Letters

Monopole Antenna Radiation into a Parallel-Plate Waveguide

Hyo J. Eom, Yong H. Cho, and Min S. Kwon

***Abstract*—A rigorous solution of coaxially fed monopole antenna radiation into a parallel-plate waveguide is obtained. The Fourier transform/series representations are used to represent scattered fields and the boundary conditions are enforced to obtain the simultaneous equations for discrete modal coefficients. Fast convergent series of the reflection coefficient is obtained and compared with other existing results.**

***Index Terms*—Monopole antennas, waveguides.**

I. Introduction

Coaxially fed monopole antenna radiation into a parallel-plate waveguide is of practical interest in microwave engineering and its characteristics are relatively well understood [1], [2]. An equivalent circuit representations for coaxially fed monopole antenna was obtained and compared with experimental data [1]. More recently, a modal-expansion analysis for coaxially fed monopole antenna has been presented and its impedance characteristics were numerically investigated [2]. In this paper, we intend to present a rigorous yet numerically efficient solution for coaxially fed monopole antenna radiation into a parallel-plate waveguide. We use Fourier series/transform representations for the scattered fields and apply the boundary conditions to obtain a rigorous solution. Note that the Fourier series/transform approach was previously used in [3], [5] to obtain an exact solution for radiation from a dielectric-filled edge slot antenna.

II. Field Analysis

Consider the problem of a coaxially fed monopole antenna radiating into a parallel-plate waveguide, as shown in Fig. 1. Assume that an incident TEM-wave with $e^{-i\omega t}$ time convention impinges upon a junction. In region (III), the total field consists of the incident, reflected, and scattered components as

$$H_\phi^i = -e^{-ik_3 z}/(\eta r) \tag{1}$$
$$H_\phi^r = -e^{ik_3 z}/(\eta r) \tag{2}$$
$$E_z^{III}(r, z) = \frac{2}{i\omega\epsilon_3\pi}\int_0^\infty R(\kappa r)\cos(\zeta z)\,d\zeta. \tag{3}$$

In regions (I) and (II), the scattered fields are

$$E_z^I(r, z) = \frac{i}{\omega\epsilon_1}\sum_{m=0}^\infty p_m \xi_{1m}\cos(h_{1m}z)J_0(\xi_{1m}r) \tag{4}$$
$$E_z^{II}(r, z) = \frac{i}{\omega\epsilon_2}\sum_{m=0}^\infty q_m \xi_{2m}\cos(h_{2m}z)H_0^{(1)}(\xi_{2m}r) \tag{5}$$

Manuscript received September 24, 1999; revised March 30, 2000.

The authors are with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, Taejon 305-701, Korea (e-mail: hjeom@ee.kaist.ac.kr).

Publisher Item Identifier S 0018-926X(00)06946-5.

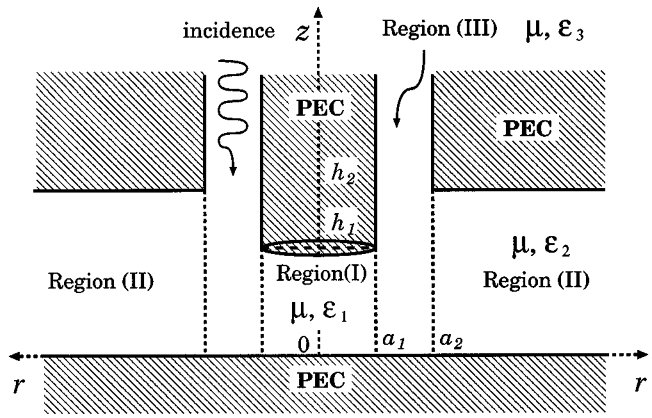

Fig. 1. Geometry of the monopole antenna.

where

h_{pm} $(m\pi/h_p)$;
ξ_{pm} $\sqrt{k_p^2 - h_{pm}^2}$;
κ $\sqrt{k_3^2 - \zeta^2}$;
k_p $\omega\sqrt{\mu\epsilon_p}$;
η $\sqrt{\mu/\epsilon_3}$;
$R(\kappa r)$ $J_0(\kappa r)\tilde{E}^+(\zeta) - N_0(\kappa r)\tilde{E}^-(\zeta)$;
$H_0^{(1)}(\cdot) J_0(\cdot) + iN_0(\cdot)$;
$J_0(\cdot)$ zeroth-order Bessel function;
$N_0(\cdot)$ zeroth-order Neumann function.

The field representations (1)–(5) are somewhat similar to [3, eqs. (1)–(7)]. To determine the modal coefficients p_m and q_m, we enforce the boundary conditions of the E_z and H_ϕ field continuities. Applying the Fourier cosine transform ($\int_0^\infty (\cdot)\cos(\zeta z)\,dz$) to the E_z continuity at $r = a_1$, $E_z^i + E_z^r + E_z^{III} = E_z^I|_{r=a_1}$ yields

$$R(\kappa a_1) = -\sum_{m=0}^\infty p_m \frac{\epsilon_3}{\epsilon_1}\xi_{1m}J_0(\xi_{1m}a_1)F_m^1(\zeta). \tag{6}$$

Similarly, from E_z continuity at $r = a_2$

$$R(\kappa a_2) = -\sum_{m=0}^\infty q_m \frac{\epsilon_3}{\epsilon_2}\xi_{2m}H_0^{(1)}(\xi_{2m}a_2)F_m^2(\zeta) \tag{7}$$

where

$$F_m^p(\zeta) = \frac{(-1)^m \zeta\sin(\zeta h_p)}{\zeta^2 - h_{pm}^2}. \tag{8}$$

Multiplying the H_ϕ continuity at $r = a_1$, for $0 < z < h_1$, by $\cos(n\pi/h_1)z$ and integrating yield

$$I_1 + p_n\frac{h_1}{2}J_1(\xi_{1n}a_1)\alpha_n = -\frac{2}{\eta}\frac{F_n^1(k_3)}{a_1}, \qquad (n = 0, 1, \ldots) \tag{9}$$

where

$$I_1 = -\frac{2}{\pi}\int_0^\infty \frac{1}{\kappa}R'(\kappa a_1)F_n^1(\zeta)\,d\zeta \tag{10}$$

$\alpha_0 = 2, \alpha_n = 1\ (n = 1, 2, \ldots)$, and $R'(\cdot) = dR(\cdot)/d(\cdot)$. Using the residue calculus, it is possible to transform (10) into a rapidly convergent series. First identifying the simple poles in the integrand of (10), applying the residue theorem [4], and performing a lengthy algebraic manipulation yields

$$I_1 = \sum_{m=0}^{\infty} (-1)^{m+n} \left[p_m \frac{\epsilon_3}{\epsilon_1} \xi_{1m} J_0(\xi_{1m} a_1) \overline{I}_1 + q_m \frac{\epsilon_3}{\epsilon_2} \xi_{2m} H_0^{(1)}(\xi_{2m} a_2) \overline{I}_{12} \right]. \tag{11}$$

Similarly, from H_ϕ continuity at $r = a_2$ for $0 < z < h_2$

$$I_2 + q_n \frac{h_2}{2} H_1^{(1)}(\xi_{2n} a_2) \alpha_n = -\frac{2}{\eta} \frac{F_n^2(k_3)}{a_2}, \qquad (n = 0, 1, \ldots) \tag{12}$$

where

$$\begin{aligned} I_2 &= -\frac{2}{\pi} \int_0^\infty \frac{1}{\kappa} R'(\kappa a_2) F_n^2(\zeta)\, d\zeta \\ &= \sum_{m=0}^{\infty} (-1)^{m+n} \left[q_m \frac{\epsilon_3}{\epsilon_2} \xi_{2m} H_0^{(1)}(\xi_{2m} a_2) \overline{I}_2 - p_m \frac{\epsilon_3}{\epsilon_1} \xi_{1m} J_0(\xi_{1m} a_1) \overline{I}_{21} \right] \end{aligned} \tag{13}$$

$$\begin{aligned} \overline{I}_p = {} & \frac{h_p}{2} \frac{\Delta_p \delta_{mn} \alpha_m}{\kappa_{pm} \Delta(\kappa_{pm})} - \frac{i(-1)^p k_3}{2a_p \ln \frac{a_2}{a_1}} \frac{1 - e^{i2k_3 h_p}}{(k_3^2 - h_{pm}^2)(k_3^2 - h_{pn}^2)} \\ & - \frac{i}{a_p} \sum_{v=1}^{\infty} \frac{\zeta_v}{\left[1 - \left(\frac{J_0^2(\kappa_v a_2)}{J_0^2(\kappa_v a_1)} \right)^{(-1)^p} \right]} \cdot \frac{1 - e^{i2\zeta_v h_p}}{(\zeta_v^2 - h_{pm}^2)(\zeta_v^2 - h_{pn}^2)} \end{aligned} \tag{14}$$

$$\begin{aligned} \overline{I}_{pq} = {} & X_{pq} - \frac{ik_3}{2a_p \ln \frac{a_2}{a_1}} \frac{e^{ik_3|h_2 - h_1|} - e^{ik_3(h_2+h_1)}}{(k_3^2 - h_{pn}^2)(k_3^2 - h_{qm}^2)} \\ & - \frac{i}{a_p} \sum_{v=1}^{\infty} \frac{\zeta_v}{\left[\frac{J_0(\kappa_v a_1)}{J_0(\kappa_v a_2)} - \frac{J_0(\kappa_v a_2)}{J_0(\kappa_v a_1)} \right]} \cdot \frac{e^{i\zeta_v|h_2 - h_1|} - e^{i\zeta_v(h_2+h_1)}}{(\zeta_v^2 - h_{pn}^2)(\zeta_v^2 - h_{qm}^2)} \end{aligned} \tag{15}$$

$$X_{12} = \begin{cases} \dfrac{2(-1)^m}{\pi a_p \kappa_{2m}^2 \Delta(\kappa_{2m})} \dfrac{h_{2m} \sin\left(\frac{h_1}{h_2} m\pi \right)}{h_{2m}^2 - h_{1n}^2} & (h_2 > h_1) \\[2ex] \dfrac{2(-1)^n}{\pi a_p \kappa_{1n}^2 \Delta(\kappa_{1n})} \dfrac{h_{1n} \sin\left(\frac{h_2}{h_1} n\pi \right)}{h_{1n}^2 - h_{2m}^2} & (h_2 < h_1) \end{cases} \tag{16}$$

δ_{mn} is the Kronecker delta, $\Delta(\kappa) = J_0(\kappa a_1) N_0(\kappa a_2) - J_0(\kappa a_2) N_0(\kappa a_1)$, $\Delta_1 = J_0(\kappa_{1m} a_2) N_1(\kappa_{1m} a_1) - J_1(\kappa_{1m} a_1) N_0(\kappa_{1m} a_2)$, $\Delta_2 = J_1(\kappa_{2m} a_2) N_0(\kappa_{2m} a_1) - J_0(\kappa_{2m} a_1) N_1(\kappa_{2m} a_2)$, $\kappa_{pm} = \sqrt{k_3^2 - h_{pm}^2}$, κ_v is given by $\Delta(\kappa_v) = 0$, and $\zeta_v = \sqrt{k_3^2 - \kappa_v^2}$. The reflected plus scattered TEM field at $z = \infty$ is

$$H_\phi^r + H_\phi^{III} = -\frac{e^{ik_3 z}}{\eta r}(1 + L_0 - M_0) \tag{17}$$

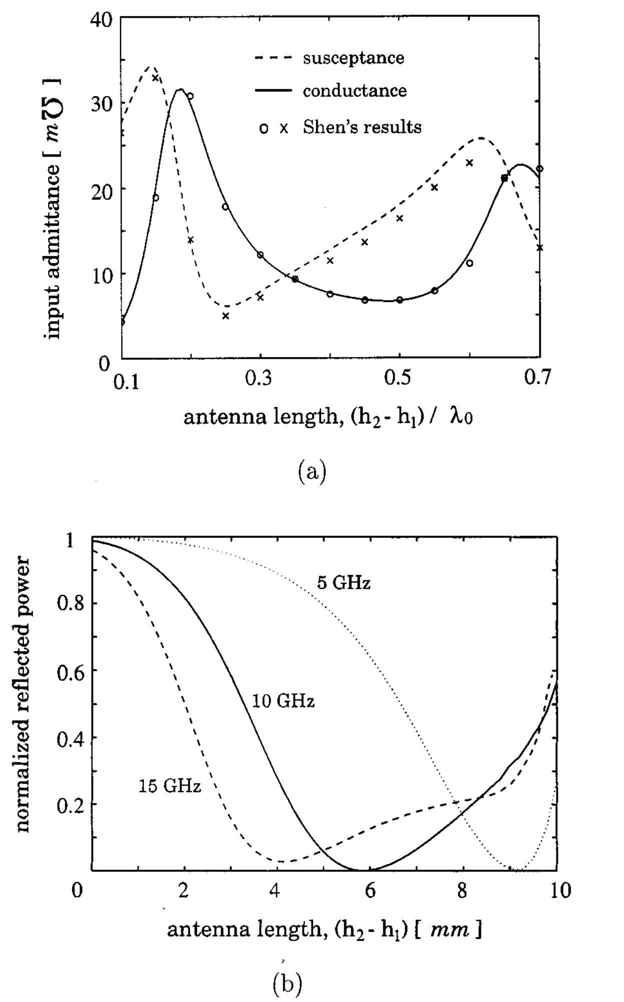

Fig. 2. (a) Input admittance of a monopole antenna fed by a coaxial line ($a_1 = 0.0509\lambda_0$, $a_2 = 1.187a_1$, $h_1 = \lambda_0$, $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_0$). (b) Normalized reflected power of a parallel-plate waveguide fed by a coaxial line ($a_1 = 0.635$ mm, $a_2 = 2.055$ mm, $h_2 = 10.2$ mm, $\epsilon_1 = \epsilon_3 = 1.99\epsilon_0$, $\epsilon_2 = \epsilon_0$).

where

$$L_0 = \frac{\eta i}{k_3 \ln \frac{a_2}{a_1}} \sum_{m=0}^{\infty} p_m \frac{\epsilon_3}{\epsilon_1} \xi_{1m} J_0(\xi_{1m} a_1) F_m^1(k_3) \tag{18}$$

$$M_0 = \frac{\eta i}{k_3 \ln \frac{a_2}{a_1}} \sum_{m=0}^{\infty} q_m \frac{\epsilon_3}{\epsilon_2} \xi_{2m} H_0^{(1)}(\xi_{2m} a_2) F_m^2(k_3). \tag{19}$$

Fig. 2(a) illustrates the comparison of antenna input admittance

$$\left(Y_{\rm in} = \frac{2\pi}{\eta \ln(a_2/a_1)} \frac{1 + (1 + L_0 - M_0) e^{i2k_3 h_2}}{1 - (1 + L_0 - M_0) e^{i2k_3 h_2}} \right)$$

between ours and [2], indicating a favorable agreement. We used ten terms (30 terms) in the series of (18) and (19) to achieve numerical convergence to within 1% error in conductance (susceptance) computation, when $h_2 < 1.4\lambda_0$. Fig. 2(b) shows the normalized reflected power ($|1 + L_0 - M_0|^2$) versus the antenna length $(h_2 - h_1)$ for three different frequencies. No reflection takes place when $(h_2 - h_1) = 4.1, 5.8, 9.3$ mm at 15, 10, 5 GHz, where the corresponding $\lambda/4$ antenna lengths are 3.5, 5.3, and 10.6 mm, respectively. The number of modes used in computation is five, indicating fast numerical convergence.

III. Conclusion

A simple series solution for monopole antenna radiating into a parallel plate is obtained. Our solution is compared with other existing one and is shown to be accurate and efficient for numerical evaluation.

References

[1] A. G. Williamson, "Radial-line/coaxial-line junctions: Analysis and equivalent circuits," *Int. J. Electron.*, vol. 58, no. 1, pp. 91–104, 1985.

[2] Z. Shen and R. H. MacPhie, "Modal expansion analysis of monopole antennas driven from a coaxial line," *Radio Sci.*, vol. 31, no. 5, pp. 1037–1046, Sept./Oct. 1996.

[3] J. K. Park and H. J. Eom, "Fourier transform analysis of dielectric-filled edge-slot antenna," *Radio Sci.*, vol. 32, no. 6, pp. 2149–2154, Nov./Dec. 1997.

[4] G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists*. New York: Academic, 1995, pp. 414–439.

[5] J. K. Park and H. J. Eom, "Fourier transform analysis of dielectric-filled edge-slot antenna," *Radio Sci.*, vol. 33, no. 3, p. 631, May/June 1998.