bar{E}_x^{\rm tot} = &\left[Zb + j\eta\frac{b}{\lambda}\left(\log\frac{b}{2\pi r_0} + \nu^{(2)} + 3\nu^{(4)}\right)\right]\bar{J} \\ &+ j\eta\frac{b}{\lambda}\Bigg[\left(\log\frac{b}{2\pi r_0} + 2\nu^{(2)} + 9\nu^{(4)}\right)\frac{1}{k^2}\frac{\partial^2\bar{J}}{\partial x^2} \\ &- \left(2\nu^{(2)} + 16\nu^{(4)}\right)\frac{1}{k^2}\frac{\partial^2\bar{J}}{\partial y^2} + \left(\nu^{(2)} + 9\nu^{(4)}\right) \\ &\times \frac{1}{k^4}\frac{\partial^4\bar{J}}{\partial x^4} + 8\nu^{(4)}\frac{1}{k^4}\frac{\partial^4\bar{J}}{\partial y^4} - \left(2\nu^{(2)} + 32\nu^{(4)}\right) \\ &\times \frac{1}{k^4}\frac{\partial^4\bar{J}}{\partial x^2\partial y^2} + 3\nu^{(4)}\frac{1}{k^6}\frac{\partial^6\bar{J}}{\partial x^6} - 16\nu^{(4)}\frac{1}{k^6}\frac{\partial^6\bar{J}}{\partial x^4\partial y^2} \\ &+ 8\nu^{(4)}\frac{1}{k^6}\frac{\partial^6\bar{J}}{\partial x^2\partial y^4} + O\left(\frac{b}{\lambda}\right)^6\Bigg]\end{aligned}\tag{21}$$

where

$$\begin{aligned}\nu^{(2)} &= \frac{\zeta(3)}{2}\left(\frac{b}{\lambda}\right)^2 \approx 0.601\left(\frac{b}{\lambda}\right)^2 \\ \nu^{(4)} &= \frac{\zeta(5)}{8}\left(\frac{b}{\lambda}\right)^4 \approx 0.1296\left(\frac{b}{\lambda}\right)^4.\end{aligned}\tag{22}$$

The boundary condition can be also written in terms of the averaged magnetic field on the two sides of the grid because $\bar{\mathbf{J}} = \mathbf{z}_0 \times (\bar{\mathbf{H}}^+ - \bar{\mathbf{H}}^-)$, where the superscripts $\pm$ mark the averaged fields on the sides $z > 0$ and $z < 0$, respectively.

III. Reflection and Transmission Coefficients

The effective higher order boundary conditions are primarily needed to solve diffraction problems for nonplane wave excitation or for limited in size grids. However, to check the accuracy of the solution, we will next calculate exact and approx-

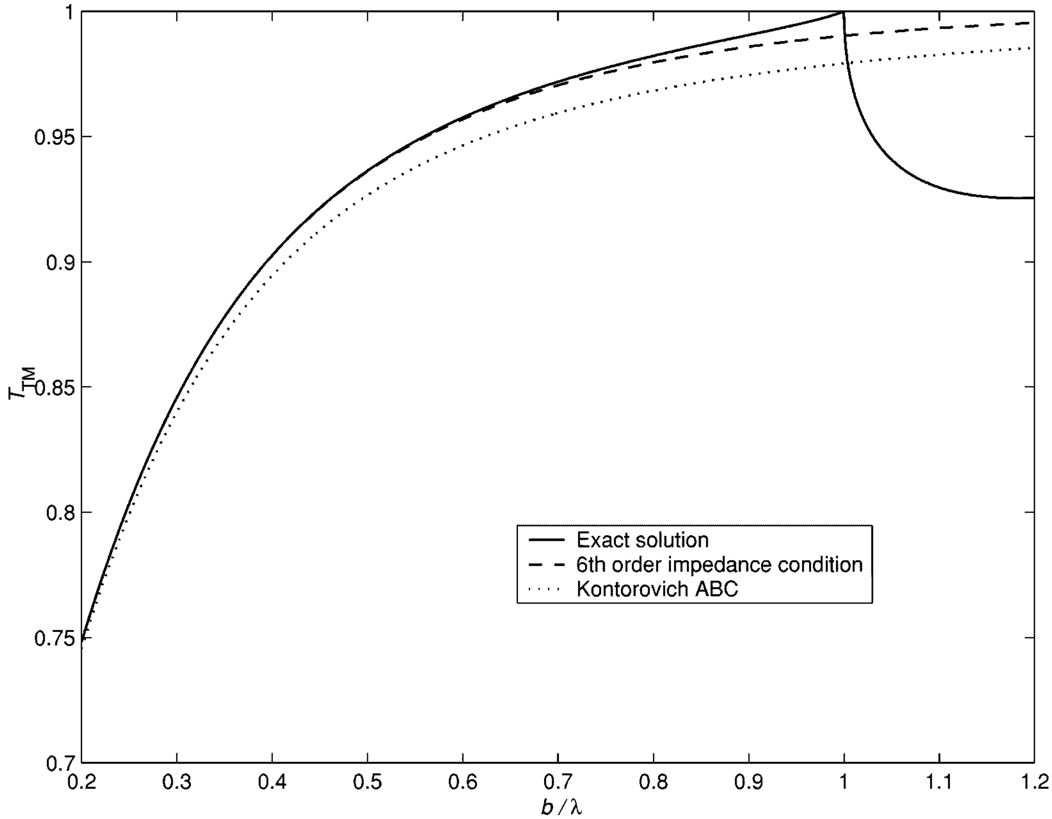

Fig. 6. Same as in Fig. 5 for the transmission coefficient.

imate reflection coefficients for infinite grids excited by plane waves. For this goal, we find the reflection and transmission coefficients in terms of the grid parameter α.

In the far zone, the grid field is a plane wave field with the components

$$E_\theta^{\rm ref} = \mathbf{e}_\theta \cdot \mathbf{E}^{\rm ref} = -\frac{\eta}{2} J \cos\varphi,$$
$$E_\varphi^{\rm ref} = \mathbf{e}_\varphi \cdot \mathbf{E}^{\rm ref} = \frac{\eta}{2} J \frac{\sin\varphi}{\cos\theta} \qquad (23)$$

where $\mathbf{e}_\theta$ and $\mathbf{e}_\varphi$ are the unit vectors connected with the unit vectors of the Cartesian coordinate system as following (see Fig. 1):

$$\mathbf{e}_\theta = \cos\theta\cos\varphi\mathbf{x}_0 + \cos\theta\sin\varphi\mathbf{y}_0 - \sin\theta\mathbf{z}_0,$$
$$\mathbf{e}_\varphi = -\sin\varphi\mathbf{x}_0 + \cos\varphi\mathbf{y}_0. \qquad (24)$$

After substitution of the current (12), the copolarized and cross-polarized reflection coefficients are found as

$$R_{\rm TM} = \frac{E_\theta^{\rm ref}}{E_\theta} = -\frac{\cos^2\theta\cos^2\varphi}{(1-\sin^2\theta\cos^2\varphi)(1+j\alpha\cos\theta)+(2/\eta)Zb\cos\theta}$$
$$R_{\rm EM} = \frac{E_\varphi^{\rm ref}}{E_\theta} = \frac{\cos\theta\sin\varphi\cos\varphi}{(1-\sin^2\theta\cos^2\varphi)(1+j\alpha\cos\theta)+(2/\eta)Zb\cos\theta} \qquad (25)$$

for TM-polarized incident wave and

$$R_{\rm TE} = \frac{E_\varphi^{\rm ref}}{E_\varphi} = -\frac{\sin^2\varphi}{(1-\sin^2\theta\cos^2\varphi)(1+j\alpha\cos\theta)+(2/\eta)Zb\cos\theta}$$
$$R_{\rm ME} = \frac{E_\theta^{\rm ref}}{E_\varphi} = \frac{\cos\theta\sin\varphi\cos\varphi}{(1-\sin^2\theta\cos^2\varphi)(1+j\alpha\cos\theta)+(2/\eta)Zb\cos\theta} \qquad (26)$$

for TE-polarized incident wave.

The transmission coefficients are determined as

$$T_{\rm TM} = 1 + R_{\rm TM}, \quad T_{\rm EM} = R_{\rm EM} \qquad (27)$$

$$T_{\rm TE} = 1 + R_{\rm TE}, \quad T_{\rm ME} = R_{\rm ME}. \qquad (28)$$

IV. Numerical Examples

In the following numerical examples shown in Figs. 2–4, we compare the exact results for the reflection coefficient with that found with the use of the sixth-order impedance boundary conditions and the second-order averaged boundary conditions by Kontorovich. The grid in these examples has the wire spacing $b = \lambda/2$, the wire radius is $r_0 = b/50$. One of the incidence angles is fixed ($\varphi = \pi/4$), the other angle varies. The grid is ideally conducting (the surface impedance $Z = 0$). On Figs. 5 and 6 the reflection and transmission coefficients as functions of the parameter b/λ are depicted. Values of the incidence angles are the following: $\theta = 0$, $\varphi = \pi/4$. We observe that the sixth-order conditions provide a very good model for the grid properties. The calculated curves go between that for the exact solution and that given by the second-order approximation, but very close to the exact solution nearly up to the first resonance of the grid when $\lambda = b$.

For such sparse grids the second-order Kontorovich boundary conditions give considerable error, which should be expected. In that theory the grid parameter $\alpha_{\rm ABC}$ is assumed to be a small quantity, but for our examples $\alpha_{\rm ABC} \approx 2$ when $\lambda \approx 2b$ and $\alpha_{\rm ABC} \approx 4$ near the first resonance.

V. Concluding Remarks

In this paper, the averaged boundary conditions for grids of parallel wires have been extended to sparse grids. The main original idea behind the averaged conditions is that the fields generated by a dense grid at a certain distance (large compared to the wire separation) from its plane become smooth functions of coordinates (for plane wave excitation, just a plane wave is generated). The smooth far field defines the reflection and transmission coefficients for the grid. This observation suggests that the smooth field at a distance from the grid can be connected with the averaged current on the grid plane by effective boundary conditions [8], [9]. The original result by Kontorovich is limited to dense grids, such that the distance between wires is much smaller compared to the wavelength. Here, using the exact solution for the corresponding plane wave diffraction problem we have found that the averaged boundary conditions can be extended to rather sparse grids, such that the wire separation can be as large as half of the wavelength. The extended conditions naturally contain higher order tangential derivatives of the averaged current.

Our method is based on the Taylor expansion of the appropriate coefficient in the exact solution and still higher order conditions can be easily generated. However, we note that the method fails when the first resonance of the grid is reached. For the normal incidence that happens when the distance between wires b equals the wavelength λ. Near that point, the grid becomes transparent. The Taylor expansion of the grid parameter α (13) fails when we reach this point (there is a branch point here). The physical reason for this limitation is the fact that at frequencies higher than the first resonance frequency, several plane waves are generated by the grid currents (in the antenna language, there exist grating lobes). Thus, the main idea of connecting the smooth field taken at a distance from the grid with the smoothed grid current cannot be directly applied anymore, and other models are needed for this regime.

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