Optimizing a Large Array Configuration to Minimize the Sidelobes

Leonid Kogan

***Abstract*—A new method of minimizing sidelobes of a large array whose element spacing is much larger than the wavelength has been developed. The analytical expression for the first derivative of the array beam in respect to the element shift of the array is obtained. Using this expression, it is possible to minimize the value of a beam pattern for a given direction. The minimization of the array's worst sidelobe is carried out iteratively. At each iteration, the worst sidelobe found is suppressed. A task in the National Radio Astronomy Observatory (NRAO) astronomical image processing system (*AIPS*) was written to apply the optimization algorithm. This task provides the optimization of the array's element position and plots the initial and optimized configurations. The optimization can be carried out under the following constraints: doughnut, two circumferences, topography, and minimum spacing between the array elements. Another constraint can be added. The area of the sidelobes' minimizing is the circle in the sky with the center at the main beam.**

***Index Terms*—Array antennas, instrumentation, interferometers, optimization methods.**

I. Introduction

THE ISSUE of minimizing the array sidelobes has been developed for linear arrays with a small spacing between antennas (order of λ). For an example, see ([1]–[5]). In principle, a simulated annealing technique ([6]) can be used to minimize sidelobes. Cornwell ([7]) used this technique to minimize the distance between samples of the array's auto convolution, with the number of elements N ranging from 3 to 12. He found that for larger numbers of elements, the computing effort required becomes prohibitive, because the work per iteration goes roughly as the fourth power of N, and also because the minima become harder to locate.

In this paper, we consider the straightforward method of minimizing the sidelobes for the two-dimensional (2-D) arrays, which can have large spacing between elements. In arrays used for radio astronomy, the distance between antennas can be measured by more than thousands of wavelengths. For an example, see very large array (VLA) ([8]) and very large baseline array (VLBA) ([9]). VLA and VLBA are operated by the National Radio Astronomy Observatory (NRAO). In such a case, the sidelobes can be very large and can even exceed 50% of the main beam amplitude. Optimizing an array's configuration to minimize sidelobes is a very important part of the array's design. Currently, the National Radio Astronomy Observatory (NRAO, USA) and the European Southern Observatory (ESO) are starting the development of the next generation array—the Atacama large millimeter array (ALMA), which will have 64 antennas and work at the millimeter wavelength. The size of the different ALMA configurations varies from 150 m to more than several kilometers. So the spacing between ALMA's antennas will be measured by more than tens of thousands wavelengths. The following sidelobe minimizing algorithm was developed for (ALMA) but can be used for any other array design.

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II. Sidelobe Minimizing Algorithm

Let us suppose the vector $\vec{r}_i$ determines the position of the element at the aperture of the array, measured at wavelengths. Then the beam pattern of the array can be verified by the following equation:

$$\begin{aligned} P(\vec{e}) &= \frac{1}{N^2}\sum_{k=1}^{N}\sum_{n=1}^{N} e^{-i2\pi(\vec{r}_k-\vec{r}_n)\cdot\vec{e}} \\ &= \frac{1}{N}\sum_{k=1}^{N} e^{-i2\pi\vec{r}_k\cdot\vec{e}} \cdot \frac{1}{N}\sum_{n=1}^{N} e^{i2\pi\vec{r}_n\cdot\vec{e}} \\ &= |U(\vec{e})|^2 \end{aligned} \tag{1}$$

where

$\vec{e}$	vector of the direction on the sky;
$\vec{r}_k-\vec{r}_n$	vector of the baseline;
$\sum_{k=1}^{N}\sum_{n=1}^{N} e^{-i2\pi(\vec{r}_k-\vec{r}_n)\cdot\vec{e}}$	Fourier transform of the auto convolution of the array's configuration;
$U(\vec{e}) = (1/N)\sum_{n=1}^{N} e^{i2\pi\vec{r}_n\cdot\vec{e}}$	voltage beam pattern;
N	number of elements in the array.

Evaluating the derivative of (1) with respect to $\vec{r}_n$, we can obtain the following expression for the beam differential as a function of the shift of the given element $\Delta\vec{r}_n$:

$$dP_{\vec{r}_n}(\vec{e}) = \frac{4\pi(\vec{e}\cdot\Delta\vec{r}_n)}{N^2}\sum_{\substack{k=1\\k\neq n}}^{N} \sin 2\pi(\vec{r}_k-\vec{r}_n)\cdot\vec{e} \tag{2}$$

where $dP_{\vec{r}_n}(\vec{e})$ is the beam value change along the direction $\vec{e}$, if $\vec{r}_n$ is changed by $\Delta\vec{r}_n$

The required shift of the nth element should be proportional to the differential in the vicinity of the minimum of $P_{\vec{r}_n}(\vec{e})$. In particular, at the minimum of $P_{\vec{r}_n}(\vec{e})$ the differential is equal to zero and, therefore, the required shift of the nth element is also equal to zero. In 2-D space, the required shift of the nth element can be found from (2) as

$$\begin{aligned} dx(n) &= -G \cdot dP_{\vec{r}_n} \cdot \cos(\alpha) \\ dy(n) &= -G \cdot dP_{\vec{r}_n} \cdot \sin(\alpha) \end{aligned} \tag{3}$$

where

α angle between the direction of vector $\vec{e}$ and axis x;

G gain at the iteration loop;

G $\in (0.0001, 0.1)$.

The correction for each array element is calculated iteratively. Starting with an initial configuration, the position in the area of optimization and value of the worst side is found for each iteration. Then, the correction for the array element positions is applied in accordance with (3). The value of the gain G should be small to conserve the value of the differential $dP_{\vec{r}_n}$ for the new position of the elements. We can start with a big gain (0.01–0.1). For such a big gain, the worst sidelobe is suppressed to a very low value. But another bad sidelobe, at times even larger than the previous one will appear in a different direction. Having jumped up and down the configuration with better worst sidelobe than the original one is found. Starting with this new configuration, we can continue the iteration process with much smaller gains. Finally, we achieve the possible minimum of the worst sidelobes in the given area of optimization.

III. Examples of Application

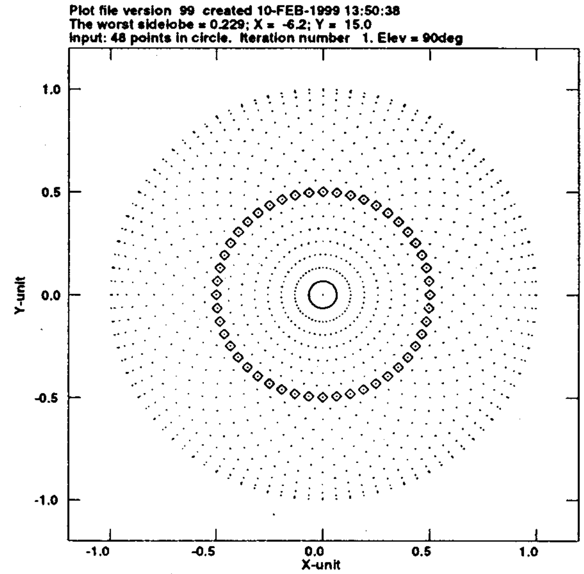
The software (task CONFI) was written in the NRAO astronomical image processing system (AIPS) based on the described algorithm. The task plots the array's configuration and its auto convolution for both the initial and the optimized configurations. The Fourier transform of the auto convolution gives the beam pattern measured in power [see (1)]. The task optimizes the array configuration, minimizing the worst sidelobe inside of the circle near the zenith—the perpendicular direction to the array's plane. It works for any other direction, however, since changing the direction changes only the position of the sidelobes, not the magnitude. The circular area of optimization for the zenith is transformed into an elliptical area with the minor axis of the ellipse equal to the radius of the circle.

So if the sidelobes are optimized at the zenith, inside of the given circle in the sky they will not be worse for any other direction, at least inside of the same circle.

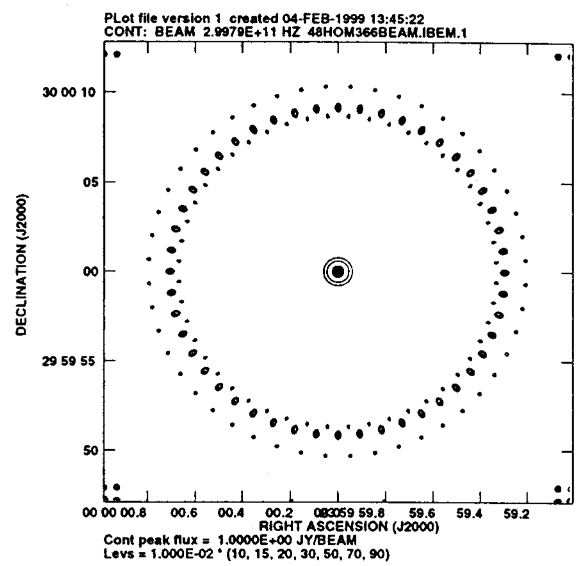
The task provides the optimization under different constraints such as:

- doughnut;
- two circumferences;
- given topography;
- minimum spacing between antennas.

It is not difficult to add another constraint. It is important to notice that the smaller the area of optimization is, the smaller are the sidelobes that can be achieved. For example, it is simple to suppress the beam value for selected directions (for instance, the directions of the interferences) to a very small value. For 500 iterations and 64 elements, about 30 min of computing is required on a SUN ULTRA-1.



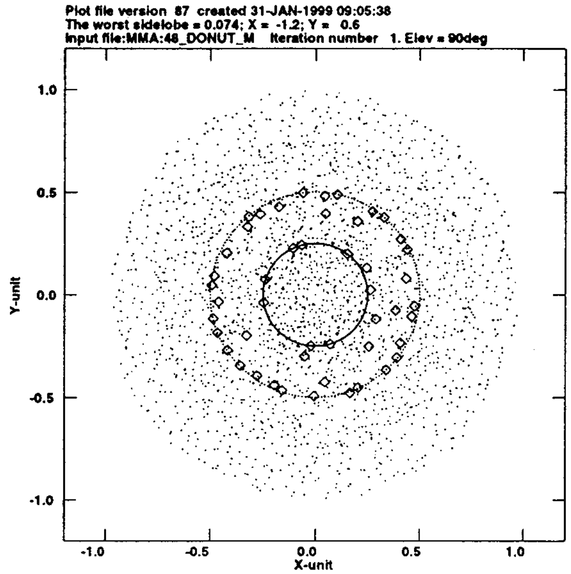
(a)



(b)

Fig. 1. Initial configuration. (a) Forty-eight antennas homogeneously distributed on a circumference. (b) Relevant beam pattern. The antenna positions are labeled with diamonds. The dots label the auto convolution of the antenna positions. The array size is normalized. The beam pattern is given for a 366-m array and a 1-mm wavelength. The minimum spacing is 15 m. The levels of isocontour lines in the beam pattern are (10, 8.2, 7, 5.2, 3, 1.5, 0.5) db

Fig. 2 shows an optimized configuration and a relevant beam pattern with a doughnut constraint. The antenna positions are labeled with diamonds. The dots label the auto convolutions of the antenna positions. The array's size is normalized. The beam pattern is given for a 366-m array and a 1-mm wavelength. The diameter of the optimization area is $\simeq$ 20 in. It is seen from the figure that the level of sidelobes is $<$10% (10 db) inside of the optimization area. For comparison, the array of antennas



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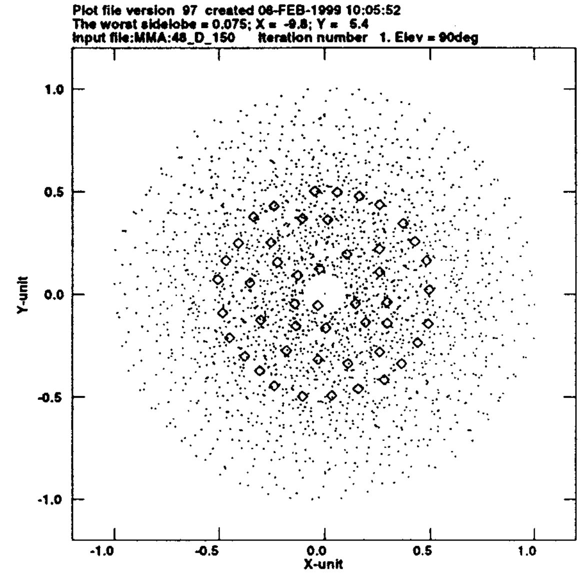

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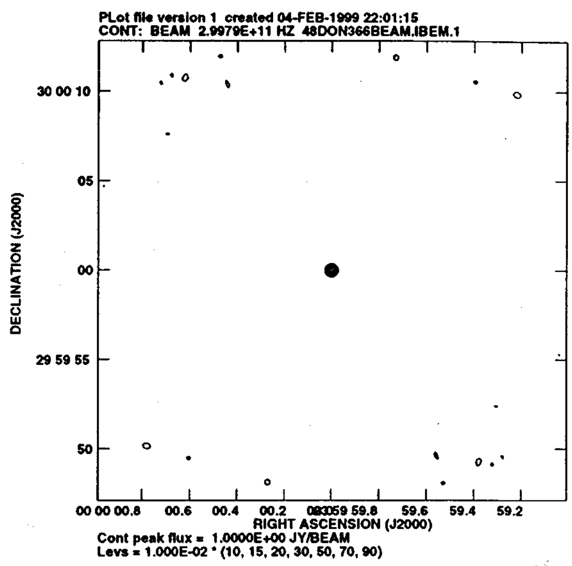
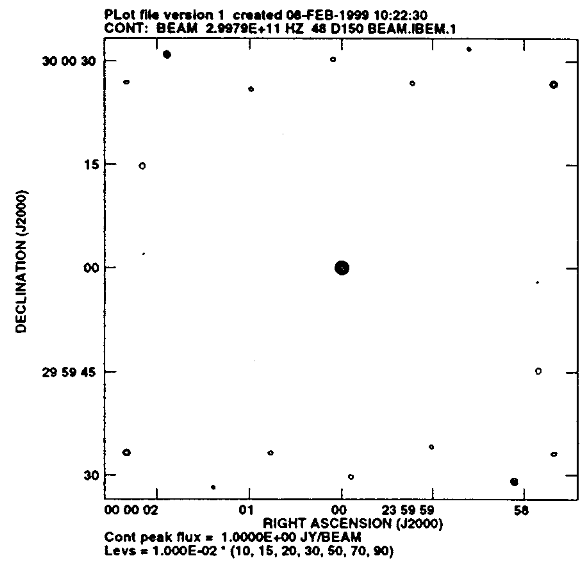


(b)

Fig. 2. (a) Optimized configuration and (b) relevant beam pattern with a doughnut constraint. The antenna positions are labeled with diamonds. The dots label the auto convolution of the antenna positions. The array size is normalized. The beam pattern is given for a 366-m array and a 1-mm wavelength. The minimum spacing is 15 m. The levels of isocontour lines in the beam pattern are (10, 8.2, 7, 5.2, 3, 1.5, 0.5) db



(b)

Fig. 3. (a) Optimized compact configuration. (b) Relevant beam pattern. The antenna positions are labeled with diamonds. The dots label the auto convolution of the antenna positions. The array size is normalized. The beam pattern is given for a 150-m array and a 1-mm wavelength. The minimum spacing is 15 m. The levels of isocontour lines in the beam pattern are (10, 8.2, 7, 5.2, 3, 1.5, 0.5) db

distributed homogeneously on a circumference gives sidelobes that are $>$20% (7 db) (Fig. 1).

Fig. 3 shows an optimized compact configuration and a relevant beam pattern with the minimum spacing constraint. The beam pattern is given for a 150-m array and a 1-mm wavelength. The minimum spacing is 15 m. The diameter of the optimization area is $\simeq$ 50 in. It is seen from the figure that the level of sidelobes is $<$10% (10 db) inside of the optimization area.

Fig. 4 shows an example of fitting and optimizing the array's configuration, constrained by the given topography. The restricted area is given as a data file, containing the coordinates of the bad points. These data correspond to the very rough survey of the future ALMA site in Chile. In the beginning, an optimization without constraints should be carried out. Then, the found configuration should be shifted and rotated until the maximum number of elements is fitted. If we are lucky enough to have all elements fit after this, the fitting process is over. If not, the unfitted elements should be moved to the nearest permitted area and the optimization of the array's configuration with the topography constraint should be carried out.

IV. Conclusion

An optimization algorithm to minimize the worst sidelobe of an array's configuration was developed for the ALMA. Even

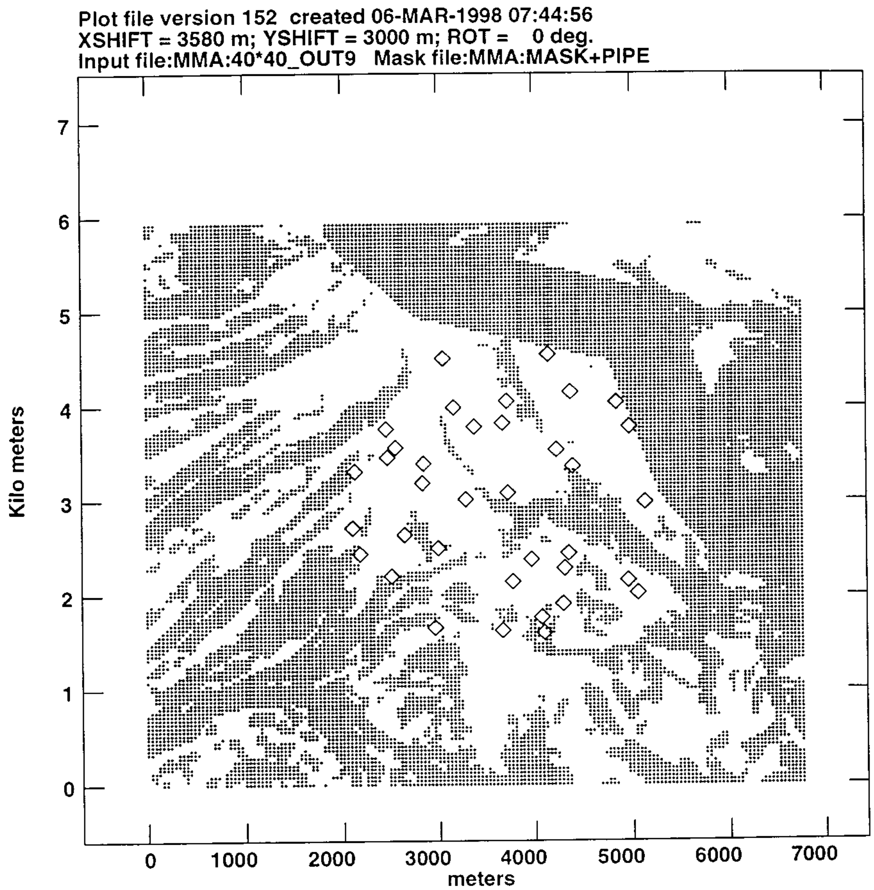


Fig. 4. The configuration fitted to the given landscape. The antenna positions are labeled with diamonds. The dots label the area restricted by topography.

though this algorithm was developed specifically for ALMA, it can be useful for the design of any array. The developed software allows minimizing sidelobes inside of a given circle in the sky with the center at the main beam. It is possible, however, to enchance the software to minimize sidelobes in any other specified area in the sky. It can be useful for supressing interference coming to the array from the selected directions.

Even if the found configuration is not accepted, the value of the found minimum sidelobe can serve as a good criterion of the quality of other configurations.

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